SPH Gleichungen

Monday, October 26, 2015 12:58 PM

$$\rho_i = \sum_{j\in NN} m_j W(|\underline{r}_i - \underline{r}_j|, h_i)$$

$$A_i = \sum_{j\in NN} \frac{m_j}{\rho_j} A_j W(|\underline{r}_i - \underline{r}_j|, h_i)$$

$$\nabla A_i = \sum_{j\in NN} \frac{m_j}{\rho_j} A_j \nabla W(|\underline{r}_i - \underline{r}_j|, h_i)$$

$$\frac{\partial W}{\partial r} = \frac{6\sigma}{h^{d+1}} \begin{cases} \left(3\left(\frac{r}{h}\right)^2 - 2\left(\frac{r}{h}\right)\right), & 0 \le \frac{r}{h} < \frac{1}{2} \\ -\left(1-\left(\frac{r}{h}\right)\right)^2, & \frac{1}{2} \le \frac{r}{h} < 1 \\ 0, & \text{sonnst} \end{cases}$$

$$\sigma = \frac{40}{7\pi} \quad \text{in } d = 2$$

$$\nabla W(|\underline{r}-\underline{r}'|, h) = \frac{\underline{r}-\underline{r}'}{|\underline{r}-\underline{r}'|} \frac{\partial W}{\partial r}$$

es gilt $\nabla W(|\underline{r}-\underline{r}'|, h) = -\nabla' W(|\underline{r}-\underline{r}'|, h)$

$$\nabla \cdot \underline{v} = \sum_b \frac{m_b}{\rho_b} \underline{v}_b \cdot \nabla W(|\underline{r}-\underline{r}_b|, h)$$

nein, man benützt lieber

$$\nabla \cdot \underline{v} = \frac{1}{\rho}\left[\nabla \cdot (\rho \underline{v}) - \underline{v} \cdot \nabla \rho\right]$$

$W_{ab} = W(|\underline{r}_a - \underline{r}_b|, h_a)$

$$\frac{1}{\rho_a}\left[\sum_b \frac{m_b}{\cancel{\rho_b}} \cancel{\rho_b} \underline{v}_b \cdot \nabla_a W_{ab} - \underline{v}_a \cdot \sum_b \frac{m_b}{\cancel{\rho_b}} \cancel{\rho_b} \nabla_a W_{ab}\right]$$

$$\rho_a (\nabla \cdot \underline{v})_a = \sum_b m_b (\underline{v}_b - \underline{v}_a) \cdot \nabla_a W_{ab}$$

$$\frac{d\underline{v}_a}{dt} = -\frac{\nabla P_a}{\rho_a}$$

$$\frac{\nabla P}{\rho} = \nabla\left(\frac{P}{\rho}\right) + \frac{P}{\rho^2}\nabla\rho$$

Ähnlich wie vorher

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$$\frac{d\underline{v}_a}{dt} = -\sum_b m_b \left(\frac{P_b}{\rho_b^2} + \frac{P_a}{\rho_a^2}\right) \nabla_a W_{ab}$$

Newton's 3ten Satz $\Rightarrow$ $F_{ab} = -F_{ba}$

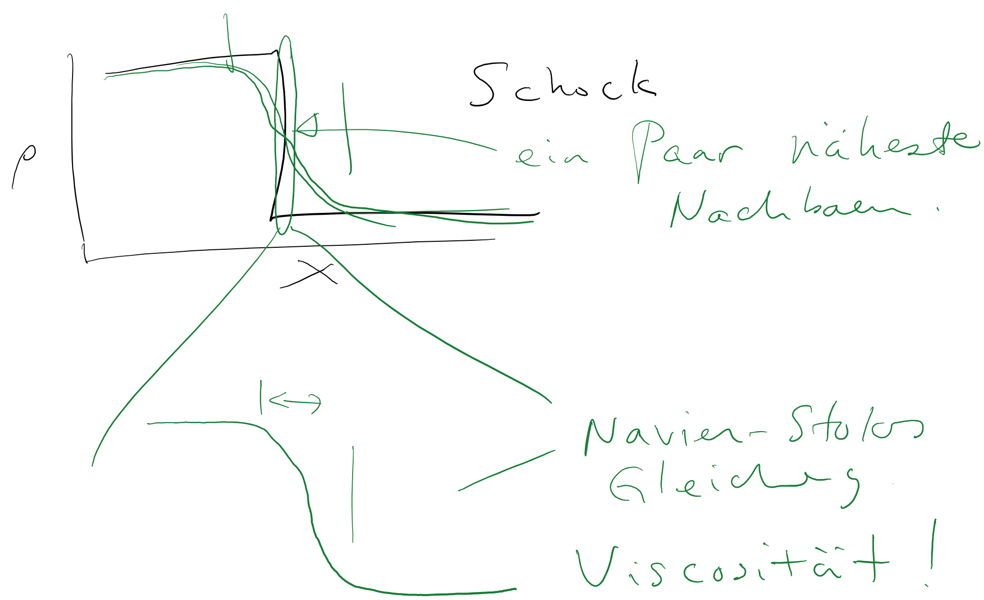
Spezifische Energie: e_a

$$\frac{de}{dt} = -\left(\frac{P}{\rho}\right) \nabla \cdot \underline{v}$$

$$\frac{de_a}{dt} = \left(\frac{P_a}{\rho_a^2}\right) \sum_b m_b \left(\underline{v}_b - \underline{v}_a\right) \cdot \nabla_a W_{ab}$$

Erhaltung der Masse $\boxed{\rho_a = \sum_b m_b W_{ab}}$

oder $$\frac{d\rho_a}{dt} = \sum_b m_b \left(\underline{v}_a - \underline{v}_b\right) \cdot \nabla_a W_{ab}$$



Künstliche Viscosität

$$\frac{d\underline{v}_a}{dt} = -\sum_b m_b \left(\frac{P_b}{\rho_b^2} + \frac{P_a}{\rho_a^2} + \Pi_{ab}\right) \nabla W_{ab}$$

$\Pi_{ab} = \{ -\alpha\, \overline{c}_{ab}\, \nu_{ab} + \beta\, \nu_{ab}^2 \ldots$

$$\Pi_{ab} = \begin{cases} \dfrac{-\alpha\, \bar{c}_{ab}\, \mu_{ab} + \beta\, \mu_{ab}^2}{\bar{\rho}_{ab}}, & \underline{v}_{ab}\cdot\underline{r}_{ab} < 0 \\ 0, & \underline{v}_{ab}\cdot\underline{r}_{ab} > 0 \end{cases}$$

$$\mu_{ab} = \frac{h\, \underline{v}_{ab}\cdot\underline{r}_{ab}}{r_{ab}^2 + \underbrace{\eta^2}_{\text{klein}}}$$

$$\bar{\rho}_{ab} = \frac{1}{2}(\rho_a + \rho_b)$$

$$\bar{c}_{ab} = \frac{1}{2}(c_a + c_b)$$

$$c = \sqrt{\frac{P\gamma}{\rho}}$$

$$c = \sqrt{\gamma(\gamma-1)e}$$

$\beta = 2\alpha \qquad \alpha \sim 1 \qquad 0.5$

LEAP FROG für SPH (Drift - Kick - Drift)

Variabeln: $\underline{r}, \underline{v}, e, c, \rho$

Zusatz Variabeln: $\underbrace{\underline{a}, \dot{e}, \underline{v}_{pred}, e_{pred}}$ ... $\underline{v}_{pred}, e_{pred}$: Leapfrog.

```
DRIFT1(Δt=0);
CALCFORCES();   gibt uns a, ė (bei t=0)
for (step=0; step<NStep; ++step) {
    DRIFT1(Δt/2);   ← braucht a, ė
    CALCFORCES();
    KICK(Δt);
```

```
KICK(Δt);
DRIFT2(Δt/2);
}
```

```
CALCFORCES (2) {                benützt { v_pred, e_pred }
    TREEBUILD();
    NN-DICHTE      (alle Teilchen rechnen ρ)
    CALC_SOUND     (alle Teilchen rechnen c)
                       c = sqrt(γ(γ-1) e_pred)
    NN_SPHFORCES   (alle rechnen a, ė)
}
```

```
DRIFT1(Δt) {                              DRIFT2(Δt) {
    r += v Δt;                                r += v Δt;
    v_prep = v + a Δt;                        }
    e_pred = e + ė Δt;
}
KICK(Δt) {
    v += a Δt;
    e += ė Δt;
}
```