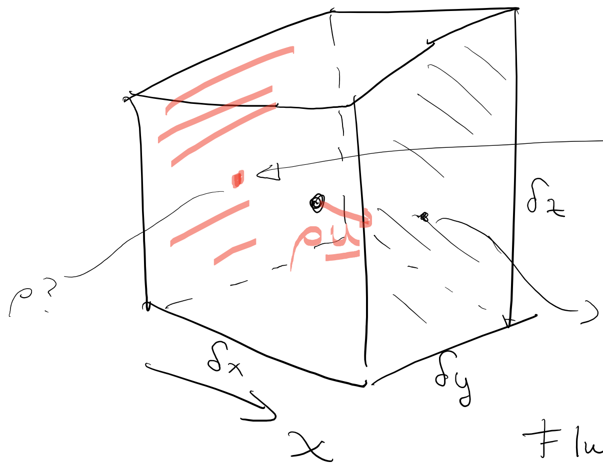
Kontinuitätsgleichung

Monday, October 12, 2015 12:54 PM

SPH

Grid Methods



"$\rho - \frac{1}{2}\frac{\partial \rho}{\partial x}\delta x$

$\left\{\cancel{\rho u} - \frac{1}{2}\frac{\partial(\rho \underline{u})}{\partial x}\delta x\right\}\delta y\,\delta z$

$-\left\{\cancel{\rho u} + \frac{1}{2}\frac{\partial(\rho \underline{u})}{\partial x}\delta x\right\}\delta y\,\delta z$

Fluss nach innen:

$$-\frac{\partial(\rho \underline{u})}{\partial x}\delta x\,\delta y\,\delta z$$

Änderung der Masse

$$-\nabla\cdot(\rho \underline{u})\,\cancel{\delta V} = \frac{\partial \rho}{\partial t}\cancel{\delta V}$$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho \underline{u}) = 0}$$

Kontinuitäts-Gleichung.

$$\frac{d\underline{u}}{dt} \quad \text{"Zeit-Ableitung mit dem Fluss"} = \frac{\underline{u}(\underline{x}+\underline{u}\,\delta t, t+\delta t) - u(\underline{x},t)}{\delta t}$$

$$\frac{dF}{dt} = \frac{1}{\cancel{\delta t}}\left\{F(\underline{x}+\underline{u}\,\delta t, t+\delta t) - F(\underline{x},t)\right\}$$

$\underline{u} = (u,v,w) \quad \underline{x} = (x,y,z)$

$$\cancel{F} + \frac{\partial F}{\partial x}u\cancel{\delta t} + \frac{\partial F}{\partial y}v\cancel{\delta t} + \frac{\partial F}{\partial z}w\cancel{\delta t} + \frac{\partial F}{\partial t}\cancel{\delta t} - \cancel{F}$$

$$\frac{dF}{dt} = (\underline{u}\cdot\nabla)F + \frac{\partial F}{\partial t}$$

! Konvektive-Ableitung

$$\frac{\partial \rho}{\partial t} + \underline{u}\cdot\nabla\rho + \rho\nabla\cdot\underline{u} = 0 \Rightarrow (1)\ \boxed{\frac{d\rho}{dt} + \rho\nabla\cdot\underline{u} = 0}$$

Lagrangische-

Formulierung.

und Momentum?

$m \underline{a} = \underline{F}$ $m \frac{d\underline{u}}{dt} = \underline{F}$ kleines Volumen δv

$$\rho \, \delta v \frac{d\underline{u}}{dt} = \underline{F} \; — \; \rho \, \delta v \, g$$

$$\boxed{\rho \, \underline{u} \cdot \nabla \underline{u} + \rho \frac{\partial \underline{u}}{\partial t} = \rho g}$$

Stimmt das?

Es fehlt der Druck!

~~Lassen~~ wir mal $g = 0$!

(2) $$\boxed{\frac{d\underline{u}}{dt} = -\frac{\nabla P}{\rho}}$$

$$\frac{\partial E}{\partial t} + \nabla \cdot \left((E + P) \underline{u} \right) = 0$$

$$E = \rho \left(\frac{1}{2} \underline{u} \cdot \underline{u} + e \right)$$

kommt nur von Thermodynamik

6 Variablen und nur 5 Gleichungen.

Zustandsgleichung um das System von Gleichungen zu schiessen!

$$e = e(P, V)$$

Ideales Gas:

$$e = \frac{P}{(\gamma - 1)\rho} \qquad \gamma = \frac{f+2}{f} = \frac{5}{3} \text{ in 3-D}$$

$$\text{in } \frac{4}{2} = 2 \text{ in 2-D}$$

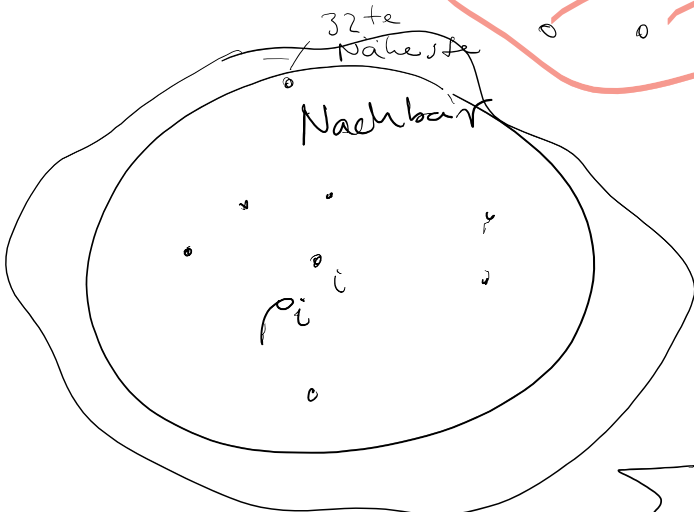
$$e = \frac{kT}{\mu m_H}$$

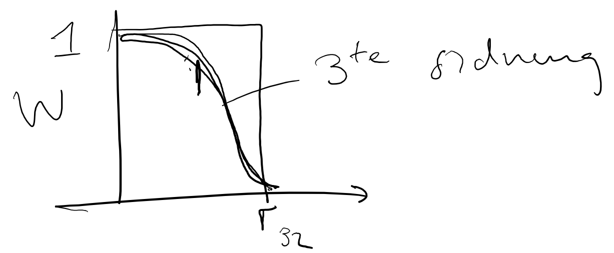
für Plasma

Non H^+, e^- $\nu \approx 0.5$

SPH

Interpoliere ohne ein Gitter!





$$\rho_i = \sum_{j \in NN} m_j W(|\underline{x}_j - \underline{x}_i|, h_i)$$

$$A_i = \sum_{j \in NN} m_j \frac{A_j}{\rho_j} W(|\underline{x}_j - \underline{x}_i|, h_i)$$

$$\nabla A_i = \sum_{j \in NN} m_j \frac{A_j}{\rho_j} \nabla W(|\underline{x}_j - \underline{x}_i|, h_i)$$

Periodische Randbedingungen: