Konstante Wachstums Rate r:

$$P_{n+1} = P_n + rP_n = (1+r)P_n$$

$$\frac{P_{n+1} - P_n}{P_n} = r$$

$$\frac{dP}{dt} = rP$$

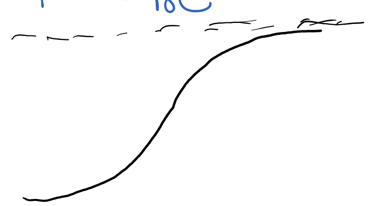
$$\frac{1}{P}\frac{dP}{dt} = r$$

$$\int_{P_0}^{P} d\ln P = \int_0^t r\, dt$$

$$\ln P - \ln P_0 = rt$$

$$\ln \frac{P}{P_0} = rt$$

$$P = P_0 e^{rt}$$



Normalizieren wir die Bevölkerung mit der "maximalen" Grösse N. Bei P=N sind die Resourcen völlig ausgelastet.

$$p = P/N$$

p	r
1	0
klein	positive
~1	klein
>1	negative!

Verhulst

$$\boxed{r \propto (1-p_n)}$$

$$\frac{p_{n+1} - p_n}{p_n} = r(1-p_n) \qquad p_{n+1} = p_n + rp_n(1-p_n)$$

p_n^2

Es ist deterministisch aber...

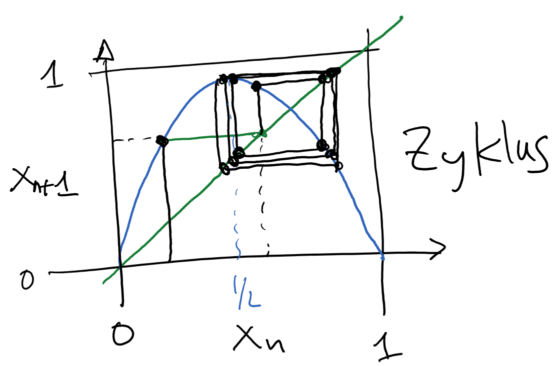
$r=3$ $\qquad p_0 = 0.01$

$\qquad p_0' = 0.00999999$

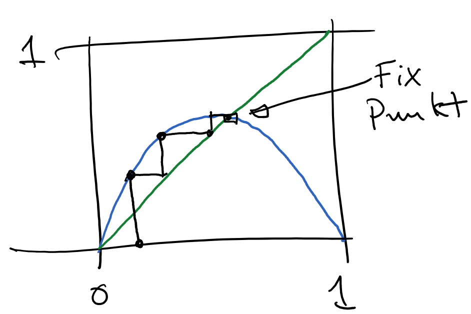
Logistische Gleichung: $\boxed{x_{n+1} = \underbrace{a x_n (1-x_n)}_{?}}$

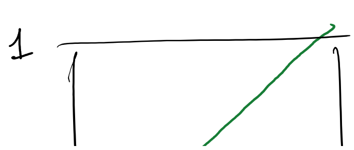
$x_n \in [0,1]$

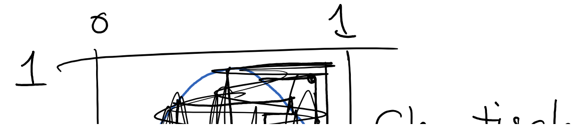
$a \in [0,4]$

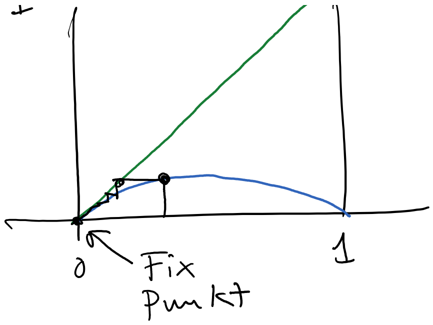


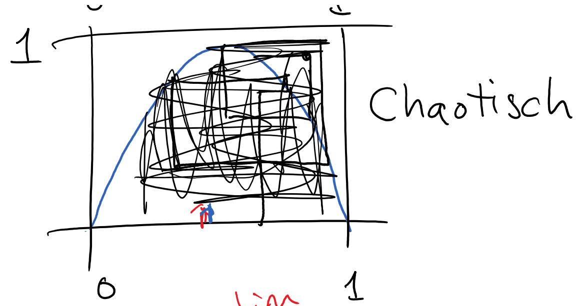
Zyklus

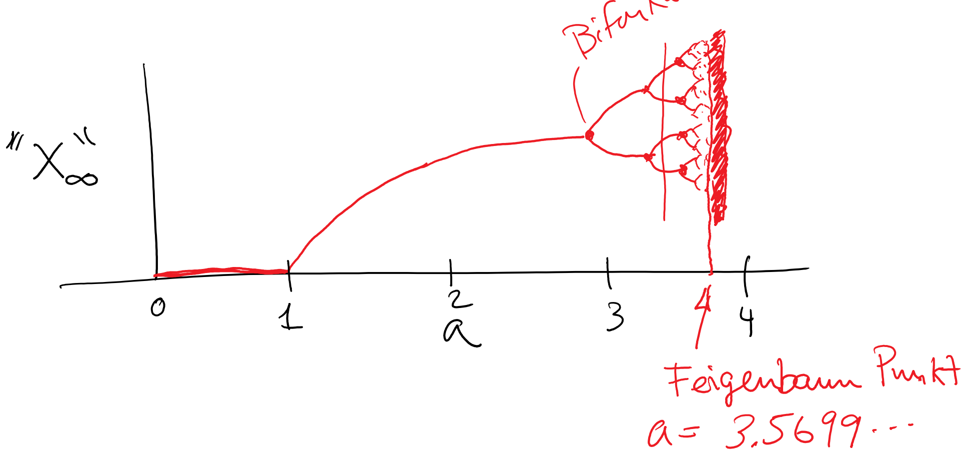




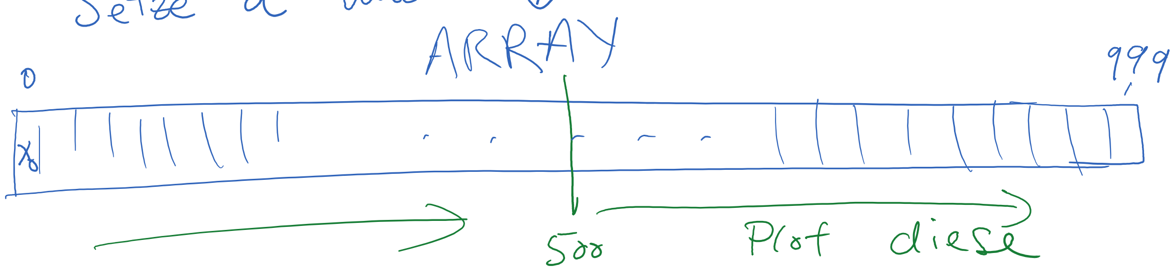


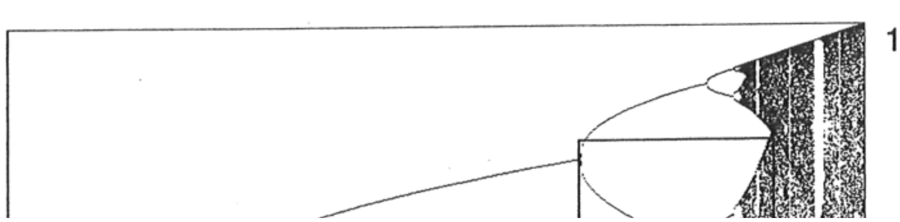

0
1
Fix
Punkt

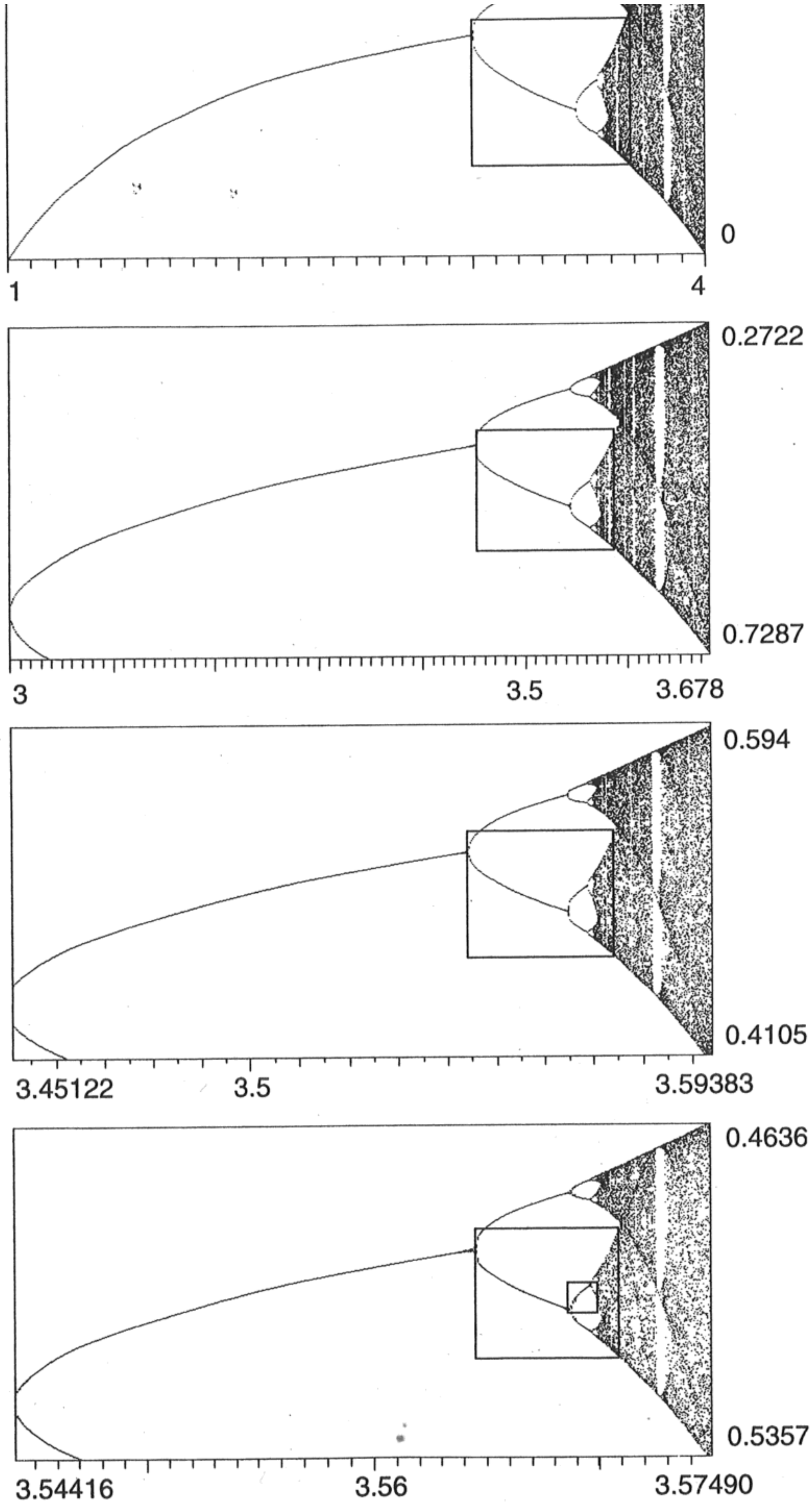

1
0
1
Chaotisch


Bifurkation
"X_∞"
0
1
2
a
3
4
Feigenbaum Punkt
a = 3.5699...

Setze a und


ARRAY
0
999
x_0
500
Plot diese

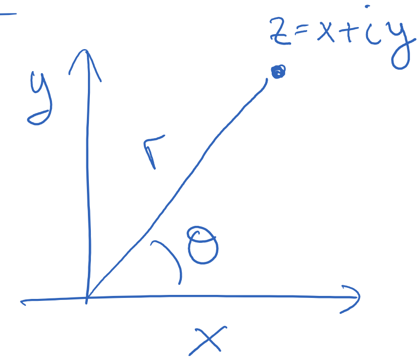

1


0
1
4
0.2722
0.7287
3
3.5
3.678
0.594
0.4105
3.45122
3.5
3.59383
0.4636
0.5357
3.54416
3.56
3.57490

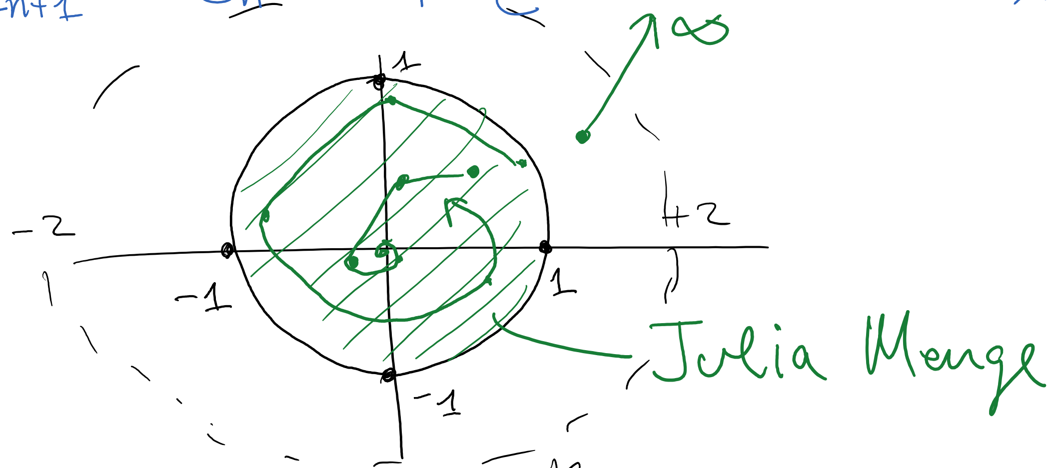
FRAKTAL

Komplexe Zahlen $\quad i^2 = -1$

$$z = (x + iy) = re^{i\theta}$$



$$z_{n+1} = z_n^2 = r^2 e^{i(2\theta)}$$



$$z_{n+1} = z_n^2 + C$$ Komplexe Konstante

$$M = \{ c \in \mathbb{C} \mid J_c \text{ geschlossen} \}$$

Mandelbrot Menge

$$= \{ c \in \mathbb{C} \mid c_{n+1} = c_n^2 + c < \infty \text{ bleibt} \}$$

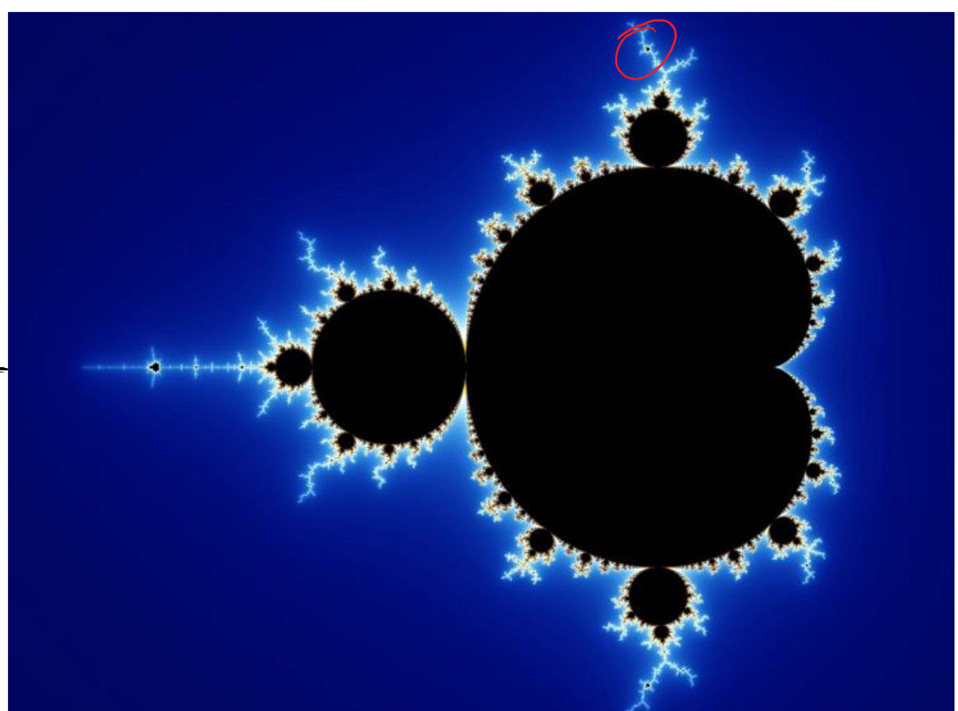
$c = -0.5 + 0.5i$

Geschlossen

"Staub"

+1

0



−2,5 −1 −2,5 1,5

K = 0

Z = C

while $(k < 100)$:

if $(|z| > r_{max})$ return $(c \notin M)$

k ist die Farbe

$z = z^2 + c$

$k = k + 1$

return $(c \in M)$ $\rightarrow$ geschlossen (Schwarz)