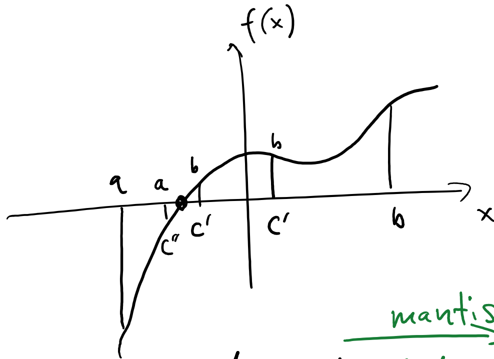
Newton's Method

27 September 2021 12:44

$$f(x) = 0$$
$$x = ?$$



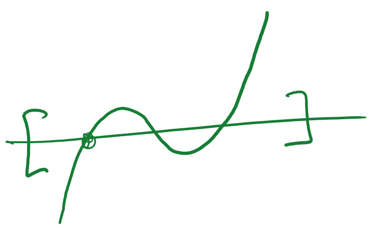
$$|a-b| < \varepsilon_{absolute}$$

$$\frac{|a-b|}{|c|} < \varepsilon_{relatie}$$

mantissa

$$x = 1.010110$$

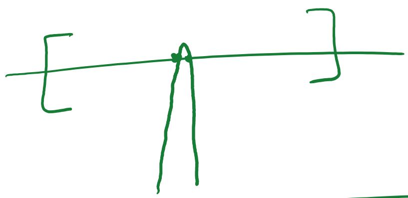
0 1 2 3 4



constant number (in this case 1) of digits with each iteration

linearly convergent.

~23x through the loop.

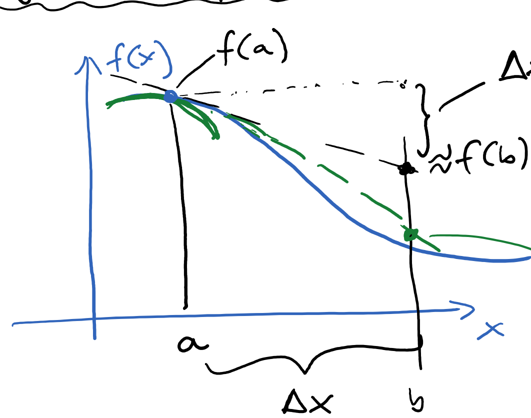


Quadratically Convergent

→ Newton's

Taylor expansion

"$f'(a)$"



$$\Delta x \cdot \frac{\partial f(x)}{\partial x}\Big|_a$$

$$f(b) \approx f(a) + \Delta x f'(a)$$

$$f(a) + \Delta x f'(a) + \frac{\Delta x^2}{2} \cdot f''(a)$$

$$f(b) \equiv f(a+\Delta x) \cong f(a) + \Delta x f'(a) + \frac{1}{2}\Delta x^2 f''(a)$$
$$+ \frac{1}{6}\Delta x^3 f^{(3)}(a) + \dots + \frac{1}{n!}\Delta x^n f^{(n)}(a)$$
$$+ \left[\frac{\Delta x^n}{\ } \cdot \int_0^1 (1-t)^{n-1} f^{(n)}(a + t\Delta x)\,dt \right]$$

$$+\left[\frac{\Delta x^n}{(n-1)!}\int_0^1 (1-t)^{n-1} f^{(n)}(a+t\Delta x)\,dt\right]$$

Sometimes it is possible to find a practical and relatively tight upper bound to this integral.

$$f(x) = f(a+\Delta x) \cong f(a) + \Delta x f'(a)$$

dropping all but the 1st order term.

Let's suppose this function has a root at x : $f(x) = 0$

$$x = a + \Delta x$$

$$f(a) + \Delta x f'(a) \cong 0$$

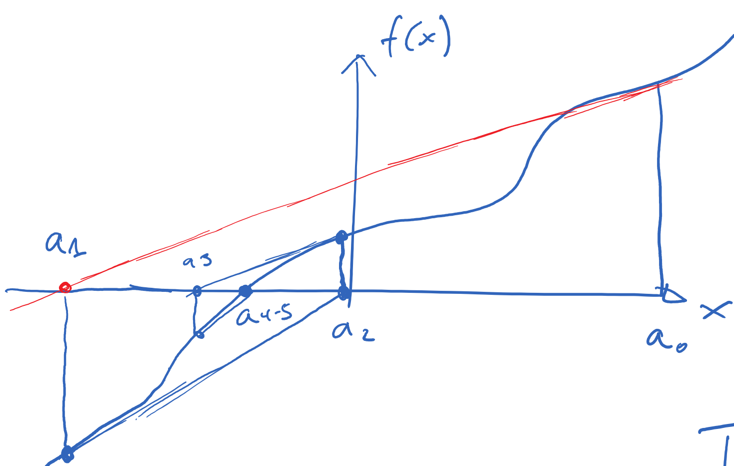
$$\Delta x \cong -\frac{f(a)}{f'(a)}$$

$$\boxed{x \simeq a - \frac{f(a)}{f'(a)}}$$

Newton's Method

Need this extra piece of information

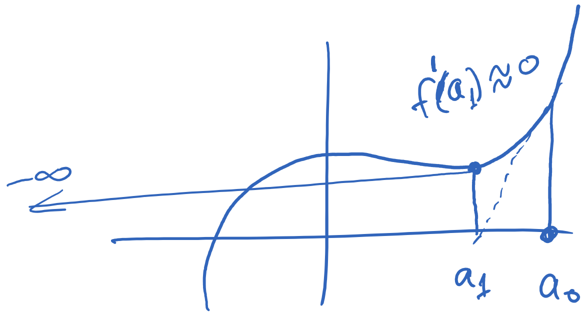
$$\boxed{a = x}$$



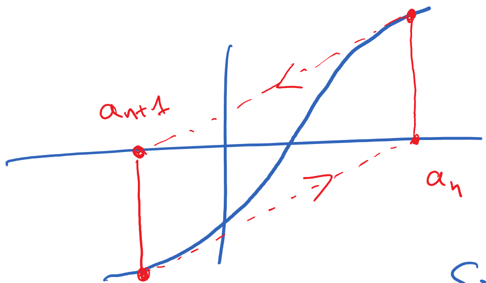
Quadratic: that the number of binary digits of accuracy doubles with each iteration.

If it is converging

= at all.

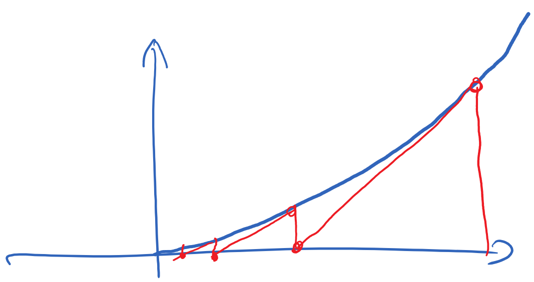


"escapes to ∞"



"Cycle"

Sometimes you can gaurantee that the algorithm converges and is stable.



Today's Assignment: Kepler's Equation

$$M = E - e \sin E$$

M: angle given function of time

E: angle

E (in $\sin E$): angle!

e: given: eccentricity of the ellipse $[0, 1]$

0: circle

1: line segment

Unknown: $E(M, e) = ?$

$f(E) = 0$! $0 = E - e\sin E - M$

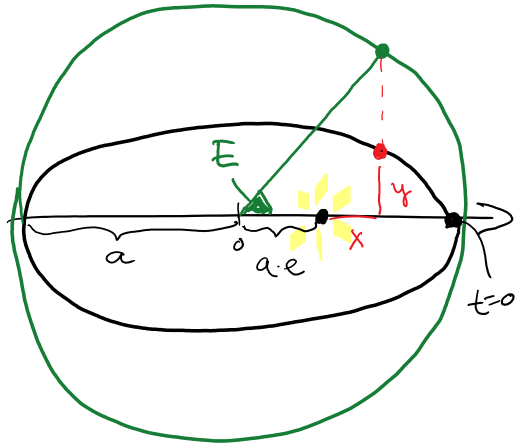
$f(E)$

$f'(E)$

$M = nt$ $n = \frac{2\pi}{T}$ — period in $T = a^{3/2}$

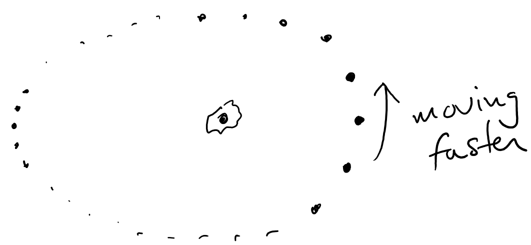
years

years AU



$$x = a \cdot \cos(E) - ae$$
$$y = b \cdot \sin(E)$$
$$y = a\sqrt{1-e^2}\,\sin(E)$$

$e = 0.5 \quad a = 1$



$f'(E) = ?$

$$\boxed{\text{Initial } E_0 = M\ ?}$$