Joachim Stadel ICS

Evaluation: 60% ORAL / 40% Exercises!

Due 2weeks after assignment (Sunday night)

How are numbers represented on a Computer?

- Integers: −ve, +ve

~~most negative # → 00000000 … 00~~ 32 bits

$\emptyset$: 000000 …… 000

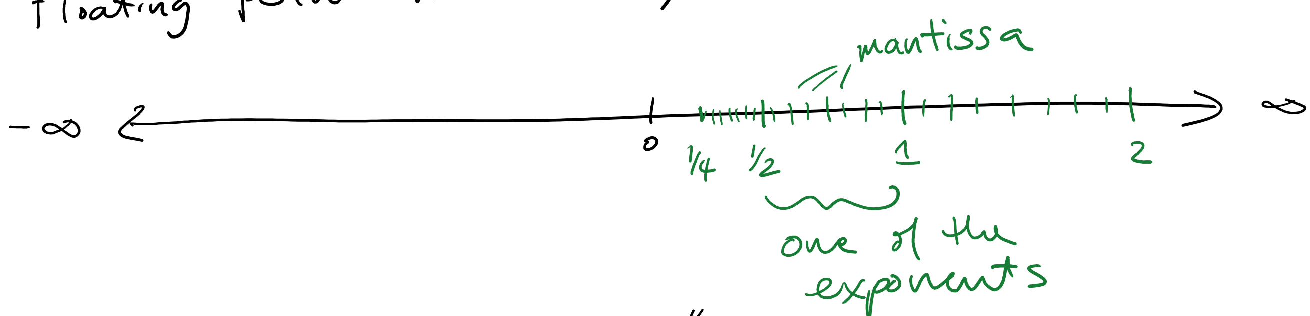
-1 : 111111 …… 111

Most negative number : $\boxed{1}$000 …. 000 $= -2^{31}$
Most positive number : $\boxed{0}$111 …. 111 $= +2^{31}-1$

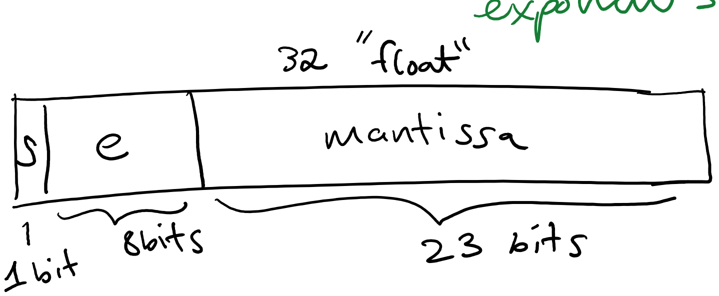
not a sign bit

2s Complement

- Floating Point numbers ≠ Real numbers!



IEEE-754



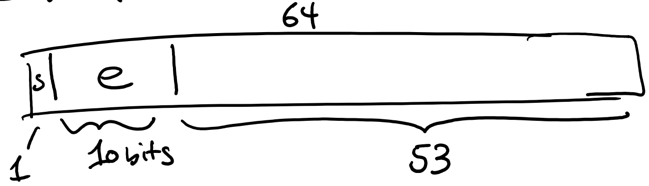
$\pm 1.23456 \times 10^{12}$ normalized

$\boxed{+}(1.\boxed{01011101}) \times 2^{\boxed{101}\,e}$

$$\underbrace{\boxed{\pm}}_{s}\ 1.\underbrace{\boxed{01011101}}_{\text{mantissa}} \times 2^{\boxed{101}\,e}$$

$$s \times 1.M \times 2^{E-127}$$

double

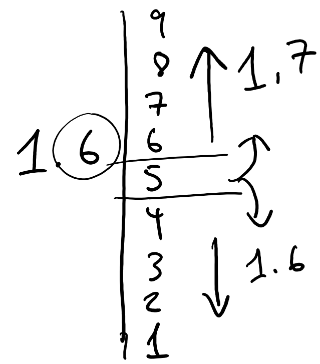


- Rounding of numbers (Random would work)

round to nearest even:

$$1.65 \Rightarrow \cancel{1.7}\ 1.6$$

$$1.75 \Rightarrow 1.8$$



± infinity, ±0, NAN

Any comparison of a NAN → False

```
r2 = x*x + y*y + z*z;
assert (r2 >= 0);
r= sqrt(r2);
```

NaN

FORMULAS

$$ax^2 + bx + c = 0$$

solve for x

$$f(x) = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \boxed{A}$$

$$x = \frac{2c}{\ldots} \quad \boxed{B}$$

$$x = \frac{2c}{-b \pm \sqrt{b^2-4ac}} \quad \boxed{B}$$

If you use only one of $\boxed{A}$ or $\boxed{B}$ then you are asking for trouble.

- when either a and/or c are very small.

$$-b \pm \sqrt{b^2 - 4ac}$$

very small

$$\boxed{\begin{gathered} q = -\frac{1}{2}\Big[b + \underset{+1,-1}{\mathrm{sign}(b)}\sqrt{b^2-4ac}\Big] \\ x_1 = \frac{q}{a} \qquad x_2 = \frac{c}{q} \end{gathered}}$$

Stability:

Golden Mean $\quad \phi = \dfrac{\sqrt{5}-1}{2} \approx 0.61803\ldots$

Powers of ϕ?

$$\phi^n = \prod_{j=1}^{n} \phi$$

stable

Suppose * is much harder than +, or −.

$$\phi^{n+1} = \phi^{n-1} - \phi^n$$

$$\phi^0 = 1\ , \quad \phi^1 = \frac{\sqrt{5}-1}{2}\ , \quad \phi^2 = \phi^0 - \phi^1$$

BAD.

$$\phi^+ = -\frac{\sqrt{5}+1}{2}$$

$$\phi^1 = \underbrace{\phi + \varepsilon\phi^+}_{\text{linear combination}}$$

$$A(x) = \frac{x-1}{e^{x-1}-1} \qquad A(1.0) = 1.0$$

Stability: small variation of the input should give a small variation of the output.

Formulas are great, but algorithms are better!

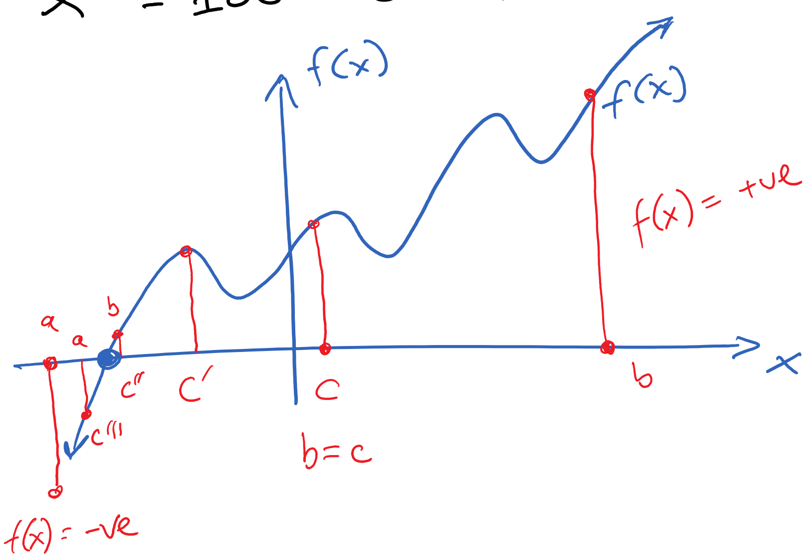
$$ax^3 + bx^2 + \cdots d = 0 \checkmark$$

$$ax^4 + bx^3 + \cdots e = 0 \checkmark$$

$$f(x) = ax^5 + \cdots \quad f = 0 \;\times$$

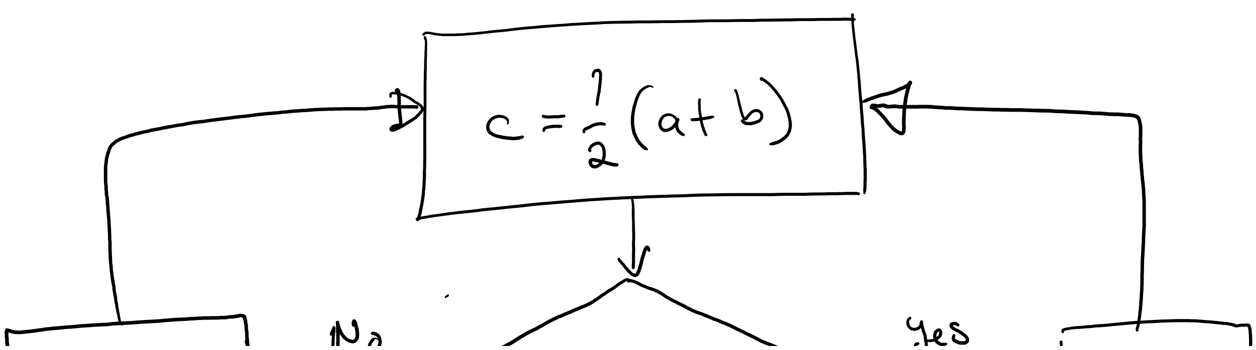
$$x = \cdots$$

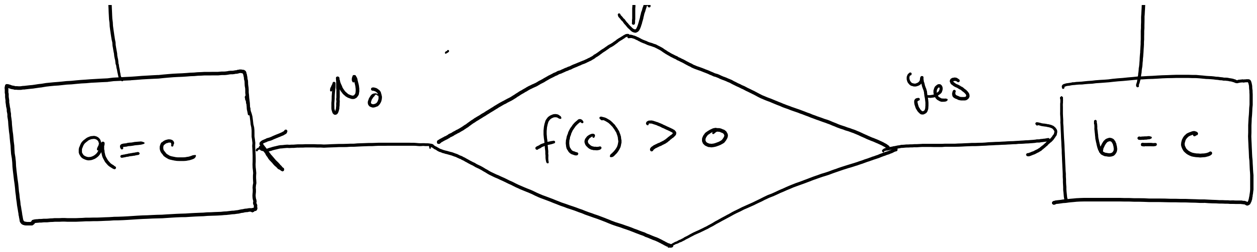
$$x^x - 100 = 0 \qquad x = ?$$



Bracketing the Root

$f(x) = 0$



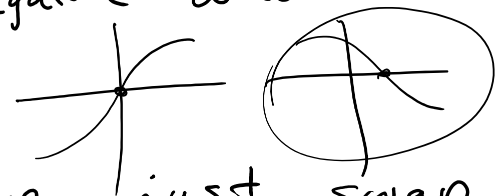


Infinite loop!

$$|a-b| < \varepsilon_{absolute}$$

$$\frac{|a-b|}{|c|} < \varepsilon_{relative}$$

Assumed $f(a)$ is negative and that $f(b)$ is positive



If not this case just swap a and b.

$$f(x) = x^x - 100 = 0$$