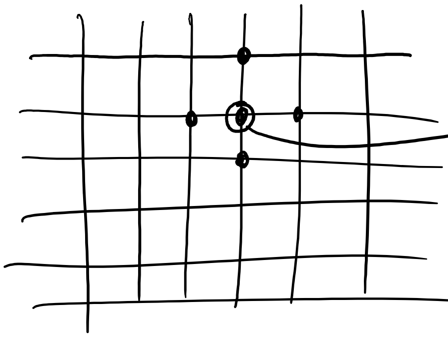
2-D Ising Model

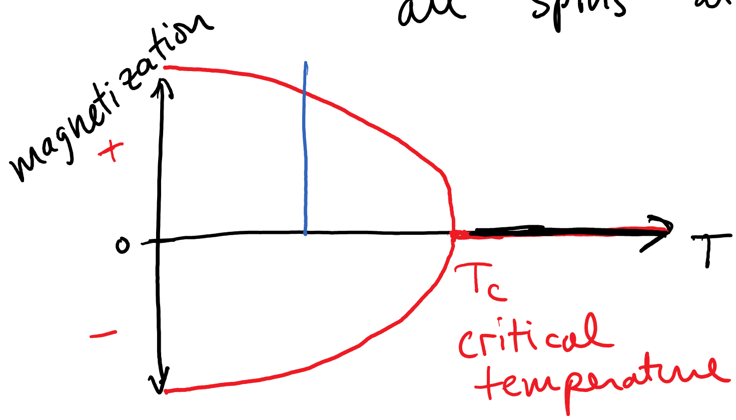
12 April 2021 12:13



N spins $\nu = 1..N$
on constant uniform grid

$S_\nu = \pm 1$

What happens as we change the temperature as $T \to 0K$
all spins at $+1$ or -1



2^{nd} Phase Transition

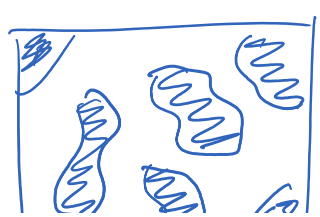
$$H = -J \sum_{\mu,\nu} S_\mu S_\nu$$

the product $S_\mu S_\nu$ is either $+1$, -1
aligned anti-aligned

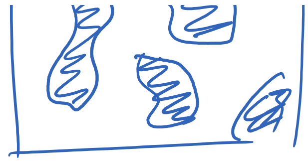
If $J > 0$ Ferromagnetic
$J < 0$ Antiferromagnetic

For this 2-D Ising Model an exact solution was found by Onsager (1944).
Critical Temperature T_c is given by:

$$\frac{J}{k_B T_c} = -\frac{1}{2}\log\left(\sqrt{2}-1\right) \cong 0.44069$$



$0 < T < T_c$



$$\langle m \rangle = \int_{\substack{\text{all } c_i \\ \text{possible} \\ \text{states}}} P(c_i)\, M(c_i)\, dc_i$$

all c_i possible states $\} \; 2^N$ states

imagine 12×12 $\quad 2^{144} \cdot 10^{-16}\,s = 10^{17}\,s$

longer than the age of the universe.

Pick a path through configuration space.

Interested in equilibrium behavior.

Special path through config space which when averaged over gives you the $\langle m \rangle$ in equilibrium.

$$C_1 \xrightarrow[\text{spin}]{\text{flip one}} C_2 \xrightarrow[\text{spin}]{\text{flip one}} C_3 \cdots \longrightarrow C_s$$

Must obey <u>detailed balance</u>.

$$\frac{W(+s_v \to -s_v)}{W(-s_v \to +s_v)} = e^{-2\beta J s_v \sum_{N \in (v)} s_N}$$

(in equilibrium)

transition probability

$$\beta \equiv \frac{1}{k_B T}$$

$\boxed{k_B = 1 \text{ for us}}$

Metropolis Algorithm

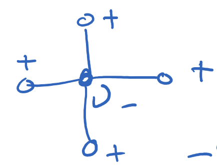
$$W(+s_v \to -s_v) = \min\left(1, e^{-\beta \Delta E}\right)$$

$$W(-s_v \to +s_v) = \min\left(1, e^{\beta \Delta E}\right)$$

If the move $+s_v \to -s_v$ goes to lower energy $W = 1$, always accept.

If the move $+s_v \to -s_v$ goes to larger energy ΔE is +ve, do $+s_v \to -s_v$ with prob $\underbrace{e^{-\beta \Delta E}}$

Pick random number $[0, 1)$



$\Delta E = +4$

ALGORITHM

pick random v

1. Pick spin s_v
2. Calculate ΔE for flipping
3. If $\Delta E < 0$ — $\boxed{s_v = -s_v}$
4. else
   - 4a: Pick a random $0 \le r < 1$
   - 4b. If $r < \exp(-\beta \Delta E)$ — $\boxed{s_v = -s_v}$
5. otherwise no change

$e^{-\beta \Delta E} < 1$

Start with high T and visualize what happens $\langle m \rangle$ (black + white pixels)

$T > T_c$ $\langle m \rangle = 0$
disorder

$T < T_c$ correlated spins and $\langle m \rangle \neq 0$.