

We used \underline{u} denote fluid velocity, but I want to switch now to using \underline{v} which will be an added field of every particle.

$$\nabla \cdot \underline{v} \cong \sum_b \frac{m_b}{\rho_b} \underline{v}_b \cdot \nabla W(|\underline{r} - \underline{r}_b|, h)$$

↑ size of the kernel.

Monahan cubic-spline kernel
we could use this, but ...

$$\begin{aligned} \nabla \cdot \underline{v} &\equiv \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) \cdot (v_x \hat{i} + v_y \hat{j} + v_z \hat{k}) \\ &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \end{aligned}$$

$$\nabla \cdot \underline{v} = \frac{1}{\rho} [\nabla \cdot (\rho \underline{v}) - \underline{v} \cdot \nabla \rho]$$

$$\nabla \cdot \underline{v}_a \cong \frac{1}{\rho_a} \left[\sum_b m_b \frac{\rho_b \underline{v}_b}{\rho_b} \cdot \nabla_a W_{ab} - \underline{v}_a \cdot \sum_b m_b \nabla_a W_{ab} \right]$$

$$\rho_a (\nabla \cdot \underline{v}_a) \cong \sum_b m_b (\underline{v}_b - \underline{v}_a) \cdot \nabla_a W_{ab}$$

W(| $\underline{r}_a - \underline{r}_b$ |, h)

$$\frac{de}{dt} = -\frac{P}{\rho} \nabla \cdot \underline{v}$$

switched $a \leftrightarrow b$ -1!

energy

$$\frac{de_a}{dt} = \left(\frac{\rho_a}{\rho_a^2} \right) \sum_b m_b (\underline{v}_a - \underline{v}_b) \cdot \nabla_a W_{ab}$$

Benz Formulation

continuity

$$\rho_a = \sum_b m_b W_{ab}$$

Momentum
(Euler equation)

$$1 \sim \frac{1}{\rho}$$

$$\frac{d\underline{v}_a}{dt} = - \frac{\rho_a \nabla P_a}{\rho_a^2}$$

$$\frac{d\underline{v}_a}{dt} = - \frac{1}{\rho_a^2} \sum_b m_b (P_b - P_a) \nabla_a W_{ab}$$

Force $\rightarrow 0$ for const. Pressure, but
linear momentum is not conserved
and angular momentum is also
not conserved.

Instead:
$$\frac{\nabla P}{\rho} = \nabla \left(\frac{P}{\rho} \right) + \frac{P}{\rho^2} \nabla \rho$$

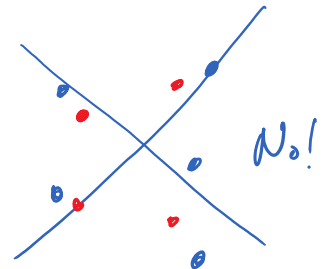
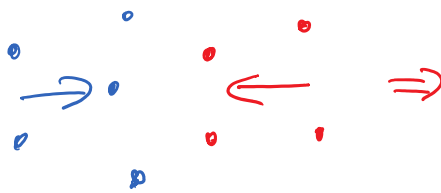
$$\Rightarrow \frac{d\underline{v}_a}{dt} = - \sum_b m_b \left(\frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2} \right) \nabla_a W_{ab}$$

Looks symmetric $a \leftrightarrow b$

Newton's 3rd Law: $F_{ab} = -F_{ba}$
momentum conserved

Caveat

However if particles a and b have
differing kernel sizes h_a, h_b then this
is not quite true.



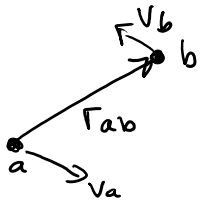
Add Artificial Viscosity



$$\frac{d\underline{v}_a}{dt} = - \sum_b m_b \left(\frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2} + \Pi_{ab} \right) \nabla_a W_{ab}$$

$\Pi_{ab} = \frac{c^2}{\rho}$

$$\frac{d \underline{v}_a}{dt} = - \sum_b m_b \left(\frac{p_a}{\rho_a^2} + \frac{p_b}{\rho_b^2} + \Pi_{ab} \right) \nabla_a W_{ab}$$



$$\Pi_{ab} = \begin{cases} -\frac{\alpha \bar{\rho}_{ab} N_{ab} + \beta N_{ab}^2}{\bar{\rho}_{ab}}, & \underline{v}_{ab} \cdot \underline{r}_{ab} < 0 \\ 0, & \underline{v}_{ab} \cdot \underline{r}_{ab} > 0 \end{cases}$$

$$\bar{\rho}_{ab} = \frac{1}{2} (\rho_a + \rho_b) \quad ; \quad N_{ab} = \frac{\frac{1}{2} (h_a + h_b) \underline{v}_{ab} \cdot \underline{r}_{ab}}{r_{ab}^2 + \eta^2}$$

$\beta = 2 \alpha$
 $\alpha = 1, \beta = 2$
 $\alpha = 0.5, \beta = 1$
 small number to keep it from exploding.

$$e_a = \frac{p_a}{\rho_a (\gamma - 1)} \quad c_a = \sqrt{\gamma (\gamma - 1) e_a}$$

$$\frac{p}{\rho^2} = \frac{c^2}{\gamma \rho}$$

Let's "write" the code:

Variables for particles: $\underline{r}, \underline{v}, e, c, \rho, (h)_{NN}$
 we need to calculate $\underline{a} = \frac{d\underline{v}}{dt}$ $\dot{e} = \frac{de}{dt}$

★ need 2 extra temporary variables → ODE "Leapfrog"

$\underline{v}_{pred}, e_{pred}$

DRIFT1(), DRIFT2(), KICK(), CALCFORCE()
 $\underline{v}_{pred}, e_{pred}$

SPH:

DRIFT1($\Delta t = 0$) $\underline{v}_{pred} e_{pred}$

CALCFORCE()

for (step = 0; step < NSTEP; ++step) {

DRIFT1($\Delta t / 2$)
 ..

CALCFORCE()
 KICK(Δt)
 DRIFT2($\Delta t/2$)
 ⋮

CALCFORCE:

TREEBUILD

NN-Density \leftarrow all particles calculate ρ_a

CALCSOUND \leftarrow all particles calculate c

NN-SPHFORCE \leftarrow All calculate \underline{a}, \dot{e}
 $c_a = \sqrt{\chi(\chi+1)} e_{\text{pred}}$

DRIFT1(Δt):

$$\underline{r} += \underline{v} \Delta t$$

$$\underline{v}_{\text{pred}} = \underline{v} + \underline{a} \Delta t$$

$$e_{\text{pred}} = e + \dot{e} \Delta t$$

DRIFT2(Δt):

$$\underline{r} += \underline{v} \Delta t$$

KICK(Δt):

$$\underline{v} += \underline{a} \Delta t$$

$$e += \dot{e} \Delta t$$

$$\bar{W}_{ab} = \frac{1}{2} (W(r_{ab}, h_a) + W(r_{ab}, h_b))$$

Symmetrized the kernel

How is this done?

