decimation in frequency

$$y_{2r} = \sum_{q=0}^{n-1} \omega^{2rq} x_q$$

$$= \sum_{q=0}^{n/2-1} \omega_{n/2}^{rq} \left(x_q + x_{\frac{n}{2}+q} \right)$$

$$y_{even} = W_{\frac{n}{2}} \left(\underline{x} + \underline{x}_{\frac{n}{2}} \right)$$

$$y_{2r+1} = \sum_{q=0}^{n-1} \omega^{(2r+1)q} x_q$$

$$= \sum_{q=0}^{n/2-1} \omega_{n/2}^{rq} \, \omega_n^{q} \left(x_q - x_{\frac{n}{2}+q} \right)$$

$$y_{odd} = W_{\frac{n}{2}} \left[\mathrm{diag}(\underline{\omega}_n) \left(\underline{x} - \underline{x}_{\frac{n}{2}} \right) \right]$$

Conventions: $(H_{-n} = H_{N-n})$

let n vary from 0 to $N-1$ in H_n

then n an k vary over the same range.

zero frequency $\longrightarrow$ $n = 0$

+ve frequency $\longrightarrow$ $1 \leq n \leq \frac{N}{2} - 1$

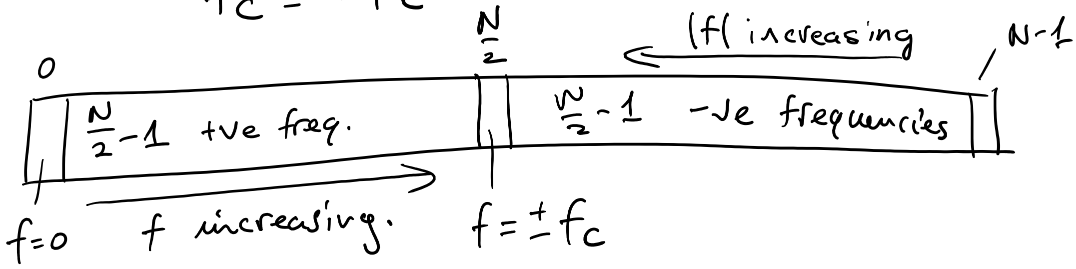
$0 < f < f_c$

-ve frequency $\longrightarrow$ $\frac{N}{2} + 1 \leq n \leq N-1$

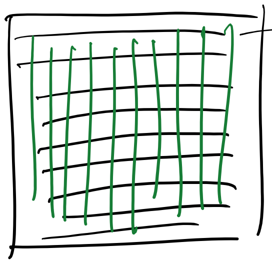
$-f_c < f < 0$

Nyquist frequency $\longrightarrow$ $n = \frac{N}{2}$

$f_c = -f_c$



1-D FFT $\longrightarrow$ 2-D FFT



1-D FFTs

$$H(n_1,n_2) = \sum_{k_2=0}^{N_2-1}\sum_{k_1=0}^{N_1-1} e^{\left(\frac{2\pi i}{N_2}\right)k_2 n_2}\, e^{\left(\frac{2\pi i}{N_1}\right)k_1 n_1}\, h(k_1,k_2)$$

$$= \sum_{k_2=0}^{N_2-1} e^{\left(\frac{2\pi i}{N_2}\right)k_2 n_2}\, \tilde{H}(n_1,k_2)$$

$$\bigcirc = \mathrm{FFT}_{①}\ \mathrm{transpose}_{①\leftrightarrow ②}\ \mathrm{FFT}_{①}\ h$$

$$\nabla^2\phi = 4\pi G\,\delta\rho$$

$$\phi = \int_{-\infty}^{\infty}\phi_k\, e^{2\pi i k x}\, dk$$

$$\left(\frac{\partial}{\partial x}\right)^2\phi(x) = \int_{-\infty}^{\infty}\phi_k\, e^{2\pi i k x}\, dk \cdot \underbrace{(2\pi i k)(2\pi i k)}_{-4\pi^2 k^2}$$

$$= \int_{-\infty}^{\infty}\left[-4\pi^2 k^2\phi_k\right] e^{2\pi i k x}\, dk$$

$$\boxed{-\cancel{4}\pi^2 k^2\phi_k = \cancel{4}\pi G\rho_k}$$

$$\phi_k = -\frac{G}{\pi k^2}\rho_k$$

?

$\hookrightarrow$ IFFT $\longrightarrow \Phi(\underline{r})$

Is exact on grid points and periodic