

10. Wavelength measurement using prism spectroscopy

10.1 Introduction

The study of emitted spectra of electromagnetic waves by excited atoms makes for one of the most important methods to investigate on the structure of atoms and the forces, which act between nuclei and electrons. The observation of discrete lines in the spectra of excited atoms played a crucial role in the development of quantum mechanics.

In this lab course, one shall study the spectral lines of helium and atomic hydrogen in the range of visible light with the help of a prism spectroscope. By measuring the wavelengths of the hydrogen lines, one shall determine one of the fundamental constants of nuclear physics, the so-called Rydberg constant R .

The decomposition of light into its spectral colors inside of a prism is caused by dispersion. Dispersion occurs, when light is being diffracted at the boundary layers between air and glass. It is designated to the fact, that the refraction index is dependent on the wavelength for most materials. In imaging systems (microscope, telescope, camera), dispersion leads to aberrations (imaging errors), which have to be minimized by using complex lens constructions and special lens coatings. In nature, dispersion, occurring when light gets reflected and diffracted on water drops, leads to the appearance of rainbows.

An alternative method for the determination of wavelengths of spectral lines is being shown in the lab course Spm. Namely the measuring of interference patterns which occur when light gets diffracted on a grid. The obtained results from this method are being used in this place to simplify the usage of the prism spectroscope.

10.2 Theory

a) Generation of spectral lines

The emergence of discrete lines in the spectrum of absorbed or emitted electromagnetic radiation by single atomic gases is only to explain sufficiently by quantum mechanics. Nevertheless, some fundamental properties can be derived from the semi-classical Bohr (atom) model. The Bohr model

rests upon three basic assumptions:

- An atom consists of a heavy nucleus and a shell of much lighter electrons
- The shell electrons can occupy only very particular orbitals, namely those, among whom their angular momentum happens to be an integer multiple of the so-called reduced Planck's constant $\hbar = h/2\pi$. While being in those orbitals, the electrons do not emit an electromagnetic radiation.
- Electromagnetic radiation is only absorbed or emitted when an electron changes from one state (1) of energy E_1 to a state (2) of energy E_2 by performing a so-called quantum jump. The energy of the absorbed or emitted electromagnetic radiation during the transition is given by the energy difference $|E_2 - E_1|$.

The second assumption is in obvious contrast with the laws of classical physics. Regarding an electron as a charged particle which moves on a circular orbit around the nucleus, it represents a constantly accelerated charge. According to the laws of classical electrodynamics, the atom should constantly emit radiation. This contradiction is being dissolved by the laws of quantum mechanics.

It follows from the third assumption, that the atom can absorb or emit electromagnetic radiation only for very distinct energies, namely

$$E = E_2 - E_1. \quad (10.1)$$

The frequency ν and the wavelength λ of the radiation arise from the energies with

$$E = h \cdot \nu \quad \text{and} \quad E = \frac{h \cdot c}{\lambda} \quad (10.2)$$

while h being the Planck's constant and c being the speed of light.

Therefore, the spectrum of electromagnetic radiation is determined by the energy spectrum of the allowed orbital states of the electrons. The correct calculation of the energy spectrum has to be done quantum mechanically. A simple classical calculation though, yields the same result for a rough description of the structure of the spectrum as the correct quantum mechanical description would give.

In the classical picture, a hydrogen atom consists of an electron of charge $-e$, which orbits along a circular path around a resting proton of charge $+e$. A Coulomb force

$$F_C = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r^2} \quad (10.3)$$

acts on the electron and its equation of motion becomes

$$\frac{m \cdot v^2}{r} = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r^2}. \quad (10.4)$$

The electrons kinetic, potential and total energies are

$$E_{kin} = \frac{m}{2} \cdot v^2 = \frac{1}{4\pi \cdot \epsilon_0} \cdot \frac{e^2}{2r} \quad (10.5)$$

$$E_{pot} = \frac{F_C}{e} = -\frac{1}{4\pi \cdot \epsilon_0} \cdot \frac{e^2}{r} \quad (10.6)$$

$$E_{tot} = E_{kin} + E_{pot} = -\frac{1}{4\pi \cdot \epsilon_0} \cdot \frac{e^2}{2r} \quad (10.7)$$

According to the Bohr model, the angular momentum can only take on the discrete values

$$L_n = m \cdot v \cdot r = n \cdot \frac{h}{2\pi} \quad \text{with } n = 1, 2, 3, \dots \quad (10.8)$$

Applying Equ. 10.4 it can be shown that the orbit radius of the electron can only take on the discrete values

$$r_n = \frac{h^2 \cdot \epsilon_0}{\pi \cdot m \cdot e^2} \cdot n^2. \quad (10.9)$$

Substituting Equ. 10.7, it follows that the electrons energy on these particular orbits becomes

$$E_n = -\frac{m \cdot e^4}{8 \epsilon_0^2 \cdot h^2} \cdot \frac{1}{n^2} \quad (10.10)$$

The resulting energy spectrum is depicted in Fig. 10.1.

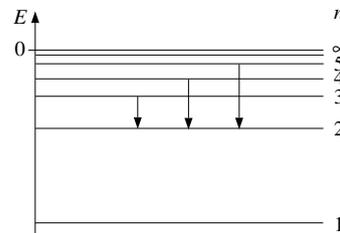


Figure 10.1: Energy levels of the hydrogen atom.

For the wavelength of the emitted radiation from a transition of a state n_1 to a state n_2 of lower energy, one has

$$\frac{1}{\lambda_{n_1 \rightarrow n_2}} = \frac{1}{h \cdot c} \cdot (E_{n_1} - E_{n_2}) = \frac{m \cdot e^4}{8 \epsilon_0^2 \cdot c \cdot h^3} \cdot \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right) = R \cdot \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right) \quad (10.11)$$

where R is the so-called Rydberg constant.

$$R = \frac{m \cdot e^4}{8 \epsilon_0^2 \cdot c \cdot h^3} \quad (10.12)$$

Only a few of the transitions generate spectral lines in the wavelength range of visible light. In particular, those are the first three lines of the so-called Balmer series, which correspond to the transitions from $n_1 = 3, 4$ and 5 to $n_2 = 2$ in Fig. 10.1.

Pure line spectra are only created by shining single-atomic gases. In multiple-atomic gases, electromagnetic radiation can also be generated for transitions between different excited states of oscillations or rotations between the atoms of a molecule. This leads to the emergence of so-called band spectra. Glowing solid bodies generate a continuous spectrum.

b) Dispersion in a prism

Inside of a prism spectrometer, the separation of light into its spectral colours is based on the fact, that the refraction index rises with increasing wavelength of the light. The optical path of the light, when passing through a prism, is depicted in Fig. 10.2.

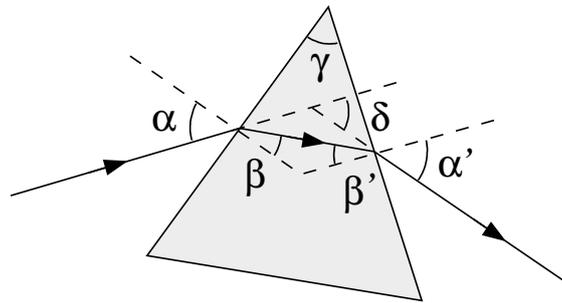


Figure 10.2: Optical path inside a prism.

According to the law of refraction one has

$$\frac{\sin \alpha}{\sin \beta} = \frac{\sin \alpha'}{\sin \beta'} = \frac{n_2}{n_1} \quad (10.13)$$

where n_1 is the refraction index of air and n_2 the refraction index of glas. For $n_1 \approx 1$ and $n_2 = n(\lambda)$ it follows that

$$\sin \alpha = n(\lambda) \cdot \sin \beta \quad (10.14)$$

The total angle of deflection δ of the ray, using Fig. 10.2, results in

$$\delta = \alpha + \alpha' - \beta - \beta' = \alpha + \alpha' - \gamma \quad (10.15)$$

where γ is the opening angle of the prism. The dependency of the deflection angle can be derived for the simplest case of a symmetric ray beam. In this case one has $\beta = \beta' = \gamma/2$ and therefore

$$\sin \alpha = n \cdot \sin \frac{\gamma}{2} \quad (10.16)$$

and

$$\frac{d}{dn}(\sin \alpha) = \sin \frac{\gamma}{2} \quad (10.17)$$

On the other hand one has

$$\frac{d}{dn}(\sin \alpha) = \cos \alpha \cdot \frac{d\alpha}{dn} \quad (10.18)$$

such that

$$\frac{d\alpha}{dn} = \frac{\sin \frac{\gamma}{2}}{\cos \alpha} \quad (10.19)$$

Furthermore, considering Equ. 10.13 and having a symmetric ray path, one has $\alpha = \alpha'$ and therefore $\delta = 2\alpha - \gamma$, thus

$$\frac{d\delta}{dn} = 2 \cdot \frac{d\alpha}{dn} = 2 \cdot \frac{\sin \frac{\gamma}{2}}{\cos \alpha} \quad (10.20)$$

After all, the dependency from the wavelength becomes

$$\frac{d\delta}{d\lambda} = \frac{d\delta}{dn} \cdot \frac{dn}{d\lambda} = 2 \cdot \frac{\sin \frac{\gamma}{2}}{\cos \alpha} \cdot \frac{dn}{d\lambda} \quad (10.21)$$

The refraction index for glass decreases with increasing wavelength, meaning $dn/d\lambda < 0$. That being said, one has $d\delta/d\lambda < 0$ as well: The shorter the wavelength, the greater the angle of deflection, blue light gets deflected stronger than red light.

To obtain a quantitative determination of the wavelength λ out of the angle of deflection δ , one has to define the dispersion function $dn/d\lambda$ first. That means, the prism spectrometer has to be calibrated by reference to various known wavelengths.

10.3 Experimental part

In this lab course one shall determine the wavelengths of the spectral lines of a hydrogen lamp in the range of visible light with the help of a prism spectrometer. They are used to calculate the Rydberg constant using Equ. 10.11. First of all, the spectrometer is being calibrated by the measurement of spectral lines of helium, whose wavelengths have been determined in the lab course Spm already.

a) Qualitative pre-examination

A pocket spectrometer is enclosed with the experiment. With its help you shall first observe the light sources qualitatively.

- Observe the spectra of the helium lamp, the hydrogen lamp and a normal light bulb using the pocket spectrometer. Take notes on your observations.

b) Experimental set-up

The set-up is depicted in Equ. 10.3.

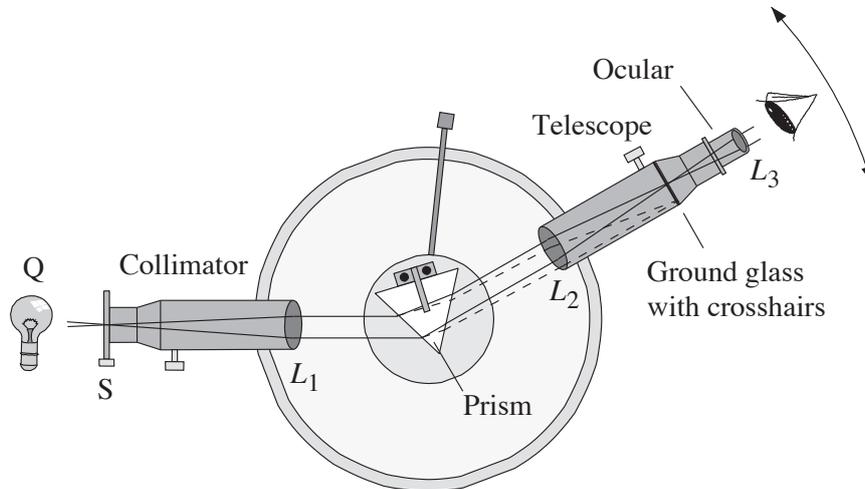


Figure 10.3: Set-up.

The light source (helium or hydrogen lamp) illuminates a narrow slit which is located in the focal plane of the collimating lens L_1 . This generates a beam of parallel light, which is then being refracted in the prism and, due to dispersion, decomposed into its spectral colors. The multicoloured reproductions of the slit emerge in the focal plane of the eyepiece lens L_2 which then can be observed on a matt screen using the ocular L_3 . The screen is equipped with a hair cross which, by turning the telescope around the rotation axis of the spectrometer, is being aligned with the line one wants to measure. The deflection angle δ can be read off from an angle scale with a precision up to one arc minute.

The scale is being calibrated by line spectra of known wavelengths. By the use of interpolation in between the measurement points, each deflection angle can be matched with a certain wavelength. Therefore one creates a calibration curve $\delta(\lambda)$ (the dispersion curve of the prism spectrometer).

c) Calibration of the prism spectrometer

The calibration curve of the prism spectrometer shall be generated by observing the spectral lines of a helium lamp. In doing so, you shall use the wavelength of those spectral lines which have been determined in the lab course Spm.

- Illuminate the entering slit of the spectrometer with a helium lamp. Adjust the telescope ocular in such a way, that both, the hair cross and the spectral lines, are being imaged sharply at the same time.
- Determine the deflection angle δ for all of the eight spectral lines. To do so, align the hair cross with the spectral line and read off the deflection angle.
- Repeat the measurement five times in total for all spectral lines and calculate the mean value and the uncertainty of the measured deflection angle.
- Plot the deflection angles and their uncertainties against the known wavelengths from the lab course Spm and interpolate the measurement points graphically. Create fitting curves of the best, highest and lowest curve through the data points in such a way that they are still compatible with the error bars, as being shown in Equ. 10.4.

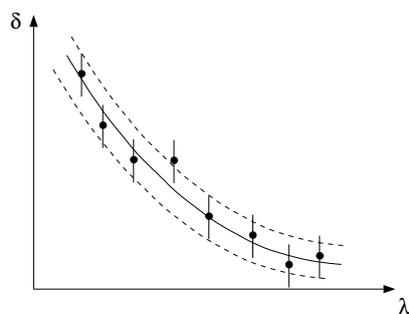


Figure 10.4: Dispersion curve.

d) Measurement of the spectral lines of a hydrogen lamp

With the help of the prepared calibration curve, you shall determine the wavelength of three spectral lines of atomic hydrogen to calculate the Rydberg constant. Hydrogen is usually bound in H_2 -molecules. In this experiment however, a hydrogen discharge lamp is being used, in which some H_2 -molecules are constantly decomposed to atomic hydrogen. Hence, three strong lines can be found in the spectrum of the discharge lamp: one in the red, one in the blue-green and one in the blue-violet wavelength range of light. At the same time, one can observe some bands which arise from the molecular hydrogen. You do not have to measure their wavelengths.

- Illuminate the entering slit of the spectrometer with the hydrogen lamp. Make sure that the hair cross and the spectral lines are being shown sharply at the same time by adjusting the ocular telescope.
- Determine the deflection angle δ for all of the three spectral lines. To do so, align the hair cross with the desired spectral line and read off the deflection angle on the angle scale.
- Repeat the measurements five times and calculate the mean and the uncertainty of the deflection angles.

With the help of the calibration curve, you shall determine the most probably value $\bar{\lambda}$ and the uncertainty m_λ of the wavelength. For the determination of the total uncertainty, one has to take into account the contribution of the uncertainty from the angle measurement itself, m_λ^δ , as well as the contribution from the uncertainty on the calibration curve, m_λ^{cal} . Both contributions can be estimated as shown in Fig. 10.5.

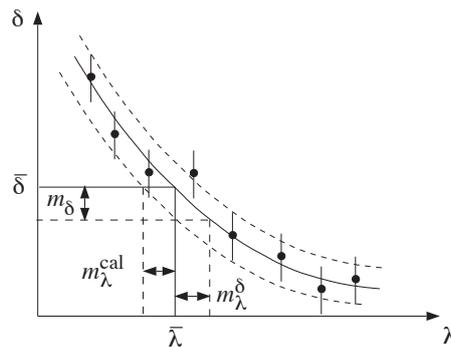


Figure 10.5: For the determination of the wavelength and its uncertainty.

- Determine $\bar{\lambda}$ by interpolating the best calibration curve at the position of the mean $\bar{\delta}$ of the angle measurement.
- Determine the uncertainty contribution m_λ^δ by interpolating the best calibration curve at the position $\bar{\delta} - m_\delta$.
- Determine the uncertainty contribution m_λ^{cal} by interpolating the lowest calibration curve at the position of the mean $\bar{\delta}$.
- Determine the total uncertainty m_λ by adding the uncertainty contributions quadratically.

The uncertainties on the three wavelengths will be very different from each other. This is due to the fact that the measurements uncertainties have a much greater impact in the flat region of the calibration curve where one has small wavelengths as compared to the steep region and large wavelengths.

e) Determination of the Rydberg constants

Subsequently you shall use your results to determine the Rydberg constant R according to Equ. 10.11. In doing so, the single measurement results shall be weighted correctly according to the uncertainties.

- To begin with, determine the single values R_i of the Rydberg constants for each one of the three spectral lines i according to Equ. 10.11. Determine the uncertainties m_{R_i} of the single measurements using error propagation.
- Determine the weighted mean of the three measurements according to

$$R = \frac{w_{R_1} \cdot R_1 + w_{R_2} \cdot R_2 + w_{R_3} \cdot R_3}{w_{R_1} + w_{R_2} + w_{R_3}} \pm \sqrt{\frac{1}{w_{R_1} + w_{R_2} + w_{R_3}}} \quad (10.22)$$

with the weighting factor $w_{R_i} = 1/m_{R_i}^2$.

- Compare the obtained result with the theoretical value of the Rydberg constant after Equ. 10.12.