



Higgs Physics

Exercise Sheet 4

ETH
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Fall semester 2018

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Issued : 10.10.2018

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Due : 17.10.2018

<https://moodle-app2.let.ethz.ch/course/view.php?id=4849>

Exercise 1 [*Application of likelihood methods in High Energy Physics*]

Likelihood methods are widely used in High Energy Physics. In this exercise we will see the application of such methods to parameters estimation and hypothesis testing.

The exercise can be solved in Python.

An python notebook with hints *LikelihoodMethods_hints.py* is attached to the course moodle website. Quick instructions to set up python and jupyter environment and run the script are reported at the end of this document and in the notebook.

a) The first part of the exercise concerns the use of maximum likelihood method for parameters estimation. The goal of the exercise is to estimate the total number of events corresponding to a resonant signal appearing on top of a falling background.

- Construct the probability density function (pdf) of the signal process in the observable $x \in [0, 10]$ as a Gaussian distribution. Take 5 for the mean and 0.1 for the width.
- Construct the background pdf as a falling exponential, with a slope of -0.35.
- Assume that fraction of the signal events is 5%.
- Generate a possible observation (“toy” or “pseudo-” experiment) from the above distributions, assuming that the total number of observed events is fixed.
- Assume that the signal model mean and width are perfectly known, while the signal fraction and the background decay length, need to be estimated from the observation. Construct the likelihood function for the data, given the signal + background model and use it to estimate the unknown parameters.
- Derive the profile likelihood function for τ and use it to derive an uncertainty on its maximum likelihood estimator $\hat{\tau}$. In this case, the profile likelihood is defined as

$$L_{\tau}(\tau) = \sup_f L(f, \tau)$$

Where τ is the background exponential slope and $L(f, \tau)$ is the likelihood function.

- Plot projection of the profile likelihood functions on τ parameter.

- Now assume that the signal model width is known with a systematic uncertainty of 0.05. Modify the likelihood function to include the corresponding nuisance parameter and obtain a new estimate of τ . Did the latter change compared to what you obtained in the previous point?
- b) The second part of the exercise concerns the use of likelihood ratios in hypothesis tests. The goal of the exercise is to construct the test statistics corresponding to a simple test of the background-only vs signal+background hypothesis and to extract exclusion limits using the CL_s prescription.
- Take the signal and background model from the first part of the exercise. Assume that all signal and background parameters are known. We will perform a simple hypothesis test: no parameter is fitted.
 - Generate 500 “pseudo-experiments” for each of the two hypotheses being tested with 100 events observed.
 - For each of the pseudo-experiments evaluate the following test statistics:

$$-lnQ = -ln \frac{L(data|s+b)}{L(data|b)}$$

- Histogram the obtained sampling distributions and use them to estimate the median expected CL_s value in the absence of a signal.
- Draw a pseudo-experiment from the background-only hypothesis and take it as the outcome of your experiment. With which CL_s can you exclude the signal based on this observation?
- Now repeat the exercise assuming that the signal width is 0.2 and 0.05. How does the median expected CL_s change?

Quick start instructions to install the needed software

Prerequisites :

Python 2

Jupyter notebook

We will use several libraries like:

Numpy, Matplotlib, scipy

Installations :

- You can create an independent python environment with anaconda.

If you do not have anaconda first download it. Miniconda will be sufficient.

<https://conda.io/docs/user-guide/install/index.html>

- Download the miniconda file for your OS system.
- In the terminal execute the file
`$ sh downloaded_file.sh`
- Follow the instructions in the terminal. When the installation is finished, log out and log in in new terminal window
- Now create a new working environment with python 2 and jupyter :
`$ conda create --name python2_7 python=2.7 jupyter`
- When environment is prepared, activate it
`$ source activate python2_7`
- To deactivate later
`$ source deactivate`
- In the new environment install the needed libraries with pip:
`$ pip install numpy`
`$ pip install matplotlib`
`$ pip install scipy`
- Now you are ready to work. To start a jupyter notebook simply type:
`$jupyter notebook`
- And a new browser window opens in your browser

In the jupyter notebook the code is divided in cells, for the rest it is just python
To execute a cell shift+Enter
Commands in File/Edit/View/Insert - have obvious meanings
Commands in Cell steer the running at the cell level
Commands in Kernel steer the running at the notebook level

To run the jupyter notebook with hints,
go to the directory with it and then run
`$jupyter notebook`
From the newly opened browser tab open the notebook
LikelihoodMethods_hints.ipynb
and start executing the code cells