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Higgs Physics
Tutorial 7

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Exercise 1: Higgs boson decay to two photons mediated by a W boson loop

The Higgs boson does not couple directly to photons, but it can decay to a photon pair via a loop-induced process. The particles in the loop can either be fermions (being the top-quark the most important contribution) or a W boson. In this exercise we will compute the leading-order matrix element for the decay of a Higgs boson into a photon pair induced by a W boson loop, working in a unitarity gauge (the calculation of the fermion contributions can be carried out analogously to the loop-induced $gg \to H$ process). We will explicitly perform the integration over the loop momentum. The result is finite in $d = 4$ dimensions, however the intermediate steps lead to divergent expressions which need be treated in dimensional regularization, and therefore we will work with $d = 4 - 2\epsilon$ dimensions.

We provide a Mathematica file to help in the manipulation of the expressions obtained in this calculation. It can be found at `www.physik.uzh.ch/~jmazzi/teaching/higgssheets/W_loop.nb`

(a) Compute the matrix element for $H \to \gamma\gamma$ via a W loop. The corresponding Feynman diagrams are shown in figure 1. Observe that there are two contributions with a triangle loop, obtained by exchanging the incoming momenta, and giving the same result.

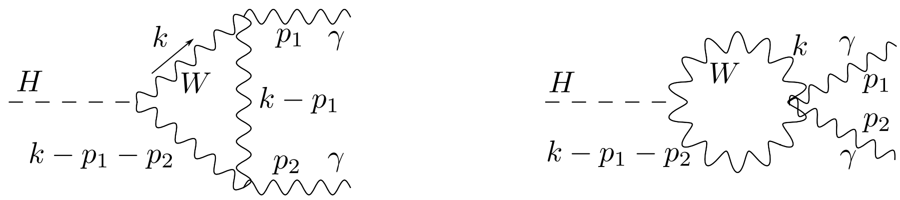

Figure 1: Feynman diagrams contributing to $H \to \gamma\gamma$ via a W loop. There is an additional triangle contribution obtained by exchanging $p_1 \leftrightarrow p_2$.

(b) Use Feynman parameterization

$$\frac{1}{ABC} = 2\int_0^1 dx \int_0^{1-x} dy \, , \frac{1}{[Ax + By + C(1-x-y)]^3} \tag{1}$$

to rewrite the loop propagators, and perform a shift in the loop momentum in order to remove the linear term on k in the denominator.

Hint: for simplicity, you might consider pulling the same common factor (including the 3 propagators) in all the contributions to the matrix element.

(c) Considering the common factor with the three propagators indicated above, group the different terms of the matrix element according to the power of the loop momentum k they have in the numerator. You should find that the term with 6 powers of k in the numerator vanishes after combining the contributions from the different diagrams.

(d) Using the results for the following d-dimensional loop integrals,

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2-\Delta)^n} = \frac{(-1)^n i}{(4\pi)^{d/2}} \frac{\Gamma(n-\frac{d}{2})}{\Gamma(n)} \Delta^{\frac{d}{2}-n} \tag{2}$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{k^\mu k^\nu}{(k^2-\Delta)^n} = \frac{(-1)^{n-1} i}{(4\pi)^{d/2}} \frac{g^{\mu\nu}}{2} \frac{\Gamma(n-\frac{d}{2}-1)}{\Gamma(n)} \Delta^{\frac{d}{2}-n+1} \tag{3}$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{k^\mu k^\nu k^\rho k^\sigma}{(k^2-\Delta)^n} = \frac{(-1)^n i}{(4\pi)^{d/2}} \frac{\Gamma(n-\frac{d}{2}-2)}{\Gamma(n)} \Delta^{\frac{d}{2}-n+2} \frac{1}{4} (g^{\mu\nu} g^{\rho\sigma} + g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}) \,, \tag{4}$$

compute the integral over the loop momentum (note that the integrals with an odd number of loop momenta in the numerator vanish).

(e) Perform the integral over the Feynman parameters to arrive to the final result. After expansion in powers on ϵ you should find that

$$\mathcal{M} = \frac{e^2 g}{(4\pi)^2 m_W} F(\beta) (p_1 \cdot p_2 g^{\mu\nu} - p_2^\mu p_1^\nu) \varepsilon_\mu(p_1) \varepsilon_\nu(p_2) \,, \tag{5}$$

with

$$F(\beta) = 2 + 3\beta + 3\beta(2-\beta) \arcsin^2(\beta^{-1/2}) + \mathcal{O}(\epsilon) \,, \tag{6}$$

where we have defined $\beta = \frac{4m_W^2}{m_H^2}$, and we consider the kinematical region $\beta > 1$.