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Higgs Physics
Tutorial 6

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Exercise 1: Higgs boson production via gluon fusion at NLO QCD

In this exercise we consider the Higgs boson production through gluon fusion. Since in the Standard Model the Higgs boson does not couple directly to the gluons, its production in this channel is mediated by triangular loops of heavy quarks (figure 1), where the top quark gives the dominant contribution.

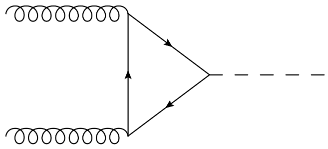

Figure 1: Feynman diagram contributing to $gg \to H$ at lowest order in QCD.

The calculation can be simplified by assuming that the top quark mass is much larger than the Higgs mass. In this effective theory the quark loop can be replaced by a ggH effective vertex (figure 2).

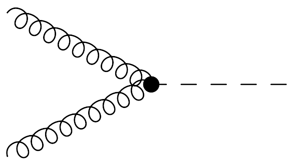

Figure 2: Feynman diagram contributing to $gg \to H$ at lowest order in the effective theory.

The effective Lagrangian at tree level is given by:

$$\mathcal{L}_{\text{eff}} = \frac{\alpha_S}{12\pi v} H G^{a,\mu\nu} G^a_{\mu\nu}\,, \tag{1}$$

where $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_S f^{abc} A^b_\mu A^c_\nu$ is the gluon field strength tensor.

In the following we will be working in the so-called conventional dimensional regularization (CDR) scheme with $d = 4 - 2\epsilon$ dimensions. In this scheme the fermions and the gluones have 2 and $2 - 2\epsilon$ degrees of freedom respectively.

(a) Using the effective Lagrangian in equation (1) derive the ggH effective vertex and compute the Higgs production cross section at LO in QCD. In the CDR scheme one finds

$$\sigma_{\text{LO}} = \sigma_0(\epsilon)\delta(1-z)\,, \tag{2}$$

where

$$\sigma_0(\epsilon) = \frac{\alpha_S^2}{\pi}\frac{1}{576 v^2}\frac{1}{1-\epsilon}\,, \tag{3}$$

and $z = \frac{m_H^2}{s}$.

At NLO one should take into account two types of corrections: real and virtual. In the case of real corrections, apart from the gluon initiated process, there are new channels opening up, namely the $qg \to Hq$ and $q\bar{q} \to Hg$.

(b) Compute the squared matrix element for the $q\bar{q} \to Hg$ process (figure 3).

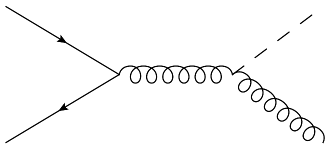

Figure 3: Feynman diagram contributing to the $q\bar{q} \to Hg$ process.

You should find that

$$\overline{|\mathcal{M}_{q\bar{q}\to Hg}|^2} = \frac{4}{81}\frac{\alpha_S^3}{\pi v^2}\mu^{2\epsilon}\frac{t^2+u^2-\epsilon(t+u)^2}{s}\,. \tag{4}$$

(c) Show that in the center of mass frame the two particle phasespace measure $d\Phi_2$ can be written in the following form,

$$d\Phi_2 = \frac{1}{8\pi}\left(\frac{4\pi}{m_H^2}\right)^{\epsilon}\frac{1}{\Gamma(1-\epsilon)}z^{\epsilon}(1-z)^{1-2\epsilon}y^{-\epsilon}(1-y)^{-\epsilon}dy\,, \tag{5}$$

where $y = (1+\cos\theta)/2$, with θ being the scattering angle.

(d) Use equations (4) and (5) to compute the cross section in the $q\bar{q}$ channel,

$$\sigma_{q\bar{q}} = \frac{1}{2s}\int \overline{|\mathcal{M}_{q\bar{q}\to Hg}|^2}d\Phi_2\,. \tag{6}$$

You should find

$$\sigma_{q\bar{q}} = \frac{1}{486\pi^2}\frac{\alpha_S^3}{v^2}(1-z)^3 + \mathcal{O}(\epsilon)\,. \tag{7}$$

(e) Compute the squared matrix element for the $qg \to Hq$ process (figure 4).

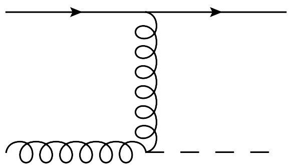

Figure 4: Feynman diagram contributing to the $qgtoHq$ process.

You should find

$$\overline{|\mathcal{M}_{qg\to Hq}|^2} = -\frac{\alpha_S^3}{54\pi v^2}\frac{1}{1-\epsilon}\mu^{2\epsilon}\frac{s^2+t^2-\epsilon(s+t)^2}{u}\,. \tag{8}$$

(f) Use equations (5) and (8) to compute the cross section in the qg channel. You should find,

$$\sigma_{qg} = (4\pi)^{\epsilon}\Gamma(1+\epsilon)\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon}\frac{\alpha_S}{2\pi}z\sigma_0(\epsilon)\left[-\frac{1}{\epsilon}P_{gq}(z) + P_{gq}(z)\ln\frac{(1-z)^2}{z} + \frac{4}{3}z - 2\frac{(1-z)^2}{z} + \mathcal{O}(\epsilon)\right], \tag{9}$$

where P_{gq} is the Altarelli-Parisi quark-gluon splitting function and it is given by:

$$P_{gq}(z) = \frac{4}{3}\frac{z^2-2z+2}{z}\,. \tag{10}$$

Hint: note that the Mandelstam invariants can be written in the following way,

$$t = -s(1-z)y\,, \qquad u = -s(1-z)(1-y)\,. \tag{11}$$

(g) In the gg channel the squared matrix element is given by

$$\overline{|\mathcal{M}_{gg\to Hg}|^2} = \frac{\alpha_S^3}{\pi v^2}\frac{1}{24(1-\epsilon)^2}\mu^{2\epsilon}\left[\frac{m_H^8+s^4+t^4+u^4}{stu}(1-2\epsilon)+\frac{\epsilon}{2}\frac{(m_H^4+s^2+t^2+u^2)^2}{stu}\right]. \quad (12)$$

Use equations (5) and (12) and perform the phasespace integration to find that the cross section in this channel can be written as

$$\begin{aligned}\sigma_{gg}^{\text{real}} &= \frac{1}{576\pi^2}\frac{\alpha_S^3}{v^2}(1-z)^{-1-2\epsilon}z^{\epsilon}\left(\frac{4\pi}{m_H^2}\right)^{\epsilon}\Gamma(1+\epsilon)\mu^{2\epsilon}\left(1-\frac{\pi^2\epsilon^2}{3}\right) \\ &\times \left\{-\frac{3}{\epsilon}\left(1+z^4+(1-z)^4\right)-\frac{11}{2}(1-z)^4-6(1-z+z^2)^2-6\epsilon f(z)+\mathcal{O}(\epsilon^2)\right\},\end{aligned} \quad (13)$$

where the function $f(z)$ satisfies $f(1)=1$.

(h) Use the expansion of $(1-z)^{-1-2\epsilon}$ in ϵ,

$$(1-z)^{-1-2\epsilon} = -\frac{1}{2\epsilon}\delta(1-z)+\left(\frac{1}{1-z}\right)_+ - 2\epsilon\left(\frac{\ln(1-z)}{1-z}\right)_+ + \mathcal{O}(\epsilon^2) \quad (14)$$

to rewrite the cross section in equation (13) in the following form:

$$\begin{aligned}\sigma_{gg}^{\text{real}} &= z\sigma_0(\epsilon)\frac{\alpha_S}{2\pi}\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon}(4\pi)^{\epsilon}\Gamma(1+\epsilon)\left\{\left[\frac{6}{\epsilon^2}+\frac{4\beta_0}{\epsilon}-2\pi^2\right]\delta(1-z)-\frac{2}{\epsilon}P_{gg}(z)\right. \\ &- \left.11\frac{(1-z)^3}{z}+12\frac{1+z^4+(1-z)^4}{z}\left(\frac{\ln(1-z)}{1-z}\right)_+ - 12\frac{(1-z)^2(1+z^2)+z^2}{z(1-z)}\ln z\right\},\end{aligned} \quad (15)$$

where P_{gg} is the gluon-gluon splitting function and is given by

$$P_{gg}(z) = 6\left[\frac{1-z}{z}+z(1-z)+\frac{z}{(1-z)_+}\right]+2\beta_0\delta(1-z), \quad (16)$$

and $\beta_0 = (11N_c - 2n_f)/12$.

When computing the virtual corrections one should take into account that the ggH effective vertex gets modified due to the exchange of virtual gluons inside the top quark loop. At one-loop order the effective Lagrangian is given by

$$\mathcal{L}_{\text{eff}} = \frac{\alpha_S}{12\pi v}HG^{a,\mu\nu}G^a_{\mu\nu}\left(1+\frac{11}{4}\frac{\alpha_S}{\pi}\right). \quad (17)$$

The squared matrix element for the virtual corrections (more specifically the interference between one-loop and tree-level amplitudes) is given by

$$\overline{|\mathcal{M}_{\text{virt}}|^2} = \frac{\alpha_S}{2\pi}\left(\frac{4\pi\mu^2}{m_H^2}\right)^{\epsilon}\frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}C_A\left(-\frac{2}{\epsilon^2}+\frac{11}{3}+\pi^2+\mathcal{O}(\epsilon)\right)\overline{|\mathcal{M}_0|}^2, \quad (18)$$

where $\overline{|\mathcal{M}_0|^2}$ is the squared Born amplitude,

$$\overline{|\mathcal{M}_0|^2} = \frac{\alpha_S^2}{576\pi^2 v^2}m_H^4\frac{1}{1-\epsilon}. \quad (19)$$

(i) Use equation (18) to compute the cross section for the virtual corrections. You should find

$$\sigma_{\text{virt}} = \frac{\alpha_S}{2\pi}\sigma_0(\epsilon)C_A\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon}(4\pi)^{\epsilon}\Gamma(1+\epsilon)\left\{-\frac{2}{\epsilon}+\frac{11}{3}+\frac{4\pi^2}{3}+\mathcal{O}(\epsilon)\right\}\delta(1-z). \quad (20)$$

After combining real and virtual corrections for the gg channel, we are still left with both UV and IR divergencies. The UV singularities are removed by renormalizing the strong coupling, which leads to a renormalized virtual amplitude,

$$\overline{|\mathcal{M}_{\rm virt}|^2}_{\rm renorm} = \overline{|\mathcal{M}_{\rm virt}|^2} - \frac{1}{\epsilon}\frac{\alpha_S}{\pi}2(4\pi)^\epsilon\Gamma(1+\epsilon)\beta_0\left(\frac{\mu^2}{\mu_R^2}\right)^\epsilon\overline{|\mathcal{M}_0|^2}\,, \tag{21}$$

and a corresponding renormalized cross section $\sigma_{\rm virt}^{\rm renorm}$. After combining $\sigma_{gg}^{\rm real}+\sigma_{\rm virt}^{\rm renorm}$ there is still a remaining IR pole, which is associated to initial state collinear singularities and is removed using the appropriate counterterm,

$$\sigma_{gg}^{\rm ct} = z\sigma_0(\epsilon)\frac{\alpha_S}{2\pi}\left(\frac{4\pi\mu^2}{\mu_F^2}\right)^\epsilon\Gamma(1+\epsilon)\frac{2}{\epsilon}P_{gg}(z)\,. \tag{22}$$

Here μ_R and μ_F are the renormalization and factorization scales, respectively.

(j) Show that the final result for the cross section in the gg channel, $\sigma_{gg}^{(1)} = \sigma_{gg}^{\rm real}+\sigma_{\rm virt}^{\rm renorm}+\sigma_{gg}^{\rm ct}$, is given by

$$\begin{aligned}\sigma_{gg}^{(1)} &= \frac{\alpha_S}{\pi}z\sigma_0\left[\left(\frac{11}{2}+\pi^2+2\beta_0\ln\frac{\mu_R^2}{\mu_F^2}\right)\delta(1-z)+12\left(\frac{\ln(1-z)}{1-z}\right)_+\right. \\ &+ \left. 6\left(\frac{1}{1-z}\right)_+\ln\frac{m_H^2}{\mu_F^2}+P_{gg}^{\rm reg}(z)\ln\frac{(1-z)^2m_H^2}{z\mu_F^2}-6\frac{\ln z}{1-z}-\frac{11}{2}\frac{(1-z)^3}{z}\right]+\mathcal{O}(\epsilon)\,,\end{aligned} \tag{23}$$

where the regular part of the splitting function is given by

$$P_{gg}^{\rm reg}(z) = P_{gg}(z) = 6\left[\frac{1-z}{z}+z(1-z)-1\right]. \tag{24}$$