ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Universität Zürich[UZH]

Higgs Physics
Tutorial 2

HS 2018
Prof. M. Grazzini
Prof. M. Donega

J. Mazzitelli, N. Chernyavskaya

Issued: 26.09.2018
Due: 03.10.2018

Exercise 1: Equivalence theorem and vector boson scattering

The equivalence theorem states that at high energies the scattering amplitudes involving vector bosons can be computed by replacing them with the corresponding Goldstone bosons

$$\mathcal{M}(W_L(p_1), W_L(p_2), \dots) \simeq \mathcal{M}(\eta(p_1), \eta(p_2), \dots) + \mathcal{O}\left(\frac{m_W}{E}\right) .$$

(a) Starting from the Lagrangian of the Standard Model (SM) Higgs sector

$$\mathcal{L}_H = (D_\mu \phi)^\dagger (D^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 ,$$

with

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_1 + i\eta_2 \\ v + H + i\eta_3 \end{pmatrix} ,$$

show that the interaction Lagrangian of the Goldstone bosons with themselves and with the Higgs boson can be written as (note that only the potential contributes)

$$\mathcal{L} = -\frac{m_H^2}{8v^2} \left(2\eta_+\eta_- + \eta_3^2 + H^2\right)^2 - \frac{m_H^2}{2v} \left(2\eta_+\eta_- + \eta_3^2 + H^2\right) H ,$$

with $\eta_1^2 + \eta_2^2 = 2\eta_+\eta_-$ (note that $v^2 = -\frac{\mu^2}{\lambda}$ and $m_H = \sqrt{-2\mu^2}$).

(b) Use the interaction Lagrangian above and the equivalence theorem to compute the scattering amplitude for $W_L^+ W_L^- \to W_L^+ W_L^-$ and $Z_L Z_L \to Z_L Z_L$. You should find

$$\mathcal{M}(W_L^+ W_L^- \to W_L^+ W_L^-) = -\sqrt{2} G_F m_H^2 \left(\frac{s}{s - m_H^2} + \frac{t}{t - m_H^2}\right) ,$$

$$\mathcal{M}(Z_L Z_L \to Z_L Z_L) = -\sqrt{2} G_F m_H^2 \left(\frac{s}{s - m_H^2} + \frac{t}{t - m_H^2} + \frac{u}{u - m_H^2}\right) .$$

(c) Use the result for the WW amplitude to derive the Lee-Quigg-Thacker unitarity bound

$$m_H^2 < 4\pi v^2 \simeq (870 \text{ GeV})^2 .$$

Hints:

Consider the partial wave expansion of the amplitude

$$\mathcal{M} = 16\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) Q_l ,$$

where $P_l(\cos\theta)$ are the Legendre polynomials which fulfill $P_l(1) = 1$ and the normalization condition

$$\int_{-1}^{1} P_l(x) P_{l'}(x) dx = \frac{2}{2l+1} \delta_{ll'} .$$

Integrate over the angles the differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|\mathcal{M}|^2 \,,$$

and use the optical theorem

$$\sigma = \frac{1}{s}\mathrm{Im}[\mathcal{M}(\theta = 0)]$$

to obtain the unitary condition $|Q_l|^2 = \mathrm{Im}(Q_l)$. Set a bound on the real part of the partial-wave amplitudes Q_l and consider $l = 0$.