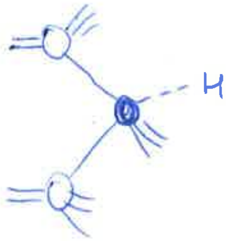
HIGGS DIFFERENTIAL CROSS SECTIONS AND DISTRIBUTIONS



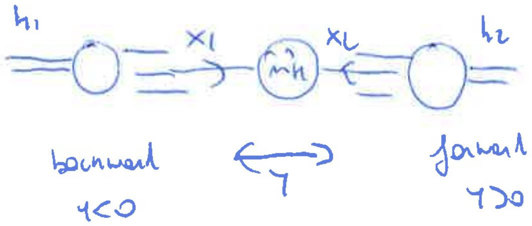
We consider the inclusive production of the Higgs boson in hadron collisions. The two hadrons provide two "beams" of incoming partons. The centre of mass of the partonic interaction is normally boosted with respect to the LAB frame

$\Rightarrow$ it is convenient to classify the final state according to variables that transform simply under longitudinal boosts. We introduce rapidity and azimuth

$$p^H_\mu = (E, \vec{p}) = (m_T \cosh y \,,\, p_T \sin\phi \,,\, p_T \cos\phi \,,\, m_T \sinh y)$$

$$y = \frac{1}{2} \ln \frac{E+p_z}{E-p_z} \qquad m_T = \sqrt{m_H^2 + p_T^2}$$

rapidity differences are boost invariant



$p_z \sim 0 \Rightarrow$ "central collision"

At Born level $\quad x_1 x_2 s = m_H^2$

$$\Rightarrow \quad x_1 = \frac{m_H}{\sqrt{s}} e^{y}$$

$$x_2 = \frac{m_H}{\sqrt{s}} e^{-y}$$

$$P_1 = x_1 \left(\frac{\sqrt{s}}{2}, 0, 0, \frac{\sqrt{s}}{2}\right)$$

$$P_2 = x_2 \left(\frac{\sqrt{s}}{2}, 0, 0, -\frac{\sqrt{s}}{2}\right)$$

$$\Rightarrow \quad y_H = \frac{1}{2} \ln \frac{(x_1+x_2)\frac{\sqrt{s}}{2} + (x_1-x_2)\frac{\sqrt{s}}{2}}{(x_1+x_2)\frac{\sqrt{s}}{2} - (x_1-x_2)\frac{\sqrt{s}}{2}}$$

$$= \frac{1}{2} \ln \frac{x_1}{x_2}$$

The maximum rapidity is obtained when there is maximum "unbalance", that is, when one of the two x, say $x_1 = 1$ $\Rightarrow$ $y_{max} = \frac{1}{2} \ln \frac{1}{x_{2\,min}} = \frac{1}{2} \ln \frac{s}{m_H^2}$

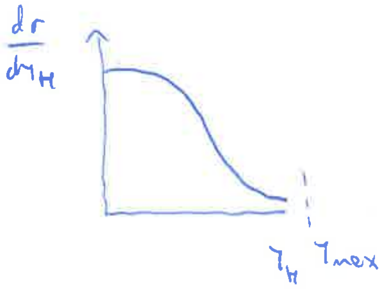
$\sqrt{s} = 8$ TeV $\Rightarrow y_{max} = 4.16$

$\sqrt{s} = 13$ TeV $\Rightarrow y_{max} = 4.64$

y_{max} smaller when the system is heavier

(at threshold there is not enough energy to boost the system $\Rightarrow y \sim 0$)

The Higgs rapidity distribution has a shape that is essentially driven by PDFs



$f(x_1)\, f(x_2)$ two competing effects when we turn x_1 very different from x_2

at $x_1 \to 0$ and $x_2 \to 1$ $f(x_1)$ is enhanced

and $f(x_2)$ is suppressed $\Rightarrow$ large x suppression always wins

$\Rightarrow$ The rapidity distribution is peaked in the central region (where $x_1 \sim x_2$) and is suppressed at large y

The rapidity distribution is weakly sensitive to RADIATIVE CORRECTIONS, which can be well approximated by an overall rescaling factor (K factor). On the contrary, the p_T distribution is highly sensitive to QCD radiative corrections. This is because QCD dynamics is "essentially" transverse.

NLO computations for differential cross sections

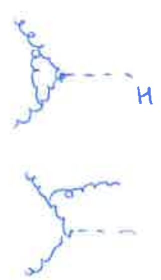
We have seen in the previous lectures how to compute the NLO QCD corrections to the total cross section in $gg \to H$.

Total cross sections, however, are ideal quantities: we would like to be able to compute the cross section at the differential level

For example: $pp \to H \to \gamma\gamma$ with $p_{T\gamma_1} > 25$ GeV $p_{T\gamma_2} > 20$ GeV $|\eta_\gamma| < 2.5$

How can one do this calculation?

We need to describe "parton level" events $d\sigma = d\sigma_R + d\sigma_V$



However, we have seen that real and virtual contributions are separately divergent! In the computation of the total cross section we have analytically computed the separate contributions and cancelled the singularities appearing as $\frac{1}{\varepsilon}$ and $\frac{1}{\varepsilon^2}$ poles.

But a computation with complicated cuts cannot be done analytically, can only be done numerically. So how to proceed?

Subtraction method

Suppose we want to compute
$$I = \lim_{\varepsilon \to 0}\left[\underbrace{\int_0^1 \frac{f(x)}{x^{1+\varepsilon}}\,dx}_{\text{"real"}} - \underbrace{\frac{1}{\varepsilon} f(0)}_{\text{"virtual"}}\right]$$

where the function $f(x)$ is too complicated to do the integral analytically.

However the integral is divergent at $\varepsilon \to 0$ and its singularity is cancelled by the term $-\frac{1}{\varepsilon} f(0)$. How to proceed? One solution is to add and subtract $f(0)$ in the "real" contribution.

$$I = \lim_{\varepsilon \to 0}\left[\int_0^1 \frac{f(x) - f(0) + f(0)}{x^{1+\varepsilon}}\,dx - \frac{1}{\varepsilon} f(0)\right]$$

$$= \lim_{\varepsilon \to 0} \int_0^1 \frac{f(x) - f(0)}{x^{1+\varepsilon}} = \int_0^1 \frac{f(x) - f(0)}{x}$$

The term $f(0)$ acts as a "subtraction counterterm" whose integral cancels the divergence of the virtual contribution. After this cancellation, the limit $\varepsilon \to 0$ can be taken and the integral can be easily computed numerically.

This idea is at the basis of the so called "subtraction method" to perform NLO calculations.

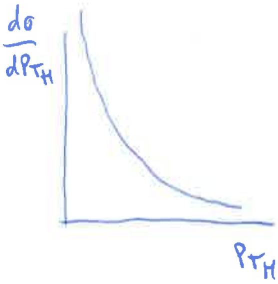
$$\int_{\bullet} d\sigma = \int_{m+1} d\sigma_R + \int_m d\sigma_V = \int_{m+1}\left(d\sigma_R - d\sigma_{CT}\right) + \overbrace{\int_1 d\sigma_{CT} + \int_m d\sigma_V}^{\text{cancel poles}} .$$

Choose the counterterm $d\sigma_{CT}$ such that it cancels the singular structure of $d\sigma_R$ and such that it can be easily integrated on the phase space of the unresolved parton. This integral will cancel the singularities of the virtual and the rest of the calculation can be done numerically.

Having discussed the method, we can now go back to the distributions. If we plot the photon p_T distributions, we get something like:

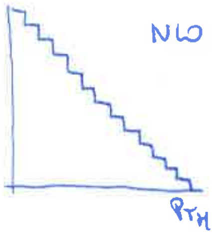
The distribution can be perturbatively computed and its integral is the NLO accepted cross section.

However, let us consider the p_T distribution of the diphoton system. At lowest order, it is concentrated at $p_T = 0$: $\frac{d\sigma^{(0)}}{dp} \sim \delta(p_T)$ but when we go to NLO we find

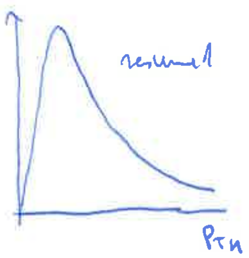


This is because of the IR (soft and collinear) singularities that appear when $p_T \to 0$. In other words, by requiring the p_T of the Higgs to become small, we "force" the radiation recoiling against the Higgs boson to be either soft or collinear.

In practice, if we do the computation with a parton level Monte Carlo we get



The integral is the NLO cross section: the virtual contributes to the first bin and its large negative contribution compensates the positive divergent contribution of the real. However the differential cross section has an unphysical behavior at $p_T \to 0$. This divergent behavior is due to the large logarithmic terms $\frac{1}{p_T^2}\alpha_S^n \ln^{2n-1}\frac{A m_H^2}{p_T^2}$ that need to be resummed to all orders and this will make the distribution well behaved.



(*) This resummation is "effectively" performed by MONTE CARLO PARTON SHOWERS

MC are indispensable tools for data analyses at high-energy colliders. They provide a complete simulation of all stages of the hadronic collisions, from the initial state radiation to the hard scattering process, to the final state radiation, to the hadronization phase.

Traditional MC generators start from an underlying process and "dress" it with multiple emissions obtained in the soft/collinear approximations.

radiation kernel

The probability for a single emission can be written as $P\, d\phi_1 \approx \frac{\alpha_S}{2\pi} K(\phi_1)\, d\phi_1$

$$P_1\, d\phi_1 \approx \frac{\alpha_S(q)}{2\pi}\, \frac{dq}{q}\, P(z,\phi)\, dz\, \frac{d\phi}{2\pi}$$

z : longitudinal momentum fraction

where $P(z,\phi)$ is the relevant DGLAP splitting function (in practice averaged over azimuth ϕ) and q is an evolution variable, which can be the transverse momentum relative to the emitter, the emission angle, or the virtuality. The divergence is regulated with a cut-off Q_0

$\boxtimes$ Here the first term gives no emission down to the IR cutoff

$\Rightarrow$ we go directly to the HADRONIZATION PHASE

the second, is then fed to the shower.

$\boxed{A}$ In this discussion we are considering for simplicity a FORWARD evolution

(relevant for example at e^+e^- collisions)

In hadronic collisions things are more complicated and we need to consider

BACKWARD evolution (from a low scale back to the hard-process scale)

we will ignore this complication here

The previous equation provides the basis for an iterative scheme for summing the logarithmically enhanced contributions to all orders. Each parton entering the hard process can emit at scales $q < Q$. The probability refers to emissions in the range $q, q+dq$. It follows that the probability of non resolvable emissions between scales q_1 and $q_2 < q_1$ is given by the Sudakov form factor $\boxed{A}$

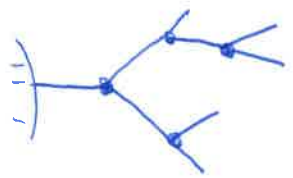
$$\Delta(q_2,q_1) = \exp\left\{ - \int_{q_2}^{q_1} \frac{\alpha_s(q)}{2\pi} \frac{dq}{q} \int P(z)\, dz \right\}$$

here we need an IR cutoff

This result can be understood by the conservation of probability (unitarity).

In a MC parton shower multiple emissions are generated by the Monte Carlo method.

In practice, one has effective diagrams, where, to each line we attach a Sudakov form factor and to each vertex, a splitting probability. The scale of the first



emission is obtained by solving the equation

$\Delta(q_1, Q) = R_1$ where R_1 is a random number in the interval $[0,1]$. The next emission is at $q_2 < q_1$ where $\Delta(q_2,q_1) = R_2$, and so on, until the chosen scale is smaller than the IR cutoff Q_0.

Below this scale the QCD coupling becomes so large that perturbation theory breaks down, and one has to provide some HADRONIZATION MODEL.

Let us now consider the hardest emission. In the shower approximation it can be written as $\boxed{B}$

$$\sigma^{(LO)}_{PS} = \int d\Phi_B\, B(\Phi_B) \left[\Delta(Q_0^2, Q^2) + \int_{Q_0^2}^{Q^2} d\Phi_1 \frac{\alpha_s}{2\pi} k(\Phi_1)\, \Delta(q^2, Q^2) \right]$$

where q is determined by the kinematics of the first emission.

Here the first term in the square bracket represents the probability of no emission, while the second term generates one emission at scale q.

It is easy to see that the square bracket integrates to unity;

Since $\Delta(q^2,Q^2) = \exp\left\{ - \int_{q^2}^{Q^2} d\Phi_1 \frac{\alpha_s}{2\pi} k(\Phi_1) \right\}$

$\boxed{*}$ NLO + PS what do we want?

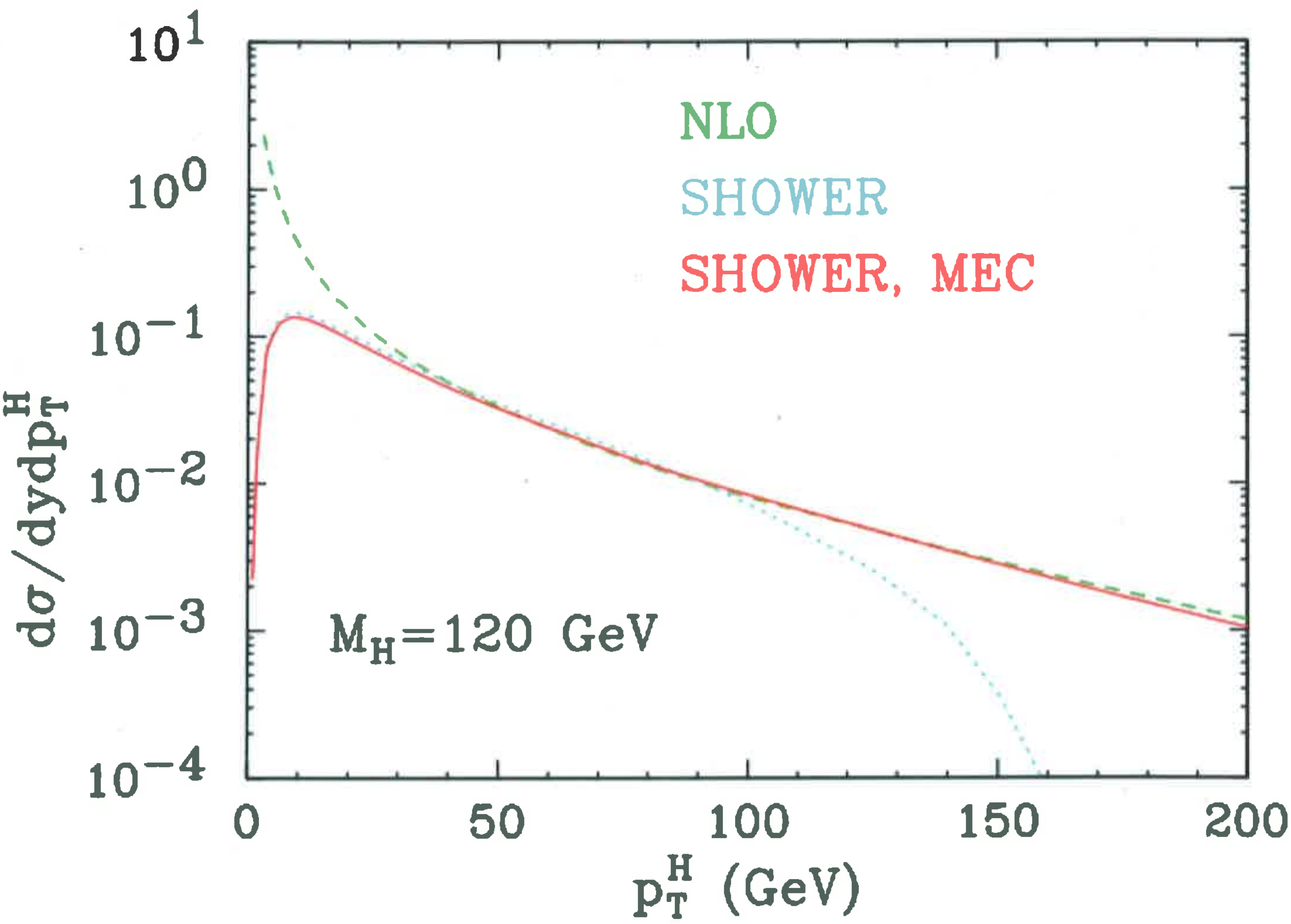
- NLO accurate total rates,
- Hard emissions treated as in NLO computations
- Soft / Collinear emissions treated as in MC
- Avoid double counting

$\Rightarrow$ Two methods exist to carry out this: MC@NLO, POWHEG

MC@NLO → Frixione, Webber

POWHEG → Nason et al.

Other methods have been proposed more recently (but they are much less developed)



NLO
SHOWER
SHOWER, MEC
M_H=120 GeV
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$d\sigma/dydp_T^H$
0
50
100
150
200
p_T^H (GeV)

$$\frac{d\Delta}{dq^2} = \Delta(q^2,Q^2)\,\frac{\alpha_s}{2\pi}\,k(\phi_1)$$

CONSERVATION OF PROBABILITY!

$$\Rightarrow \quad \Delta(Q_0^2,Q^2) + \int_{Q_0^2}^{Q^2} d\phi_1\,\frac{\alpha_s}{2\pi}\,k(\phi_1)\,\Delta(q^2,Q^2) = \Delta(Q_0^2,Q^2) + \int_{Q_0^2}^{Q^2}\frac{d\Delta}{dq^2}\,d\phi_1$$

$$= \cancel{\Delta(Q_0^2,Q^2)} + \underbrace{\Delta(Q^2,Q^2)}_{=1} - \cancel{\Delta(Q_0^2,Q^2)}$$

$\Rightarrow$ this implies that the shower does not change the LO normalization $\Rightarrow$ NLO corrections often included as overall K factors in the past!

When applied to hadronic collisions, this construction provides a p_T to the Higgs

however, the shower is able to describe the spectrum only in the very low p_T region. At high p_T, the important contribution comes from the EXACT MATRIX elements with one additional parton emission, which here are only approximated in the soft/collinear region.

One can remedy to this situation by replacing the kernel k with the full matrix element

$$k \to \frac{R}{B} \qquad \Delta \to \bar{\Delta} = \exp\left\{-\int d\phi_1\,\frac{\alpha_s}{2\pi}\,\frac{R(\phi_1)}{B}\right\}$$

$$\sigma^{(LO)}_{MEC} = \int d\phi_B\, B(\phi_B)\left[\bar{\Delta}(Q_0^2,Q^2) + \int_{Q_0^2}^{Q^2} d\phi_1\,\frac{R(\Phi_B,\Phi_1)}{B(\phi_B)}\,\bar{\Delta}(q^2,Q^2)\right]$$

As before, the square bracket integrates to unity and we have LO normalization. However, in this way we obtain the correct tail of the distribution. $\boxed{*}$ (but not the shape and normalization!)

Powheg

To illustrate how one can match NLO calculations to PS simulations, we recall the expression of the NLO cross section obtained through the subtraction method

$$\sigma^{(NLO)} = \int d\phi_B \left[B(\phi_B) + V(\phi_B) + I^S(\phi_B)\right] + \int d\phi_R\left[R(\phi_R) - CT(\phi_R)\right]$$

virtual — integral of subtraction term — subtraction term

One can then write the first emission as

$$\sigma^{(NLO)}_{POWHEG} = \int d\phi_B \, \bar{B}(\phi_B) \left[\bar{\Delta}(Q_0^2, Q^2) + \int_{Q_0^2}^{Q^2} d\phi_1 \frac{\alpha_s}{2\pi} k(\phi_1) \bar{\Delta}(q^2, Q^2) \right]$$

$$\bar{B}(\phi_B) = B(\phi_B) + V(\phi_B) + I^S(\phi_B) + \int d\phi_1 \left[R(\phi_R) - CT(\phi_R) \right]$$

In order to achieve NLO accuracy we have to perform the replacement $B \to \bar{B}$. This replacement, due to the unitarity condition, makes sure that we achieve the NLO normalization through a "local" k factor. The disadvantage of this method is that it exponentiates the full real emission matrix element. The advantage is that it produces only positive weights.

MC@NLO

An alternative formulation of the matching procedure is to write $R = R_S + R_F$ where R_S is the singular part of the real matrix element and R_F is finite. Then we can write

$$\sigma^{(NLO)} = \int d\phi_B \left[B(\phi_B) + V(\phi_B) + I^S(\phi_B) \right] + \int d\phi_R \left[R_S - CT \right] + \int d\phi_R R_F$$

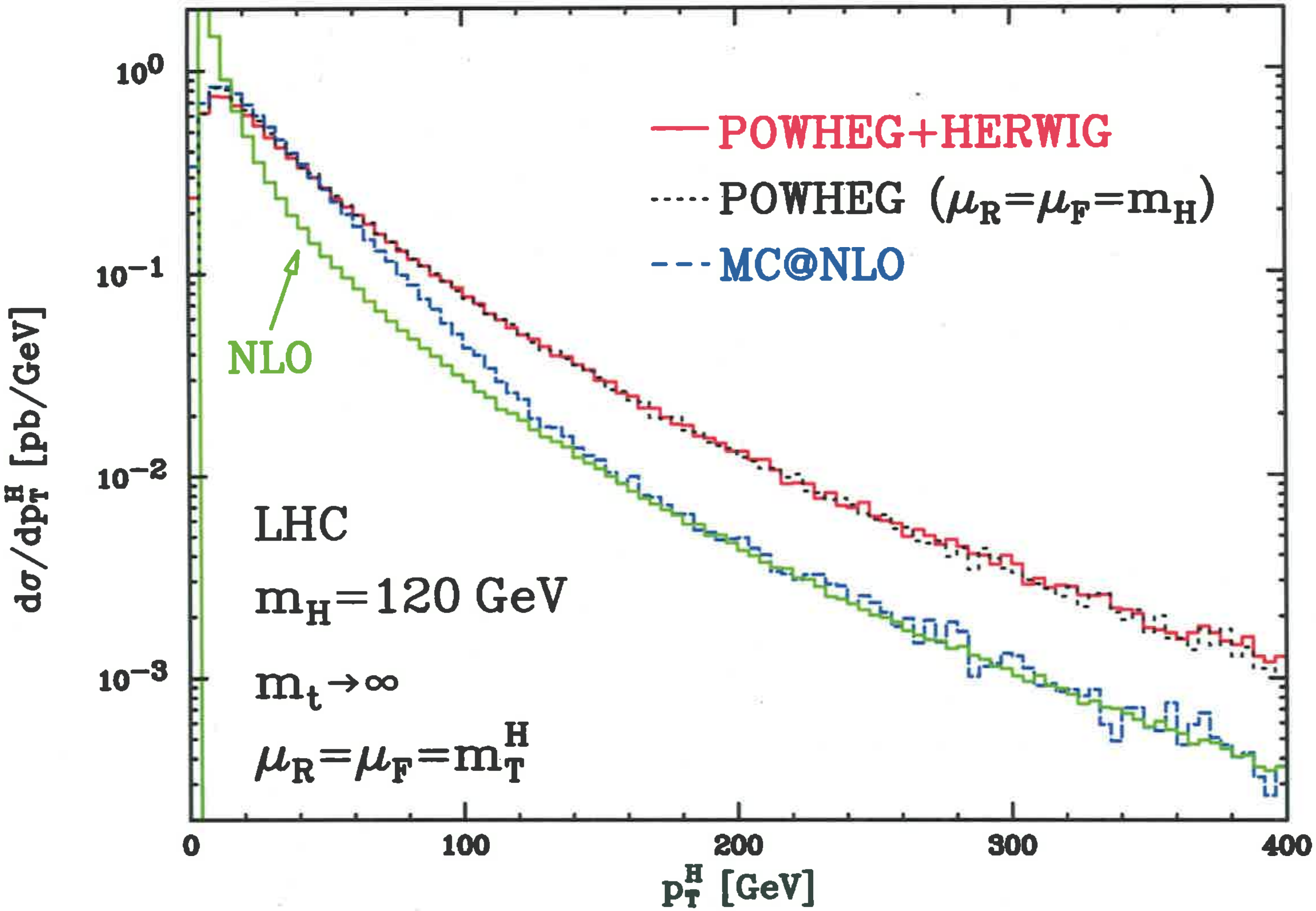
and then the matching is obtained as

$$\sigma^{(NLO)}_{MC@NLO} = \int d\phi_B \, \bar{B}'(\phi_B) \left[\bar{\Delta}'(Q_0^2, Q^2) + \int_{Q_0^2}^{Q^2} d\phi_1 \frac{R_S}{B} \bar{\Delta}'(q^2, Q^2) \right] + \int d\phi_R R_F(\phi_R)$$

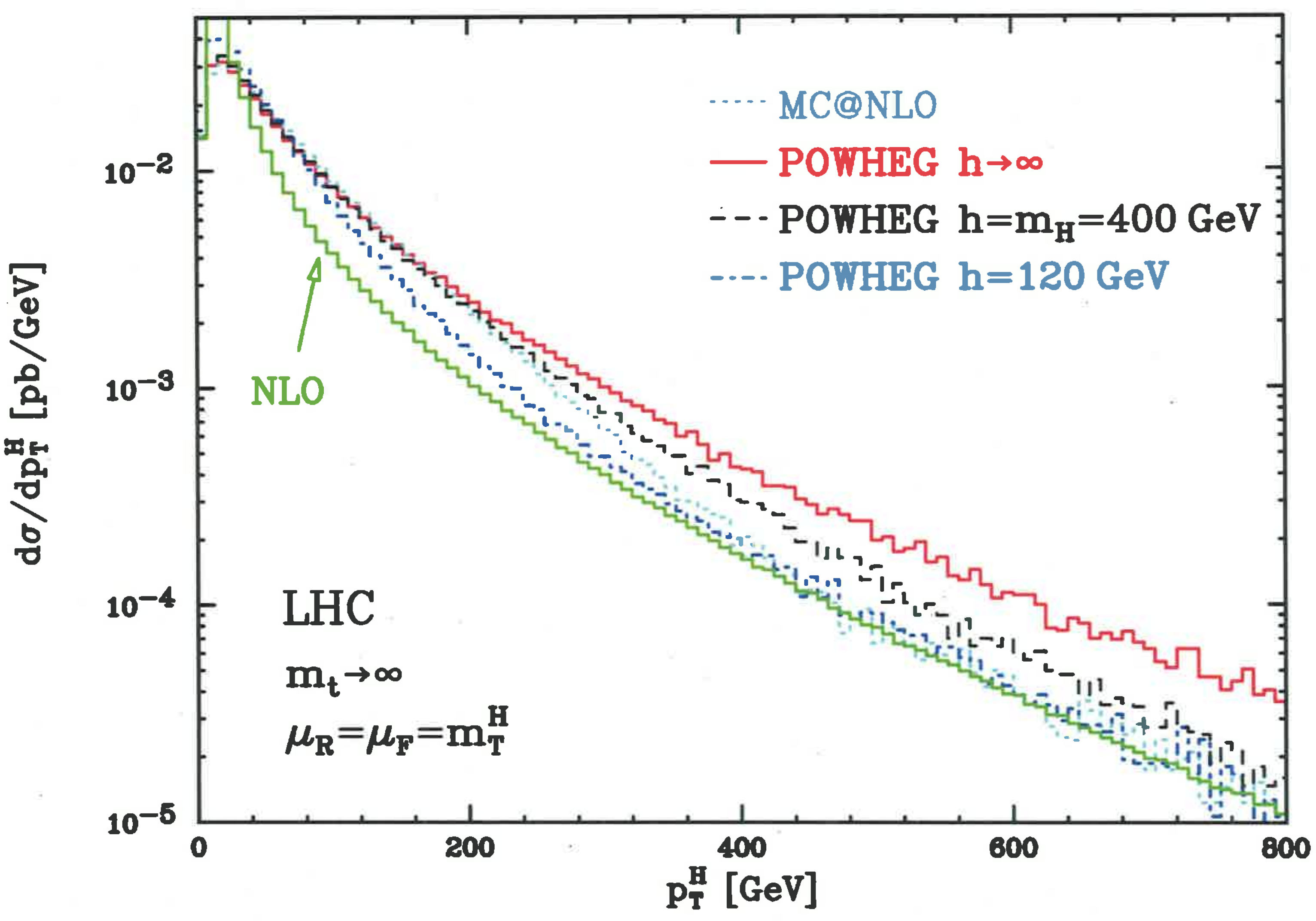
where $\bar{B}'(\phi_B) = B(\phi_B) + V(\phi_B) + I^S(\phi_B) + \int d\phi_1 \left[R_S(\phi_R) - CT(\phi_R) \right]$

and $\bar{\Delta}'(Q_0^2, Q^2) = \exp\left\{ - \int_{Q_0^2}^{Q^2} d\phi_1 \frac{\alpha_s}{2\pi} \frac{R_S}{B} \right\}$

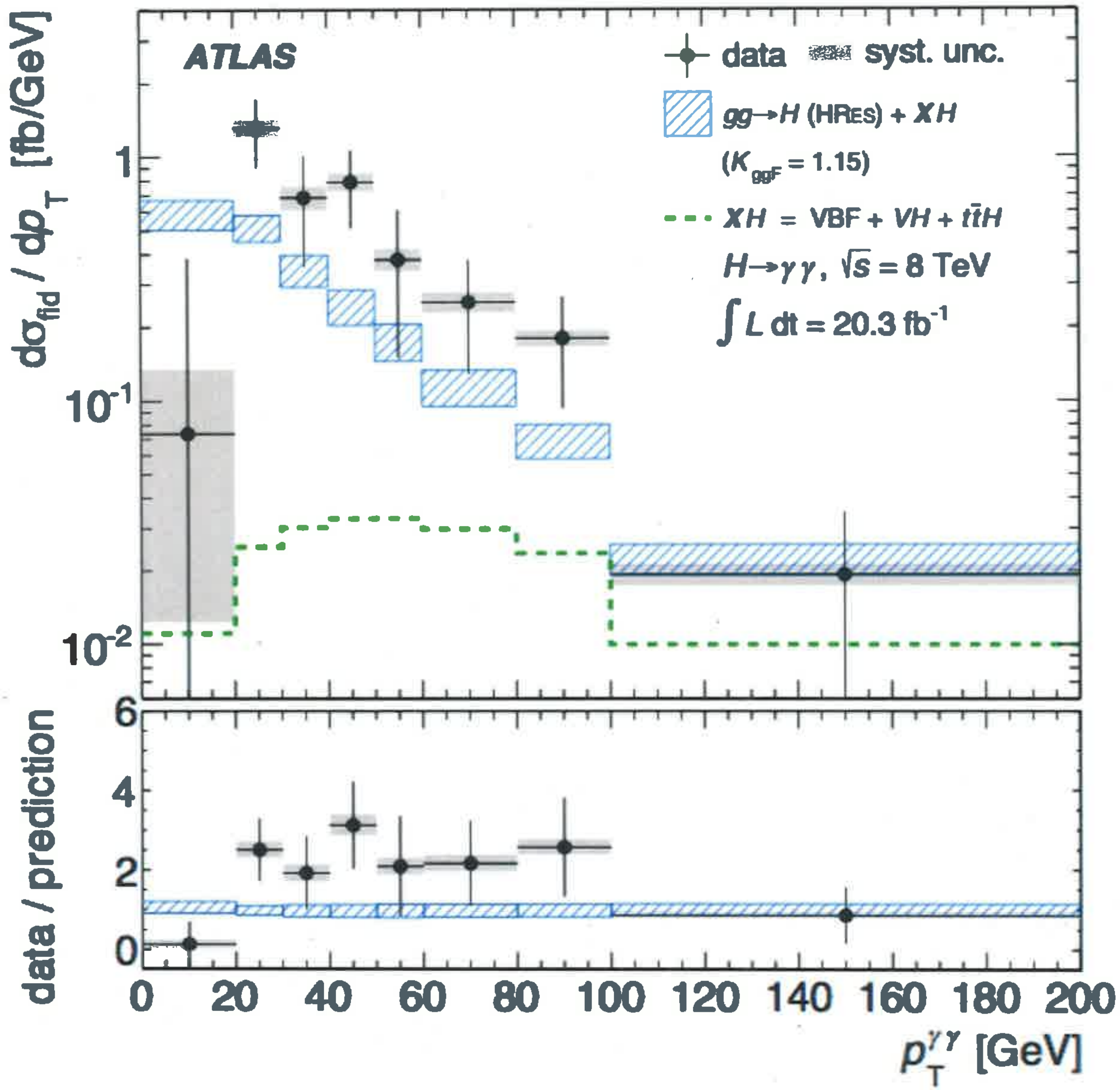
In this way only the singular (universal) part of the real matrix element is exponentiated. The term $\int d\phi_R R_F(\phi_R)$ may lead to negative weights.



POWHEG+HERWIG
POWHEG ($\mu_R=\mu_F=m_H$)
MC@NLO
NLO
LHC
$m_H=120$ GeV
$m_t\to\infty$
$\mu_R=\mu_F=m_T^H$
$d\sigma/dp_T^H$ [pb/GeV]
p_T^H [GeV]
10^0
10^{-1}
10^{-2}
10^{-3}
0
100
200
300
400



$$R_S = \frac{h^2}{h^2 + p_T^2} R \qquad R_F = \frac{p_T^2}{h^2 + p_T^2} R$$


ATLAS
data
syst. unc.
gg→H (HRES) + XH
(K_ggF = 1.15)
XH = VBF + VH + tt̄H
H→γγ, √s = 8 TeV
∫L dt = 20.3 fb⁻¹
dσ_fid / dp_T [fb/GeV]
data / prediction
p_T^γγ [GeV]