Advanced Field Theory: Exercise Sheet 9

Due: Thursday, 8.5.2014

Exercise 13

a) Start from the Lagrangian of chiral perturbation theory at order p^2 as stated in the lecture:

$$\mathcal{L}_{\chi\mathrm{PT},p^2} = \frac{v^2}{4}\mathrm{tr}\,\left(D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U\right), \tag{1}$$

where

$$U = \exp(i\sqrt{2}\Phi/v), \qquad \Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \overline{K}^0 & -2\frac{\eta_8}{\sqrt{6}}. \end{pmatrix}. \tag{2}$$

Insert $D_\mu = \partial_\mu$, $\chi = 2BM$, $M = \mathrm{diag}(m_u, m_d, m_s)$ and expand the Lagrangian up to the second order in Φ. Up to a constant, you should find

$$\mathcal{L} = \frac{1}{2}\mathrm{tr}\,\left(\partial_\mu\Phi\partial^\mu\Phi\right) - \mathrm{tr}\,\left(2BM\Phi^2\right). \tag{3}$$

b) Write the Lagrangian in the form

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\pi^0)(\partial^\mu\pi^0) - \frac{1}{2}m^2_{\pi^0}(\pi^0)^2 + \frac{1}{2}(\partial_\mu\eta_8)(\partial^\mu\eta_8) - \frac{1}{2}m^2_{\eta_8}(\eta_8)^2 + (\partial_\mu\pi^+)(\partial^\mu\pi^-) - m^2_{\pi^+}\pi^+\pi^- \tag{4}$$

$$+ (\partial_\mu K^0)(\partial^\mu\overline{K}^0) - m^2_{K^0}K^0\overline{K}^0 + (\partial_\mu K^+)(\partial^\mu K^-) - m^2_{K^+}K^+K^- + \frac{2B}{\sqrt{3}}(m_d - m_u)(\pi^0\eta_8) \tag{5}$$

and identify the masses of the scalar particles appearing in the Lagrangian.

c) Verify the Gell-Mann–Okubo mass formula

$$4m_K^2 - 3m_\eta^2 - m_\pi^2 = 0 \tag{6}$$

and the Weinberg ratio of quark masses

$$\frac{2m_K^2 - m_\pi^2}{m_\pi^2} = \frac{2m_s}{m_d + m_u}, \tag{7}$$

where $m_\pi^2 \equiv \frac{1}{3}(m^2_{\pi^+} + m^2_{\pi^-} + m^2_{\pi^0})$, $m_K^2 \equiv \frac{1}{4}(m^2_{K^0} + m^2_{\overline{K}^0} + m^2_{K^-} + m^2_{K^+})$.