Advanced Field Theory: Exercise Sheet 6

Due: Thursday, 3.4.2014

Exercise 8 The decay width $\Gamma\left(\tau^+ \to \overline{\nu}_\tau \pi^+\right)$ can be obtained from the smileptonic pion decay $\pi^+(p) \to \mu^+(k)\nu_\mu(q)$ considered in Ex. 7.

a) Start from

$$\sum_{spins} |\mathcal{M}_{\pi^+\to\mu^+\nu_\mu}|^2 = 8G_F^2 f_\pi^2 \left(2(q\cdot p)(k\cdot p) - p^2(q\cdot k)\right) \tag{1}$$

and cross the lepton to the final state and the pion to the initial state via $k \to -k$ and $p \to -p$. Note that there is an additional overall factor of -1, as we have to cross a fermion line. Also, the τ is now in the initial state and hence we should average over the spins. Show that after exploiting the kinematics of the process, we can write

$$\frac{1}{2}\sum_{spins} |\mathcal{M}|^2 = 2G_F^2 f_\pi^2 m_\ell^4 \left(1 - \frac{m_\pi^2}{m_\ell^2}\right). \tag{2}$$

b) Show that after integration over the phasespace, we obtain

$$\Gamma\left(\tau^+ \to \overline{\nu}_\tau \pi^+\right) = \frac{1}{8\pi} G_F^2 f_\pi^2 m_\ell^3 \left(1 - \frac{m_\pi^2}{m_\ell^2}\right)^2. \tag{3}$$

Exercise 9 The ρ meson is an isospin triplet of massive spin 1 particles. To compute $\Gamma(\tau^+ \to \overline{\nu}_\tau \rho^+)$, we parametrize the matrix element of the vector current between the vacuum and the rho meson as

$$\langle 0|J^{\mu a}|\rho_\lambda^b(p)\rangle = g_\rho \varepsilon_\lambda^\mu e^{-ipx}, \tag{4}$$

where g_ρ is the rho decay constant, a and b are isospin indices and λ and p are the polarisation index and momentum of the rho meson. The matrix element can be written as

$$\mathcal{M} = 2G_F g_\rho \overline{v}(k)\not{\varepsilon}_\lambda(p) P_L v(q). \tag{5}$$

a) Show that

$$\langle|\mathcal{M}|^2\rangle = 2G_F^2 g_\rho^2 \mathrm{tr}\left((\not{k} - m_\ell)\gamma^\mu P_L \not{q}\gamma^\nu \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{m_\rho^2}\right)\right). \tag{6}$$

Hint: The polarisation sum for a massive vector particle is

$$\sum_\lambda \varepsilon_\lambda^\mu(p)\varepsilon_\lambda^{\nu*}(p) = -g^{\mu\nu} + \frac{p^\mu p^\nu}{m^2}. \tag{7}$$

b) Show that only two terms in the trace actually contribute and we can write

$$\langle|\mathcal{M}|^2\rangle = 2G_F^2 g_\rho^2 \mathrm{tr}\left(-\frac{1}{2}\not{k}\gamma^\mu \not{q}\gamma_\mu + \frac{1}{2m_\rho^2}\not{k}\not{p}\not{q}\not{p}\right). \tag{8}$$

c) Evaluate the trace to arrive at

$$\langle|\mathcal{M}|^2\rangle = 2G_F^2 g_\rho^2 \left(2(k\cdot q) + \frac{4(p\cdot q)(k\cdot p)}{m_\rho^2}\right) \tag{9}$$

$$= 2G_F^2 g_\rho^2 \frac{m_\ell^4}{m_\rho^2}\left(1-\frac{m_\rho^2}{m_\ell^2}\right)\left(1+2\frac{m_\rho^2}{m_\ell^2}\right), \tag{10}$$

i.e.

$$\Gamma(\tau^+ \to \overline{\nu}_\tau \rho^+) = \frac{1}{8\pi} G_F^2 g_\rho^2 \frac{m_\ell^3}{m_\rho^2}\left(1-\frac{m_\rho^2}{m_\ell^2}\right)^2\left(1+2\frac{m_\rho^2}{m_\ell^2}\right). \tag{11}$$