Advanced Field Theory: Exercise Sheet 4

Due: Thursday, 20.3.2014

Exercise 6 We will redo the computation of the triangle diagram in QED using Pauli-Villars regularization instead of dimensional regularization. This reguarization scheme works by replacing the divergent loop integral by the difference of the original loop integral (where a fermion of mass m circulates in the loop) and the same loop integral with a different fermion mass M:

$$I(\{p_i\}, m) \to I(\{p_i\}, m) - I(\{p_i\}, M). \tag{1}$$

In the end of the computation, M is removed by sending it to infinity, leaving a finite result.

Let $\Gamma^5_{\mu\nu\lambda}(p,k)$ denote the Fourier transform of $\langle 0|T j^5_\mu(z) j_\nu(y) j_\lambda(x)|0\rangle$, given by

$$\Gamma^5_{\mu\nu\lambda}(p,k,m) = i\int \frac{d^4\ell}{(2\pi)^4}\mathrm{tr}\left[\frac{1}{\not\ell - \not k - m}\gamma_\lambda \frac{1}{\not\ell - m}\gamma_\nu \frac{1}{\not\ell + \not p - m}\gamma_\mu\gamma_5\right] + (p,\nu) \leftrightarrow (k,\lambda)\,. \tag{2}$$

We denote by $\Gamma^5_{\mu\nu\lambda}(p,k,m,M)$ the regularized amplitude:

$$\Gamma^5_{\mu\nu\lambda}(p,k,m,M) = \Gamma^5_{\mu\nu\lambda}(p,k,m) - \Gamma^5_{\mu\nu\lambda}(p,k,M). \tag{3}$$

a) Convince yourself (by expanding the integrand in powers of $1/\ell^2$) that the linear divergence in Eq. (3) cancels out.

b) Show that

$$(p+k)^\mu \Gamma^5_{\mu\nu\lambda}(p,k,m,M) = 2m\Gamma^5_{\nu\lambda}(p,k,m) - 2M\Gamma^5_{\nu\lambda}(p,k,M), \tag{4}$$

where

$$\Gamma^5_{\nu\lambda}(p,k,m) = 2i\int \frac{d^4\ell}{(2\pi)^4}\mathrm{tr}\left[\frac{1}{\not\ell - \not k - m}\gamma_\lambda \frac{1}{\not\ell - m}\gamma_\nu \frac{1}{\not\ell + \not p - m}\gamma_5 - (m \to M)\right]. \tag{5}$$

Hint: Use $\not p + \not k = (\not\ell + \not p - m) - (\not\ell - \not k + m) + 2m$ and the fact that second rank pseudotensors depending on only one momentum have to vanish.

c) Show that the Pauli-Villars regularization scheme preserves vector current conservation, i.e.

$$p^\nu \Gamma^5_{\mu\nu\lambda}(p,k,m,M) = 0 = k^\lambda \Gamma^5_{\mu\nu\lambda}(p,k,m,M). \tag{6}$$

Hint: Use $\not p = (\not\ell + \not p - m) - (\not\ell - m)$ and the same argument as above. Note that the integrals are finite and thus shifting the integration variable is allowed.

d) Show that $\Gamma^5_{\nu\lambda}(p,k,m)$ can be written as

$$\Gamma^5_{\nu\lambda}(p,k,m) = 2\cdot 4mi\varepsilon^{\nu\lambda\rho\sigma}p_\rho k_\sigma F(p,k,m), \tag{7}$$

where

$$F(p,k,m) = i\int \frac{d^4\ell}{(2\pi)^4}\frac{1}{(\ell-k)^2 - m^2}\frac{1}{\ell^2 - m^2}\frac{1}{(\ell+p)^2 - m^2}. \tag{8}$$

e) Use the Feynman parameterization to show that

$$\lim_{M\to\infty} 2M\Gamma^5_{\nu\lambda}(p,k,M) = \frac{i}{2\pi^2}\varepsilon^{\nu\lambda\rho\sigma}p_\rho k_\sigma, \tag{9}$$

i.e.

$$(p+k)^\mu \Gamma^5_{\mu\nu\lambda}(p,k,m) = 2m\Gamma^5_{\nu\lambda}(p,k,m) - \frac{i}{2\pi^2}\varepsilon_{\nu\lambda\rho\sigma}p^\rho k^\sigma. \tag{10}$$

Setting $m = 0$, we recover the result derived in the lecture.