Advanced Field Theory: Exercise Sheet 2

Due: Thursday, 5. 3. 2014

Exercise 2 In dimensional regularization (DREG), the Clifford algebra of γ-matrices,

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\mathbb{1}, \tag{1}$$

is maintained and $\mathrm{tr}\,\mathbb{1} = 4$, but the indices are now d-dimensional:

$$g^\mu{}_\mu = g^{\mu\nu}g_{\mu\nu} \equiv d. \tag{2}$$

a) Prove the following contraction identities:

$$\gamma^\mu\gamma_\mu = d\,\mathbb{1} \tag{3}$$
$$\gamma^\mu \not{a}\gamma_\mu = -(d-2)\not{a} \tag{4}$$
$$\gamma^\mu \not{a}\not{b}\gamma_\mu = 4a\cdot b\,\mathbb{1} + (d-4)\not{a}\not{b} \tag{5}$$
$$\gamma^\mu \not{a}\not{b}\not{c}\gamma_\mu = -2\not{c}\not{b}\not{a} - (d-4)\not{a}\not{b}\not{c}. \tag{6}$$

In four dimensions, γ_5 can be defined by its two properties:

$$\mathrm{tr}\,(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5) = 4i\,\varepsilon_{\mu\nu\rho\sigma} \tag{7}$$
$$\{\gamma_\mu, \gamma_5\} = 0. \tag{8}$$

b) Show that those two properties cannot be maintained simultaneously in d dimensions.

Hint: Consider $\frac{1}{d}\mathrm{tr}\left(\gamma_\lambda\gamma^\lambda\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5\right)$ *and use Eq.* (8) *to show that the trace in Eq.* (7) *vanishes.*

Exercise 3

a) Prove the following identities:

$$\int \frac{d^dk}{(2\pi)^d}\frac{1}{(k^2-\Delta)^n} = \frac{(-1)^n i}{(4\pi)^{d/2}}\frac{\Gamma(n-d/2)}{\Gamma(n)}\Delta^{d/2-n} \tag{9}$$
$$\int \frac{d^dk}{(2\pi)^d}\frac{k^\mu k^\nu}{(k^2-\Delta)^n} = \frac{(-1)^{n-1} i}{(4\pi)^{d/2}}\frac{g^{\mu\nu}}{2}\frac{\Gamma(n-d/2-1)}{\Gamma(n)}\Delta^{1+d/2-n}. \tag{10}$$

b) Convince yourself that any integral with an odd number of loop momenta vanishes.

Hint: You will need to use a Wick rotation, the integral representation of the beta function

$$\int_0^1 dx\, x^a(1-x)^b = B(a+1, b+1) = \frac{\Gamma(a+1)\Gamma(b+1)}{\Gamma(a+b+2)} \tag{11}$$

and the integral over the d-dimensional solid angle

$$\int d\Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}. \tag{12}$$

Exercise 4 We consider a general triangle topology shown in Fig 1 in massless ϕ^3 theory.

a) Write down the expression for this diagram.

b) Use Feynman parameters and a momentum shift to combine the denominators.

c) Use Eq. (9) and integrate the Feynman parameters to obtain an expression for the diagram as a function of $s = (p_1 + p_2)^2$.

d) Writing $d = 6 - 2\varepsilon$, convince yourself that this expression has a pole in ε.

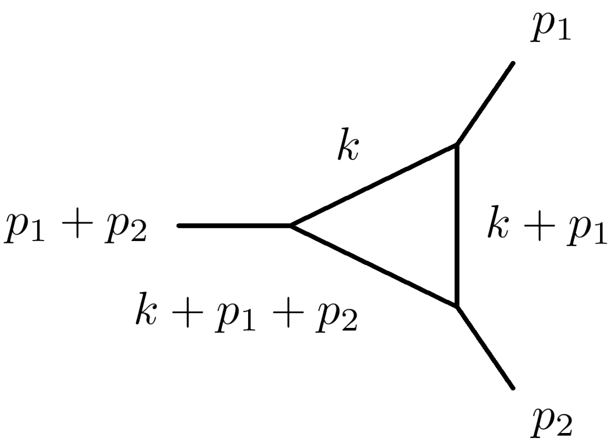

Figure 1: Triangle diagram.