Higgs Physics

Exercise Sheet 4

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

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Lecturers: M. Donega, M. Grazzini
Assistants: P. Musella, H. Sargsyan
https://moodle-app2.let.ethz.ch/course/view.php?id=1720

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Exercise 1 [*Application of likelihood methods in High Energy Physics*]

Likelihood methods are widely used in High Energy Physics. In this exercise we will see the application of such methods to parameters estimation and hypothesis testing.

The exercise can be solved using the ROOT analysis framework package, which can be downloaded from: https://root.cern.ch/content/release-60502

An example python script *LikelihoodMethods.py* (and the equivalent ipython notebook *LikelihoodMethods.ipynb*) showing how to use ROOT to solve the exercise is attached to the course website. Quick instructions to download ROOT and run the script are reported at the end of this document.

a) The first part of the exercise concerns the use of maximum likelihood method for parameters estimation. The goal of the exercise is to estimate the total number of events corresponding to a resonant signal appearing on top of a falling background.

- Construct the probability density function (pdf) of the signal process in the observable $x \in [0, 10]$ as a Gaussian distribution. Take 5 for the mean and 0.1 for the width.
- Construct the background pdf as a falling exponential, with a slope of -0.2.
- Assume that the expected number of events for the signal and background p processes are 80 and 10000 respectively.
- Generate a possible observation ("toy" or "pseudo-" experiment) from the above distributions, assuming that the total number of observed events follows a Poisson distribution.
- Assume that the signal model mean and width are perfectly known, while the total number of signal (n_{sig}) and background events (n_{bkg}), as well as the background decay length, need to be estimated from the observation. Construct the extended likelihood function for the data, given the signal + background model and use it to estimate the unknown parameters.
- Derive the profile likelihood function for n_{sig} and use it to derive an uncertainty on its maximum likelihood estimator $\hat{n}_{sig}$. In this case, the profile likelihood is defined

as

$$L_{n_{sig}}(n_{sig}) = sup_{(n_{bkg},\tau)} \quad L(n_{bkg}, \tau, n_{sig})$$

Where τ is the background exponential slope and $L(n_{bkg}, \tau, n_{sig})$ is the likelihood function.

- Now assume that the signal model width is known with a systematic uncertainty of 0.05. Modify the likelihood function to include the corresponding nuisance parameter and obtain a new estimate of n_{sig} and its uncertainty. Did the latter change compared to what you obtained in the previous point?

b) The second part of the exercise concerns the use of likelihood ratios in hypothesis tests. The goal of the exercise is to construct the test statistics corresponding to a simple test of the background-only vs signal+background hypothesis and to extract exclusion limits using the CL_s prescription.

- Take the signal and background model from the first part of the exercise. Assume that all signal and background parameters are known. Fixed them to the values given above, but assume that $n_{sig} = 40$ (and not 80).
- Generate 5000 "pseudo-experiments" for each of the two hypotheses being tested.
- For each of the pseudo-experiments evaluate the following test statistics:

$$-lnQ = -ln\frac{L(data|s+b)}{L(data|b)}$$

- Histogram the obtained sampling distributions and use them to estimate the median expected CL_s value in the absence of a signal.
- Draw a pseudo-experiment from the background-only hypothesis and take it as the outcome of your experiment. With which CL_s can you exclude the signal based on this observation?
- Now repeat the exercise assuming that the signal width is 0.2 and 0.05. How does the median expected CL_s change?

Quick start instructions to install the needed software

ROOT website https://root.cern.ch/content/release-60502

Optional: to install IPython notebook, go to http://ipython.org/. You will need the file root-notes.py to see ROOT canvases in the notebook https://gist.github.com/mazurov/6194738 in this case.

```
mkdir nll_and_hyptest
cd nll_and_hyptest

TARBALL=root_v6.05.02.Linux-ubuntu14-x86_64-gcc4.8.tar.gz
## for MacOS uncomment the following line
```

```
# TARBALL=root_v6.05.02.macosx64-10.10-clang61.tar.gz

wget https://root.cern.ch/download/${TARBALL}

tar zxf ${TARBALL}
source root/bin/thisroot.sh

## to install ipython notebook uncomment the following lines
# pip install --user -U jupyter
# export PATH=PATH=${PATH}:~/.local/bin/
# wget https://gist.githubusercontent.com/mazurov/6194738/raw/23189b976931db88a4cae29374a9f9e
# jupyther notebook
## then open the notebook LikelihoodMethods.ipynb

## otherwise just run
python -i LikelihoodMethods.py
```