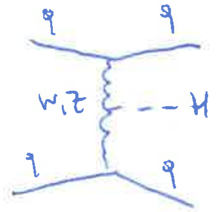
# WEAK BOSON FUSION (VBF)




Weak boson fusion is the second important production channel of the SM Higgs boson at the LHC, and it has several distinctive features. It has the largest cross section among the processes that occur at tree-level to lowest order, and it has a distinctive signature of two forward jets, which makes it possible to tag them and to study Higgs boson decays that are normally difficult to be isolated, like $H \to \tau\tau$.

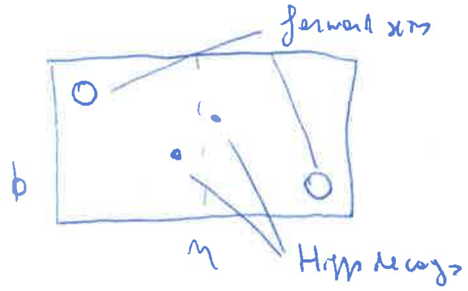
The process occurs through the scattering of two quarks, that exchange a W or a Z boson, which in turn radiates the Higgs boson. When we pick up two valence quarks from the proton their PDF is peaked at $x \sim 0.1-0.2$. The outgoing quarks, after exchanging the vector boson will have a transverse momentum of the order of a fraction of the vector boson mass. This means that this process tends to produce two highly energetic jets with a large rapidity interval between them.

This conclusion can also be reached by noting that if $P_i$ and $P_j$ are the momenta of the incoming and outgoing quarks say in the upper part of the diagram, there is a propagator factor $\frac{1}{Q^2 - m^2_V}$ for the vector boson with $Q^2 = (P_i - P_j)^2 < 0$.

The suppression is smaller if $Q^2$ is smaller but

$$Q^2 = (P_i - P_j)^2 = -2P_iP_j = -2E_iE_j(1-\cos\theta) \approx -E_i^2(1-\varepsilon)\theta^2$$

where $\varepsilon$ is the fraction of the quark momentum carried away by the vector boson. We conclude that the amplitude is larger when $\theta$ is smaller, and that the two outgoing quarks end up in the forward region of the detector, whereas the Higgs decay products will generally go central



Another distinctive feature of VBF is QCD radiation.

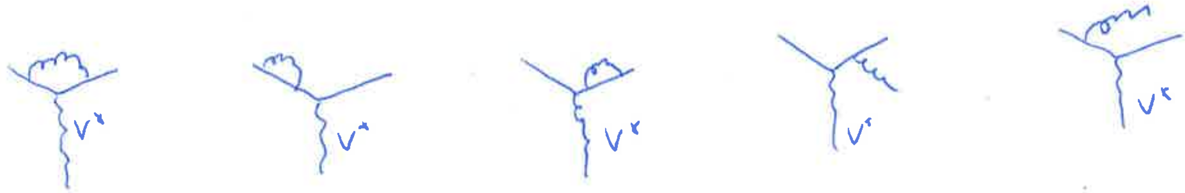
Beyond the lowest order the additional jet activity comes from radiations off the quarks, and is thus typically forward. By contrast, the major backgrounds, like $t\bar{t}$, will involve QCD radiation between the incoming partons, and thus the central region is usually covered. Central jet activity can be vetoed, thus leading to a good background rejection.

Radiative corrections to VBF are known at the NLO (QCD+EW) and NNLO QCD (neglecting color connections between the scattered quarks), and are generally small, thus this channel is a very good candidate for precision studies.

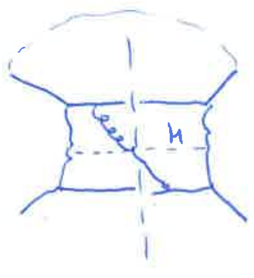
One important point is that the genuine VBF process is in principle not unambiguously defined, since one has to consider also the gluon fusion contribution to H+2 jets.

Nevertheless there are variables which are sensitive to the production mechanism. The first is the azimuthal separation of the tagging jets, which has a rather different shape in ggF and VBF, and the other is the rapidity of the third jet with respect to the average of the two tagging jets (the "Zeppenfeld variable"), which, due to the very peculiar radiation pattern in VBF, is completely different from what we see in H+2 jets from gluon fusion.

We now discuss the computation of QCD radiative corrections at NLO. They consist of virtual and real emission diagrams

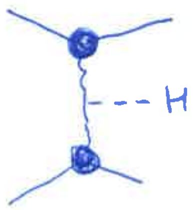


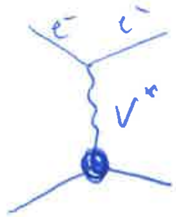

however there are no diagrams with gluons connecting the upper and lower part




a diagram like this would in fact vanish due to <u>color conservation</u>

This implies that NLO QCD corrections for VBF can be obtained by using the STRUCTURE FUNCTION approach, in which one can directly apply the well known results from Deep Inelastic Scattering




The NLO calculation gives corrections of O(5%) which are partially compensated by the EW corrections. NLO QCD+EW calculations are available in the VBFNLO and HAWK programs

In summary, the VBF process has a smaller cross section than ggF, but it provides a rather clean environment, with theoretical predictions under very good control (Recently also NNLO corrections have been computed in the structure function approach: the effect is at the 1% level).

VBF can thus be used for precision studies of the Higgs boson properties when enough luminosity will be accumulated.

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"ZEPPENFELD VARIABLE"

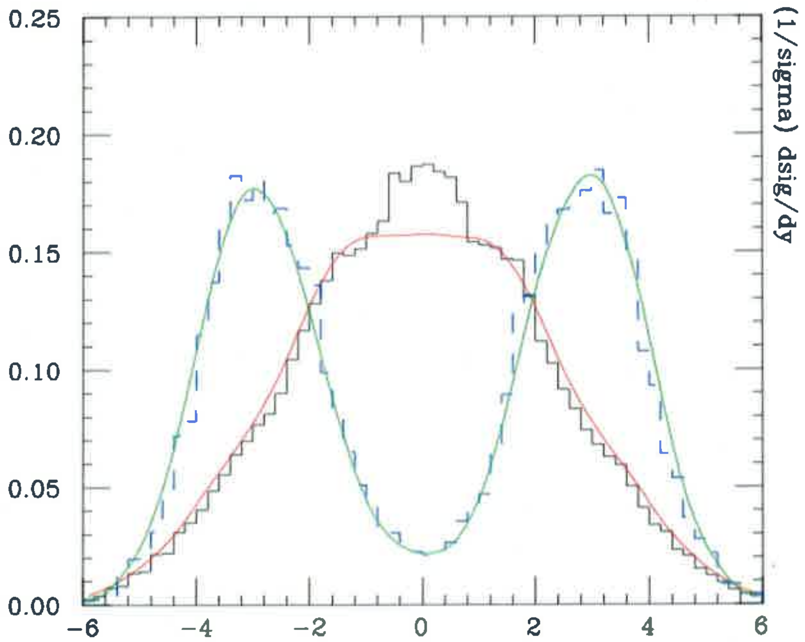


**Figure 7:** Normalised distribution of the rapidity of the third jet, measured with respect to the rapidity average of the two tagging jets in Higgs + 3 parton events after the parton shower, via gluon fusion (solid) and via VBF (dashed histogram). Also shown are the pure parton level expectations, generated with Higgs + 3 parton matrix elements, for gluon fusion (red curve) and VBF (green curve).

$y_3 - \frac{y_1+y_2}{2}$

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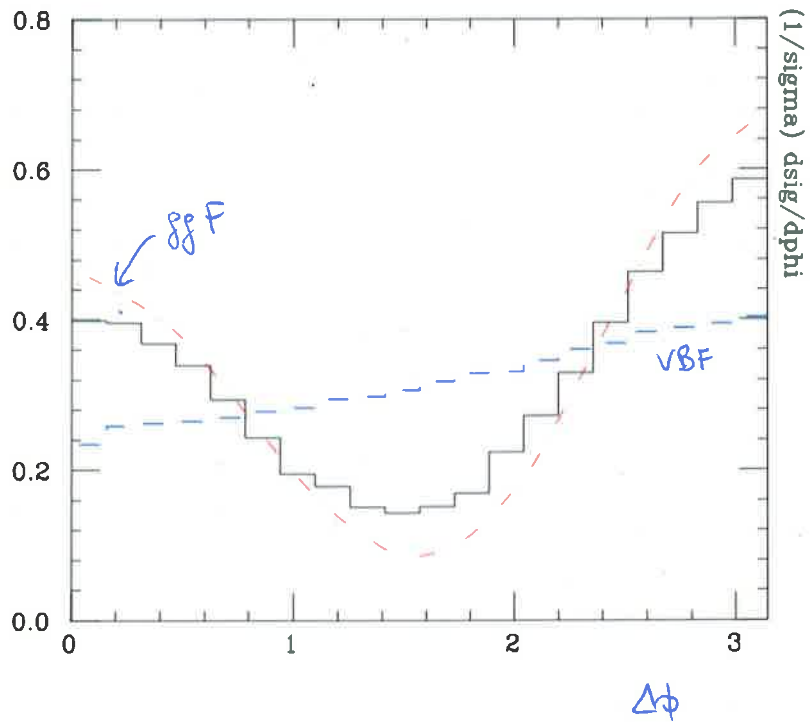
AZIMUTHAL SEPARATION OF THE TAGGING JETS




The matrix element squared $|M_{q\bar{q}\to qqH}|^2$ in the forward scattering region is proportional to $m_{jj}^2$ and it is thus roughly independent on $\Delta\phi$ $\Rightarrow$ the $\Delta\phi$ distribution for VBF is thus rather flat.

In the case of gg fusion, the matrix element squared becomes proportional to $\underline{p_{T,1}} \cdot \underline{p_{T,2}}$

$\Rightarrow$ The cross section is strongly suppressed at $\Delta\phi \sim \frac{\pi}{2}$

see hep-ph/0105325

## VH

The associated production of the Higgs boson with a vector boson is the third channel as far as the cross section is concerned at the LHC, while it was the most important channel at the Tevatron, where its cross section is larger than the VBF cross section for a light Higgs. This is due to the fact that in $p\bar{p}$ collisions $q\bar{q} \to VH$ can proceed through two valence quarks, which carry more momentum with respect to the sea quarks on average.

At the lowest order VH production ($V = W^{\pm}, Z$) proceeds as a Drell-Yan process, where the off shell vector boson $V^*$ radiates the Higgs boson.

The inclusive cross section can be obtained from the $e^+e^- \to VH$ cross section by using the appropriate couplings to the quarks and adding a factor $\frac{1}{3}$ to average over the number of colors:

$$\hat{\sigma}_{LO}^{VH}(\hat{s}) = \frac{G_F^2 m_V^4}{288\pi \hat{s}^2} (v_q^2 + a_q^2)\, \lambda^{1/2}(m_V^2, m_H^2, \hat{s})\, \frac{\lambda(m_V^2, m_H^2, \hat{s}) + 12 \hat{s} m_V^2}{(\hat{s} - m_V^2)^2}$$

The total hadronic cross section is obtained through the factorization theorem as

$$\sigma_{LO}^{VH} = \sum_{a,b = q_j, \bar{q}_j'} \int_0^1 dx_1 dx_2\, f_a(x_1, M_F^2)\, f_b(x_1, M_F^2)\, \hat{\sigma}_{LO\,ab}^{VH}(x_1 x_2 s)$$

The cross section with WH final states ($W = W^+ + W^-$) is typically a factor of 2 larger than the one with ZH final states.

Since the VH final state is colourless, the NLO corrections to VH production are identical to those of the Drell-Yan process

By writing

$$\sigma^{VH} = \sum_{a,b} \int_0^1 dx_1 dx_2 dz\, f_a(x_1,\mu_F^2)\, f_b(x_2,\mu_F^2)\, \hat{\sigma}_{ab}^{VH}(z, \alpha_s(\mu_R^2), M_R, M_F)$$

we have

$$\hat{\sigma}_{ab} = \sum_{c,d=q,g,\bar{q},g} \hat{\sigma}_{LO\,c,d}(x_1 x_2 z s) \left( \delta(1-z)\,\delta_{ac}\delta_{bd} + \frac{\alpha_s(\mu_R)}{\pi}\, g^{(1)}_{ab}(z; M_F, M_R) + \dots \right)$$

where

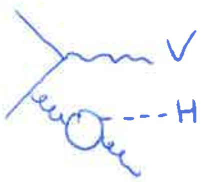
$$g^{(1)}_{q\bar{q}} = P_{qq}(z) \log \frac{M^2}{\mu_F^2} + C_F \left[ \left( \frac{\pi^2}{3} - 4 \right) \delta(1-z) + 2(1+z^2)\, D_1(z) \right]$$

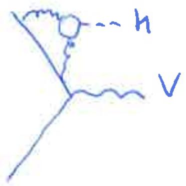
$$g^{(1)}_{gq} = \frac{1}{2} P_{qg}(z) \log \frac{(1-z)^2 M^2}{\mu_F^2} + \frac{1}{8}\left(1 + 6z - 7z^2\right)$$

$$g^{(1)}_{gg} = 0 \qquad P_{qq} = C_F \left( \frac{1+z^2}{1-z} \right)_+ \qquad P_{qg} = \frac{1}{2}\left(z^2 + (1-z)^2\right)$$

The above coefficient functions, which are the same entering in the NLO corrections to the Drell-Yan process, are completely analogous to the corrections we have evaluated for $gg \to H$ (just in the $q\bar{q}$ channel instead of the $gg$ channel) $K_{NLO} \sim 1.3$

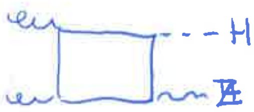
At NNLO, besides the corrections of Drell-Yan type, one has to consider additional diagrams mediated by a heavy quark loop




These diagrams contribute to the 1-3% level.

When $V=Z$ there are other non Drell-Yan like diagrams induced by $gg$ fusion




These contributions are at the few percent level on the total cross section.

At the Tevatron the VH channel was the most important for a light Higgs, because the lepton(s) from the V decay provides the necessary background rejection, and because the $H \to \gamma\gamma$ decay mode offers too small a rate to be observed.
At the LHC the situation is different, and this channel has been considered for many years too difficult to be observed, due to the large backgrounds.
The $t\bar{t}$ background can easily mimic a $Wb\bar{b}$ signal if one of the W from the top decay go in the beam direction. Moreover, in the decay of a top quark at rest ($t \to Wb$) the energy of the b quark is about 65 GeV, and thus rather close to $m_H/2$. Another issue is that the relatively small typical invariant mass of the VH pair ($m_{HV} \gtrsim 200$ GeV) implies that the system is produced at medium rapidities (Remember that $Y_{max} = \frac{1}{2} \ln \frac{M^2}{s}$ !), thus a certain fraction of events is lost because the V and H decay products go beyond the rapidity coverage of the detector. The idea proposed by Butterworth et al. (2008) was to study the process at high transverse momenta. In this region the cross section (e.g. for $p_{TH} > 200$ GeV) is only about 5% of the inclusive cross section, but there are several advantages. The high $p_T$ implies that $m_{HV}$ is large, and thus the HV system is more central. The V and H decay products will have sufficiently large $p_T$ to be tagged. The background is under better control because, for example, it is impossible for a $t\bar{t}$ event to produce a high $p_T$ $b\bar{b}$ system and a leptonically decaying W without a jet recoiling. Moreover, a high $p_T$ Higgs decaying into a $b\bar{b}$ pair gives two b-jets which are rather collimated (fat jet). This proposal has essentially resurrected the VH channel at the LHC.

# HIGGS PRODUCTION AND DECAY

The Higgs boson can only be detected through its decay products. Being the SM Higgs a scalar particle, and with a rather small width ($\Gamma \sim 4\,MeV$), one can treat to a very good approximation the production and decay mechanisms as factorized. Indeed we can write

$$d\sigma(I \to H \to F) = \frac{1}{2s} |\mathcal{M}(I \to H)|^2 \frac{1}{(s - m_H^2)^2 + \Gamma_H^2 m_H^2} |\mathcal{M}(H \to F)|^2 \, d\Phi_F$$

In the narrow width approximation we can write

$$\frac{1}{(s-m_H^2)^2 + \Gamma_H^2 m_H^2} \simeq \frac{\pi}{\Gamma_H m_H} \delta(s - m_H^2)$$

[the constant is fixed by imposing that the normalization is the same

$$\int_{-\infty}^{+\infty} \frac{1}{(s-m_H^2)^2 + \Gamma_H^2 m_H^2} = \frac{\pi}{\Gamma_H m_H}$$

We can thus write

$$\sigma(I \to H \to F) = \frac{1}{2s} \int |\mathcal{M}(I \to H)|^2 \frac{\pi}{\Gamma_H m_H} |\mathcal{M}(H \to F)|^2 \delta(s - m_H^2) \, d\Phi_F$$

$$= \frac{1}{2s\,\Gamma_H} \int |\mathcal{M}(I \to H)|^2 \, 2\pi \, \delta(s - m_H^2) \int \frac{1}{2m_H} |\mathcal{M}(H \to F)|^2 d\Phi_F$$

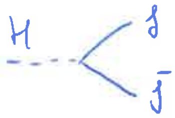
$$= \frac{1}{\Gamma_H} \sigma(I \to H) \, \Gamma(H \to F) = \sigma(I \to H) \, Br(H \to F)$$

where we have defined the BRANCHING RATIO $\quad Br(H \to F) = \dfrac{\Gamma(H \to F)}{\Gamma_H}$

# HIGGS BOSON DECAYS

As discussed at the beginning of this course, in the SM, once the Higgs boson mass is fixed, its profile is uniquely determined. The couplings to the gauge bosons and the fermions, in particular, are uniquely determined, and are proportional to their masses. As such, the Higgs decays into the heaviest SM particles allowed by kinematics. The case $m_H \simeq 125$ GeV is particularly interesting, because it will allow to study many different (and interesting) decay modes.

## $H \to f\bar{f}$

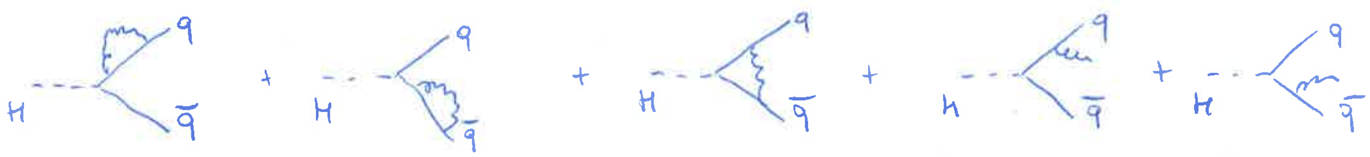


The decay into a fermion pair is a simple two-body decay and the corresponding width is

$$\Gamma = \frac{G_F}{4\sqrt{2}\pi}\, m_H\, m_f^2\, \beta_f^3 \qquad\qquad \beta_f = \sqrt{1-\frac{4m_f^2}{m_H^2}}$$

In the case in which the fermion is a quark this result has to be multiplied by a factor of 3. The quantity $\beta_f$ is the fermion velocity, and it vanishes in the threshold limit. The suppression factor $\beta_f^3$ is typical of a scalar particle (for a pseudoscalar the width would be proportional to $\beta$). We now focus on $f=q$.

The QCD corrections involve as usual virtual and real diagrams



It is interesting to consider the limit $m_q \to 0$. According to the KLN theorem one expects that, at each order in perturbation theory, there are no collinear singularities in the UNRENORMALIZED decay rate. However the collinear singularity is reintroduced through the renormalization procedure. In a spontaneously broken gauge theory in which the fermion acquires mass from the Higgs mechanism, the mass of the fermion and its coupling to the Higgs boson cannot be renormalized independently [Braaten-Leveille, 1980]

The collinear singularity is thus reintroduced through the renormalization procedure.

One finds $$\Gamma_{NLO} \approx \Gamma_{LO}\left(1 + C_F \frac{\alpha_s}{\pi}\left(\frac{9}{4} + \frac{3}{2}\log\frac{m_q^2}{m_H^2}\right)\right)$$

The limit $m_q \to 0$ is still smooth (due to the overall Yukawa factor present in $\Gamma_{LO}$) but at small $m_q$ the large logarithmic term can be important and can turn the width to negative values. However the large logarithm can be absorbed in the redefinition of the quark mass $m_q \to m_q(m_H)$. After that the effect of NLO corrections is to increase the LO rate by about 20%.

The QCD corrections are now known to $O(\alpha_s^4)$ and can be written as [Baikov et al 2005]

$$\Gamma(H \to q\bar{q}) = \frac{3 G_F m_H}{4\sqrt{2}\pi} \, m_q^2(m_H) \, \tilde{R}$$

where

$$\tilde{R} = 1 + 5.6666\frac{\alpha_s}{\pi} + (35.94 - 1.359\, n_F)\left(\frac{\alpha_s}{\pi}\right)^2 + (164.14 - 25.77\, n_F + 0.259\, n_F^2)(\alpha_s/\pi)^3$$
$$+ (39.34 - 220.9\, n_F + 9.685\, n_F^2 - 0.0205\, n_F^3)\left(\frac{\alpha_s}{\pi}\right)^4$$

## $H \to WW, ZZ$


The width for the Higgs decay into on-shell vector bosons is

$$\Gamma(H \to ZZ) = \frac{G_F m_H^3}{16\sqrt{2}\pi}\sqrt{1 - x_Z}\left(1 - x_Z + \frac{3}{4}x_Z^2\right) \qquad x_V = \frac{4 m_V^2}{m_H^2}$$

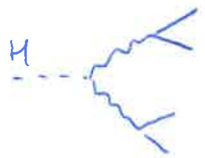
$$\Gamma(H \to WW) = \frac{G_F m_H^3}{8\sqrt{2}\pi}\sqrt{1 - x_W}\left(1 - x_W + \frac{3}{4}x_W^2\right)$$

The leading term as $x_V \to 0$ (large Higgs mass) goes like $m_H^3$ $\Rightarrow$ that's why a heavy Higgs boson becomes obese! This is a consequence of the fact that the decay is dominated by longitudinally polarized gauge bosons.

One finds $$\frac{\Gamma(H\to V_T V_T)}{\Gamma(H\to V_L V_L)} = \frac{\frac{1}{2}x_V^2}{(1-x_V/2)^2}$$

At high $m_H$, when the phase space factors can be ignored, one simply has $\Gamma_{WW}\sim 2\Gamma_{ZZ}$

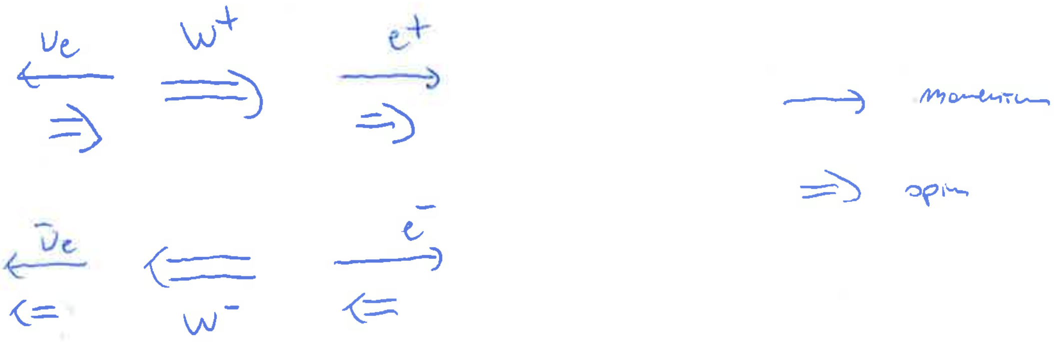
In the region $m_H \sim 125$ GeV we are clearly below the WW and ZZ threshold and one has to consider the full 4 fermion decay. NLO QCD+EW corrections to this decay process have been computed, and are included in the PROPHECY4f program.




The decays $H\to WW^*\to \ell\nu\ell\nu$ and $H\to ZZ^*\to 4\ell$ are characterized by a definite pattern of spin and angular correlations that allow to discriminate the spin/CP properties of the Higgs boson.

For the moment we consider the $H\to WW\to \ell\nu\ell\nu$ decay and assume $m_H \sim 2m_W$.

In this case the scalar nature of the Higgs, together with the V-A decay structure of the W bosons allow us to conclude that the two charged leptons must be very close in angle




as a consequence, a boost invariant quantity which turns out to be very sensitive to this behavior is the $\Delta\phi_{ll}$ : azimuthal separation of the charged leptons in the transverse plane. The potential of spin correlations in the $H\to WW\to \ell\nu\ell\nu$ decay mode was already pointed out by C. Nelson [1988]

The first realistic study using $\Delta\phi_{ll}$ was carried out by Dittmar-Dreiner [1996]

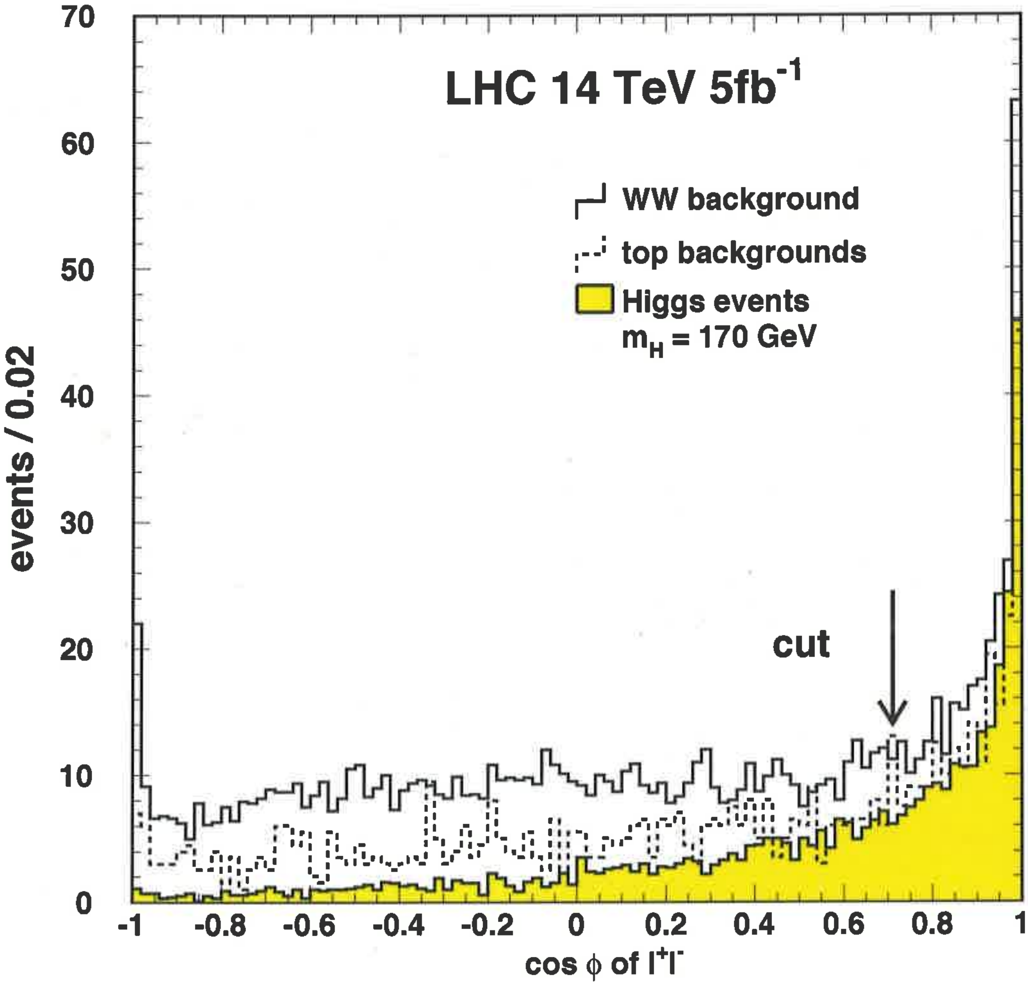
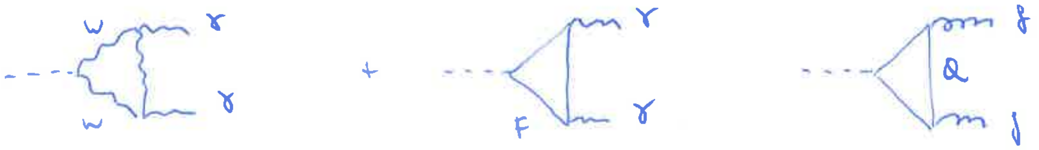



Figure 2: $\cos\phi$ distribution of the dilepton system in the plane transverse to the beam direction for Higgs signal and background events, cut number 6 has not yet been applied.

## Loop-induced decays

The Higgs boson does not directly couple to $\gamma\gamma$ and $gg$. However the $H\gamma\gamma$, $Hgg$ and also $HZ\gamma$ vertices are generated at quantum level.




The contribution of heavy particles in the triangular loops does not decouple

(the conditions of the DECOUPLING THEOREM are not satisfied since the couplings with the Higgs grow with the masses)

$\Rightarrow$ this implies that these decays are very interesting as they are sensitive to scales well above $m_H$

- $H \to \gamma\gamma$

The $H \to \gamma\gamma$ decay is sensitive to both colored and colorless particles in the loop. The corresponding width can be written as

$$\Gamma(H\to\gamma\gamma) = \frac{G_F\,\alpha^2 m_H^3}{128\sqrt{2}\,\pi^3}\left|\sum_q 2N_c Q_q^2 A_{1/2}(\tau_q) + A_1(\tau_W)\right|^2$$

where we have considered only the contributions of heavy-quarks and of the W boson.

The function $A_{1/2}(\tau_q)$ is the same encountered in $gg\to H$ and has the form

$$A_{1/2}(\tau_q) = \tau_q\left(1+(1-\tau_q)f(\tau_q)\right) \qquad \tau_q = \frac{4m_q^2}{m_H^2}$$

$$f(\tau_q) = \begin{cases} \arcsin^2\sqrt{\dfrac{1}{\tau_q}} & \tau_q \geq 1 \\ -\dfrac{1}{4}\left[\log\dfrac{1+\sqrt{1-\tau_q}}{1-\sqrt{1-\tau_q}} - i\pi\right]^2 & \tau_q < 1 \end{cases}$$

The function $A_1(\tau_W)$ is instead

$$A_1(\tau_W) = -\left[2 + 3\tau_W\left(1 + (2-\tau_W) f(\tau_W)\right)\right] \qquad \tau_W = \frac{4m_W^2}{m_H^2}$$

The contribution of the top quark interferes destructively with the one of the W boson.

For $m_H \sim 125$ GeV the W contribution is about 4.5 times larger than the top one.

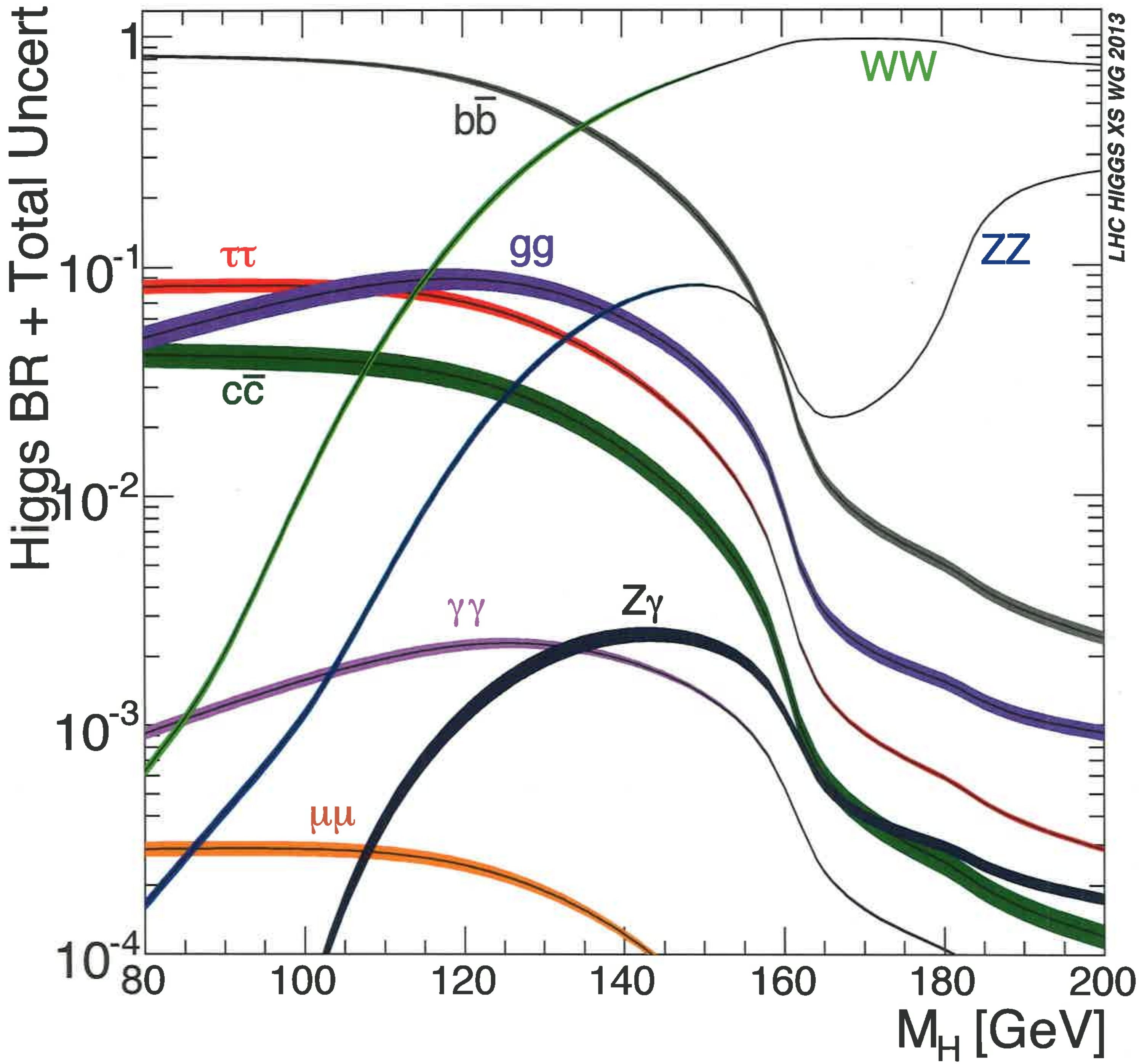
The NLO corrections to the $H\to\gamma\gamma$ width consist only of virtual corrections ($H\to\gamma\gamma+g$ is forbidden by color conservation). For $m_H = 125$ GeV their effect is completely cancelled by electroweak corrections. The QCD corrections have been evaluated up to 3 loop order but the complete effect (QCD+EW) is well below 1%

The $H\to\gamma\gamma$ decay mode is a discovery mode at the LHC due to the very clean signature. However the corresponding branching ratio is extremely small ($BR\sim10^{-3}$)

- $H\to Z\gamma$


The $H\to Z\gamma$ decay mode is less important than $\gamma\gamma$. It provides a comparable rate with respect to $\gamma\gamma$ but pays the additional suppression of the $Z\to \ell^+\ell^-$ decay mode, and will thus be important only at high luminosity.

The NLO QCD corrections are known and are below 1%.


Higgs BR + Total Uncert
1
10^-1
10^-2
10^-3
10^-4
80
100
120
140
160
180
200
M_H [GeV]
LHC HIGGS XS WG 2013
bb̄
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