A synthetic model of the gravitational wave background from evolving binary compact objects

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[arXiv:1607.06818]

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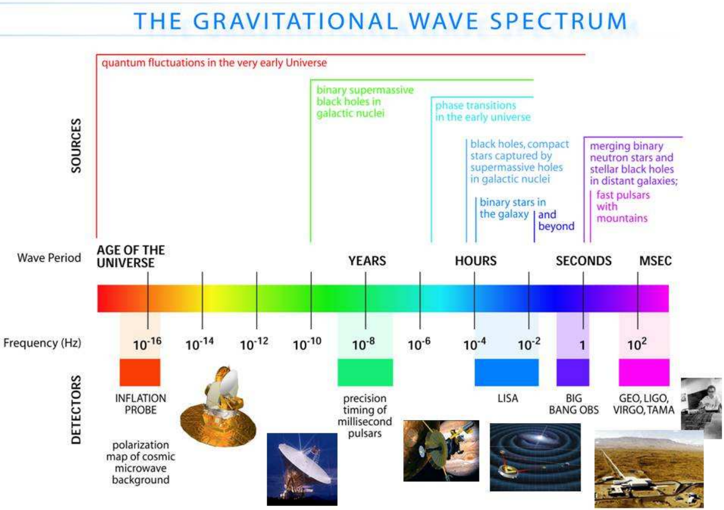
Institut d'astrophysique de Paris

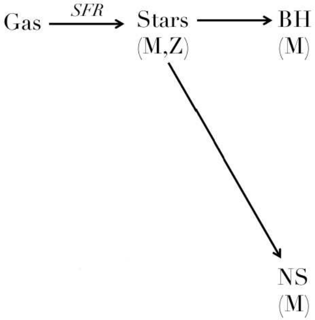
Sources of Gravitational Waves



Model framework

$$\text{Gas} \xrightarrow{SFR} \underset{\text{(M,Z)}}{\text{Stars}}$$

Model framework



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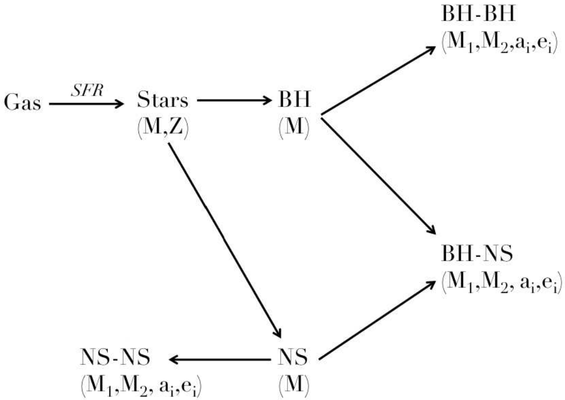

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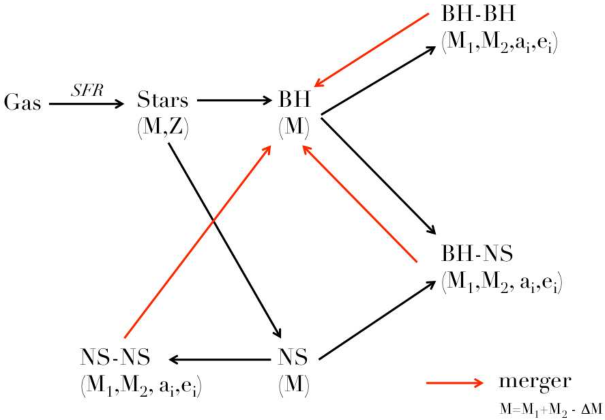

BH binaries number density (simple case)

If all BH are in binaries, and all merger products remain single, the number density n_X of binaries in a certain mass M and orbital parameters bin is set by: [where $\mathbf{w} = (a, e)$]

- The formation rate of BH (determined from stellar physics) $R_X(M, t)$
- The initial distribution of orbital parameters $\mathcal{P}_X(\mathbf{w})$
- The evolution in time of the orbital parameters of the binary $\mathrm{d}\mathbf{w}/\mathrm{d}t$

Evolution of the orbital parameters

General case ($\mathbf{w} = (a, e)$):

$$\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}t} = \mathbf{f}(\mathbf{w}, M)$$

A merger occurs when $\mathbf{w} = \mathbf{w}_{\mathrm{merger}}$

Evolution of the orbital parameters

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Example: evolution due to emission of GW [Peters & Mathews (1963)]

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -\frac{64}{5}\frac{G^3 \mu m^2}{c^5 a^3}\frac{\left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right)}{(1 - e^2)^{7/2}}$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = -\frac{304}{15}\frac{G^3 \mu m^2}{c^5 a^4}\frac{e\left(1 + \frac{121}{304}e^2\right)}{(1 - e^2)^{5/2}}$$

Continuity equation

Hydrodynamics (matter density ρ, coordinate x, velocity $u = \mathrm{d}x/\mathrm{d}t$):

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \frac{\mathrm{d}}{\mathrm{d}\mathbf{x}}.[\rho\mathbf{u}] = 0$$

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Binaries (number density n_X, coordinate $\mathbf{w}$, velocity $\mathbf{f} = \mathrm{d}\mathbf{w}/\mathrm{d}t$):

$$\frac{\mathrm{d}n_X}{\mathrm{d}t} + \frac{\mathrm{d}}{\mathrm{d}\mathbf{w}}.[n_X\mathbf{f}] = 0$$

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Assuming $\partial/\partial t = 0$ (stationary distribution of binaries in the galaxy) $\longrightarrow$ stochastic GW emission from coalescing binary NS

Buitrago, Moreno-Garrido & Mediavilla (1994); Moreno-Garrido, Mediavilla & Buitrago (1995); Ignatiev et al. (2001)

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$$\frac{\mathrm{d}n_X}{\mathrm{d}t} + \frac{\mathrm{d}}{\mathrm{d}\mathbf{w}}.[n_X\mathbf{f}] = R_X$$

No stationarity
Source function R_X is given by astrophysics

Continuity equation (single population)

$$\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}t} = \mathbf{f}(\mathbf{w}, M)$$

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$$\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}t} = \mathbf{f}(\mathbf{w}, M)$$

$$\frac{\mathrm{d}n_X^{(1)}(M,t)}{\mathrm{d}t} = S\left(M', M', t\right)$$

Continuity equation (single population)

$$\begin{aligned}
\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}t} &= \mathbf{f}(\mathbf{w}, M) \\
\frac{\mathrm{d}n_X^{(1)}(M,t)}{\mathrm{d}t} &= S\left(M', M', t\right) \\
\frac{\mathrm{d}n_X^{(2)}(M,M,\mathbf{w},t)}{\mathrm{d}t} &= \frac{1}{2}R_X(M,t)\mathcal{P}_X(\mathbf{w}) \\
&- \frac{\partial}{\partial \mathbf{w}}.\left[\mathbf{f}\left(\mathbf{w}, M\right) n_X^{(2)}\left(M, M, \mathbf{w}, t\right)\right]
\end{aligned}$$

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S is source term due to mergers

All the merger products remain single, all objects are in binaries

Continuity equation (single population)

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\begin{aligned}
\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}t} &= \mathbf{f}(\mathbf{w}, M) \\
\frac{\mathrm{d}n_X^{(1)}(M,t)}{\mathrm{d}t} &= (1-\beta_X) S\left(M', M', t\right) \\
\frac{\mathrm{d}n_X^{(2)}(M,M,\mathbf{w},t)}{\mathrm{d}t} &= \frac{1}{2} R_X(M,t)\mathcal{P}_X(\mathbf{w}) + \frac{1}{2}\beta_X S\left(M', M', t\right)\mathcal{P}_X(\mathbf{w}) \\
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&- \frac{\partial}{\partial \mathbf{w}}.[\mathbf{f}\left(\mathbf{w}, M\right) n_X^{(2)}\left(M, M, \mathbf{w}, t\right)]
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&- \frac{\partial}{\partial \mathbf{w}}.\left[\mathbf{f}\left(\mathbf{w}, M\right) n_X^{(2)}\left(M, M, \mathbf{w}, t\right)\right] \\
S\left(M', M', t\right) &= \int_{C_m} \mathbf{f}\, n_X^{(2)}\left(M', M', \mathbf{w}, t\right) .\mathrm{d}\boldsymbol{\ell} \\
M &= 2M' - \Delta M(M')
\end{aligned}
$$

Initial distribution of orbital parameters

Sana et al. (2012); de Mink & Belczynski (2015)

Joint distribution: $\mathcal{P}_X(\mathbf{w}) = P(e)P(a)$

- $P(e) \propto e^{-0.42}$
- $P(\log T) \propto (\log T)^{-0.5}$ in $T \in (T_{min}, T_{max})$

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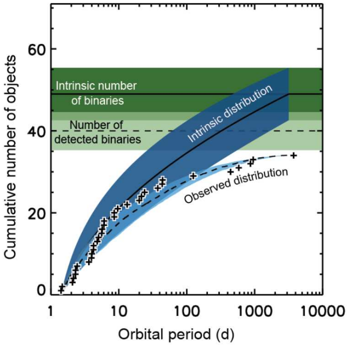
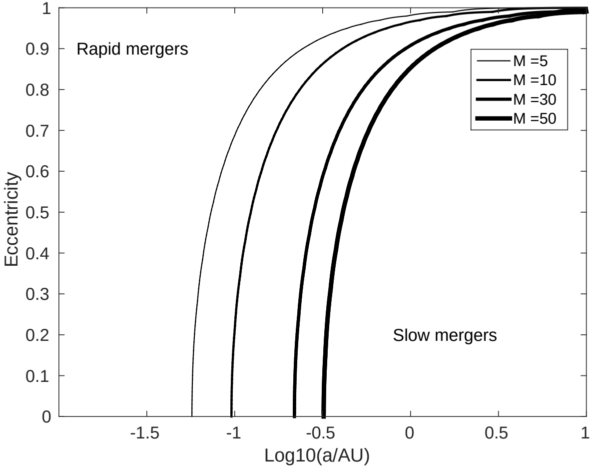


Time to coalescence

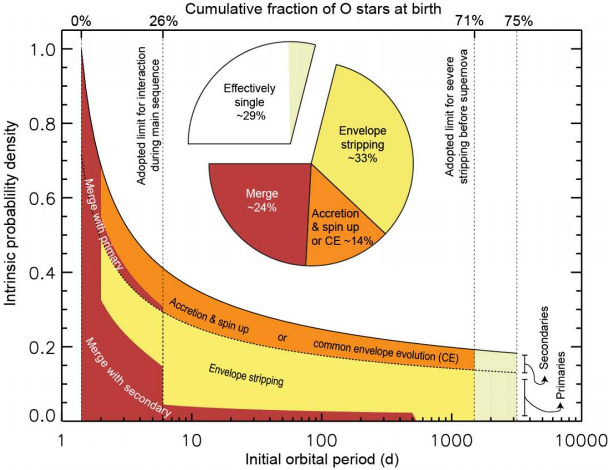
a_{min} chosen so as to fit the observed merger rate ($\tau_{merger} \propto a^4$)



Initial distribution of orbital parameters

Sana et al. (2012)

$a_{min} = 0.2$ AU $\rightarrow T \simeq 10$ days for $M \simeq 10 M_{\odot}$



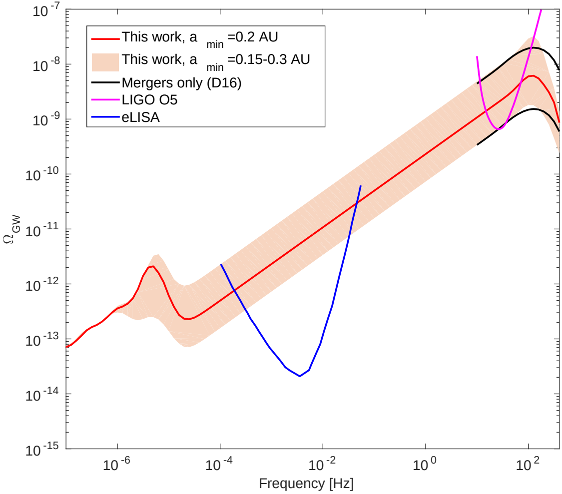
Complete model

Dvorkin et al. (2016) [1604.04288], [1607.06818]

- Galaxy evolution (gas inflow/outflow, SFR, chemical evolution) [Daigne et al. (2004, 2006), Vangioni et al. (2015), Dvorkin et al.(2015)]
- BH formation [Fryer et al. (2012)]
- Distribution of masses and orbital parameters of BH binaries
- Evolution of the binaries due to emission of GW

GW background from BH binaries

Dvorkin et al. [1607.06818]



Summary

An exciting time for astrophysics:

- Gravitational wave astronomy will provide constraints on:
  - Stellar evolution
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What we plan to do next:

- Include BH-NS mergers
- Explore a full merger-tree based galaxy evolution model
- Test different BH formation scenarios
- Use formalism for SMBH

Additional slides

Astrophysics with gravitational waves

- GW150914: The most massive stellar black holes ever observed!
  - Masses: $36^{+5}_{-4} M_\odot$ and $29^{+4}_{-4} M_\odot$

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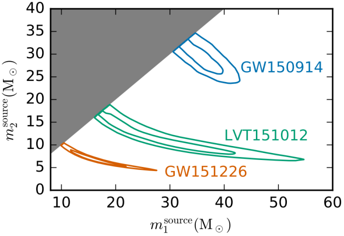

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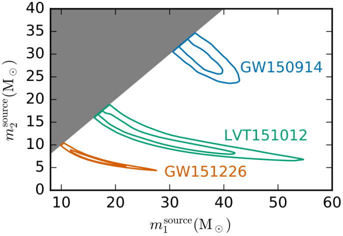
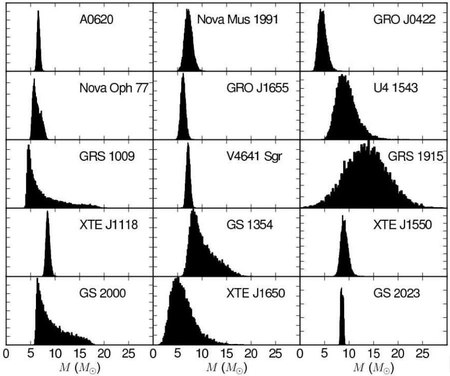


What can we learn about stellar evolution and black hole formation?

Stellar mass black holes - masses

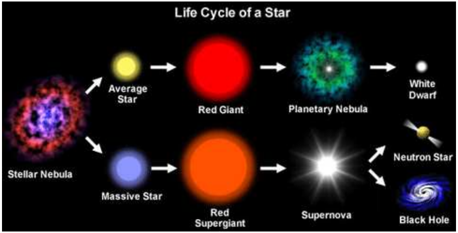
- All previous estimates of BH masses are from X-ray binaries
- GW150914 is the first direct evidence of the existence of 'heavy' stellar mass BHs (M$\gtrsim 20 M_\odot$)



Farr et al. (2007)

How to make a black hole

- BHs form at the end of the nuclear burning phase of massive stars



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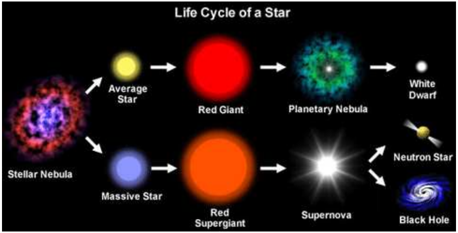
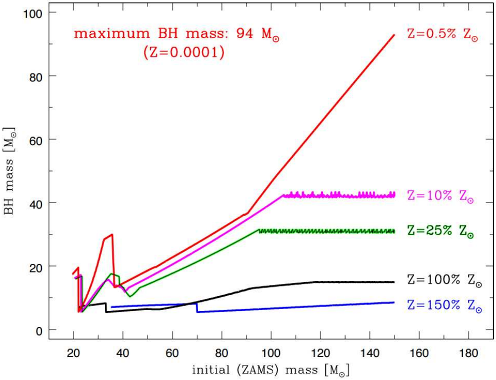


- How to relate the initial stellar mass to the BH mass?

From massive stars to black holes

Mass prior to core collaps is determined by stellar winds



Belczynski et al. (2016)

From massive stars to black holes

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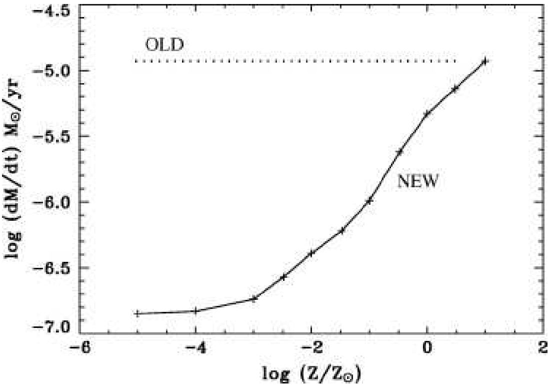
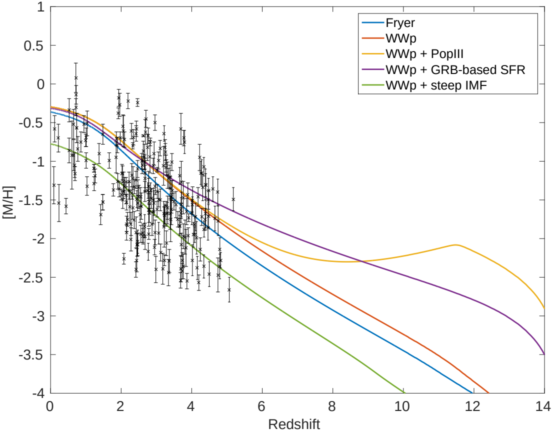
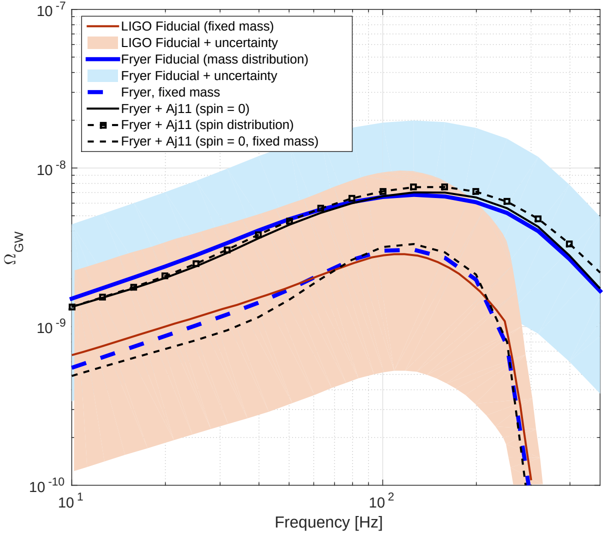


Vink (2008)

Metallicity

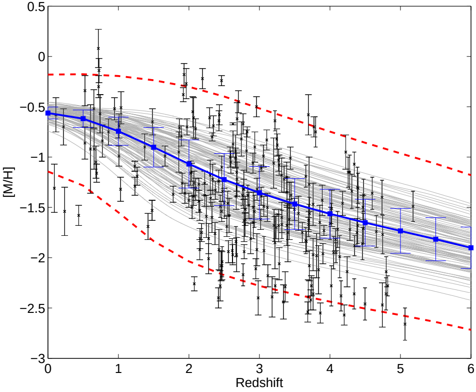


Stochastic gravitational wave background



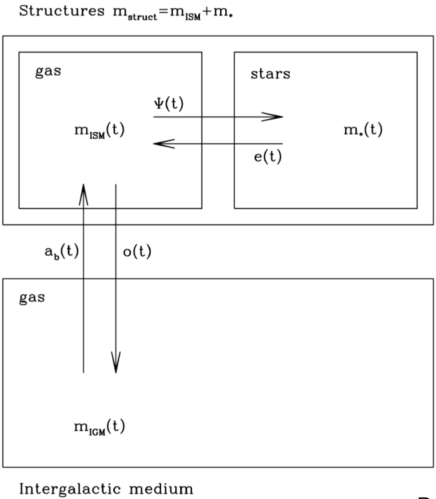
Cosmic metallicity evolution

Damped Ly-α systems data from Rafelski et al. (2012)



Dvorkin et al. (2015)

Galaxy evolution model



Daigne et al. (2006)

Model summary

Baryon flow:

- $\dot{M}_{struct} = a_b(t) + e(t) - \psi(t) - o(t)$
- $\dot{M}_{*} = \psi(t) - e(t)$

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- $\dot{M}_* = \psi(t) - e(t)$

Chemical evolution (X_i is the mass fraction of element i):

- $\dot{X}_i^{ISM} = \frac{1}{M_{ISM}(t)} \left\{ e_i(t) - e(t) X_i^{ISM} + a_b(t) \left[X_i^{IGM} - X_i^{ISM} \right] \right\}$
- $\dot{X}_i^{IGM} = \frac{1}{M_{IGM}(t)} o(t) \left\{ X_i^{ISM}(t) - X_i^{IGM}(t) \right\}$

Self-consistent model of BBH birth rate: overview

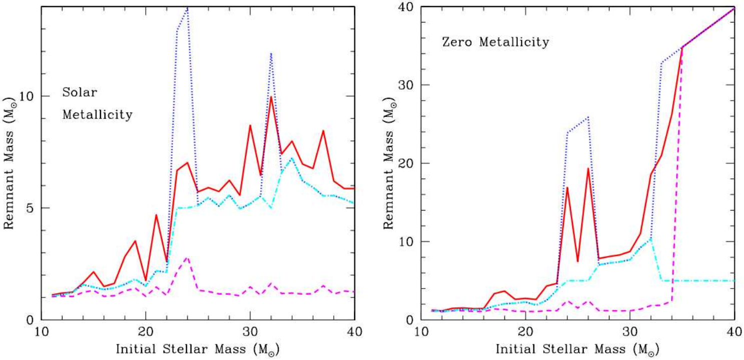
Input

- Galaxy growth (inflow and outflow) prescriptions
- Cosmic star formation rate
- Stellar initial mass function
- Stellar yields
- Black hole mass as a function of initial stellar mass and metallicity

	Model name	Ref.	Parameters	Parameter values
BH masses	*WWp*	Woosley & Weaver (1995)	A, β, γ	0.3, 0.8, 0.2
	Fryer	Fryer et al. (2012)	-	-
	WWp+K	Kinugawa et al. (2014)	$Z_{\rm limit}/Z_{\odot}$	0.001 or 0.01
	Fryer+K			
SFR	*Fiducial*	Vangioni et al. (2015)	ν, z_m, a, b	0.178, 2.00, 2.37, 1.8
	PopIII			0.002, 11.87, 13.8, 13.36
	GRB-based			0.146, 1.72, 2.8, 2.46
IMF	*Fiducial*	Salpeter (1955)	x	2.35
	Steep IMF	Chabrier, Hennebelle & Charlot (2014)		2.7

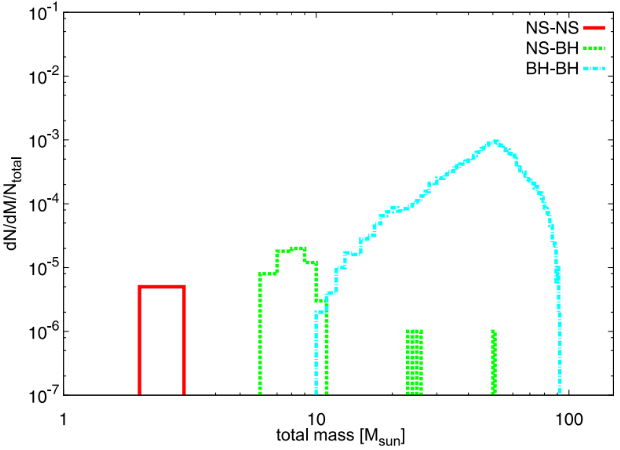
Dvorkin et al. [1604.04288]

BH masses



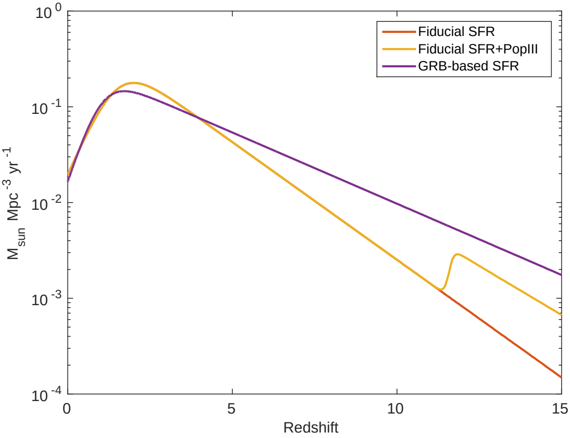
Fryer et al. (2012)

BH masses

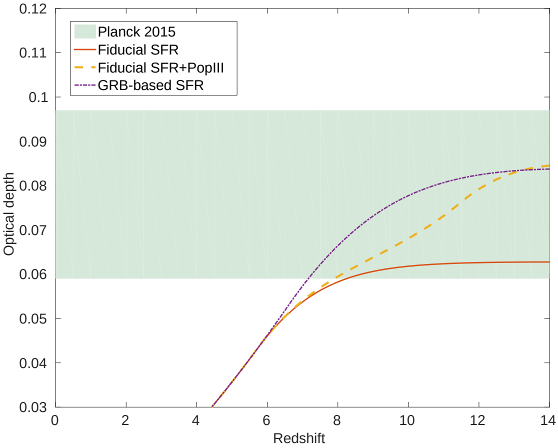


Kinugawa et al. (2014)

Star formation rate



Optical depth to reionization



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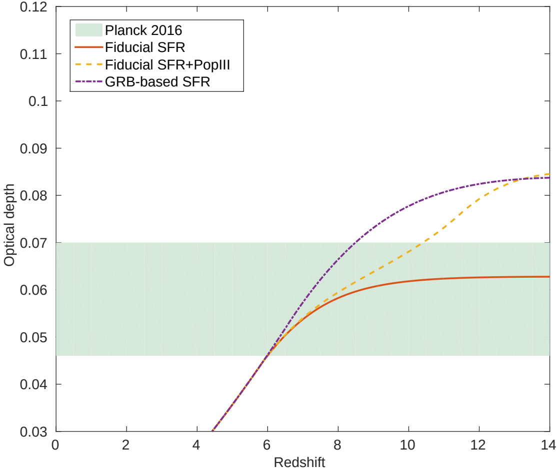

Self-consistent model of BBH birth rate: overview

Dvorkin et al. (2015) [1506.06761]

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Constraints

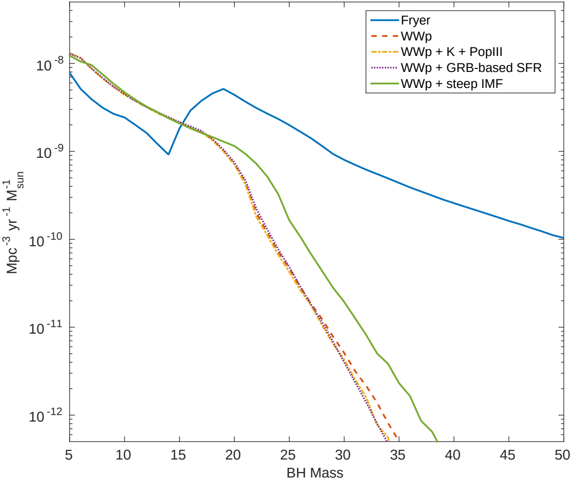
- Cosmic chemical evolution
- Optical depth to reionization

Output

- Birth rate of black holes per unit mass

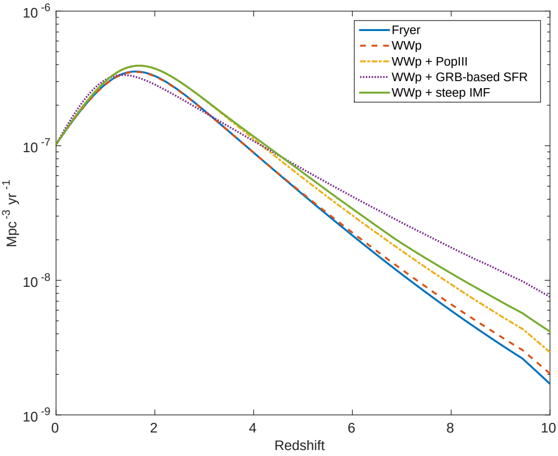
Merger rates vs. mass

Normalized to the observed merger rate: Dvorkin et al. (2016) [1604.04288]



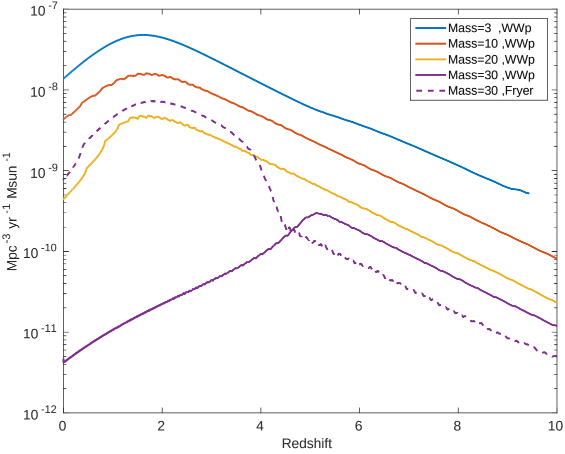
Total merger rates

Normalized to the observed merger rate: $R = 10^{-7}\ \mathrm{Mpc}^{-3}\ \mathrm{yr}^{-1}$



Merger rates vs. redshift

Normalized to the observed merger rate: $R = 10^{-7}\ \mathrm{Mpc}^{-3}\ \mathrm{yr}^{-1}$



Stochastic gravitational wave background

- The background due to unresolved mergers of binary BHs
- Emission of gravitational waves during SN collapse

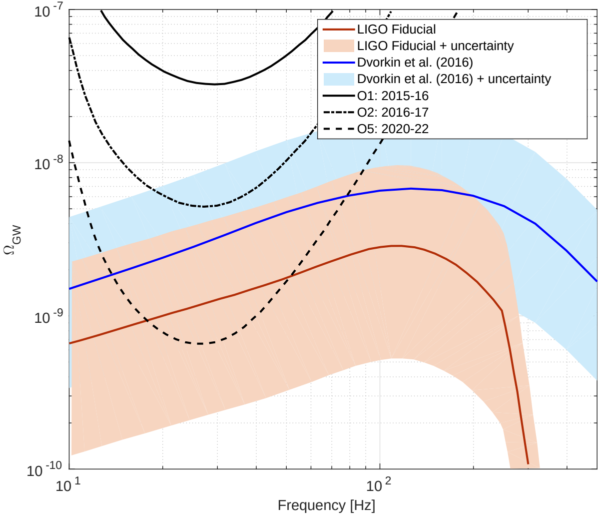
Stochastic gravitational wave background

- The background due to unresolved mergers of binary BHs
- Emission of gravitational waves during SN collapse
- Dimensionless density parameter (energy density in units of ρ_c per unit logarithmic frequency)

$$\Omega_{\rm gw}(f_o) = \frac{8\pi G}{3c^2 H_0^3} f_o \int dm_{bh} \int dz \frac{R_{\rm source}(z, m_{bh})}{(1+z)E_V(z)} \frac{dE_{\rm gw}(m_{bh})}{df}$$

$R_{\rm source}(z, m_{bh})$ is the merger rate, dE_{gw}/df is the emitted spectrum

Stochastic gravitational wave background



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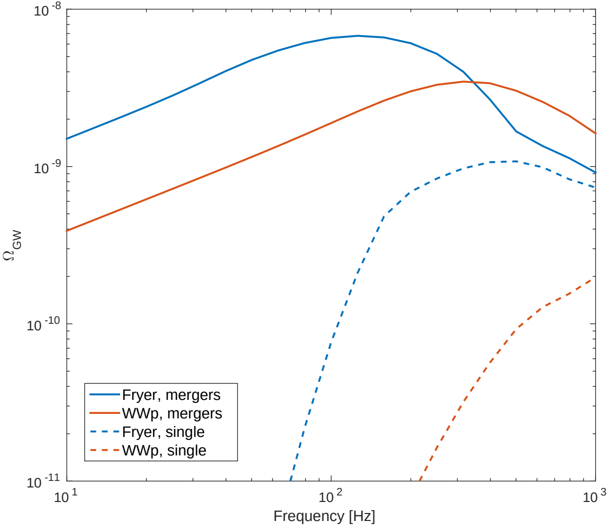

Initial distribution of orbital parameters

Sana et al. (2012); de Mink & Belczynski (2015)

$P(\log T) \propto (\log T)^{-0.5}$ in $a \in (a_{min}, a_{max})$