Primordial Gravitational Waves
&

eLISA Mission

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OUTLINE

- Introduction
- Inflation and Primordial GW
- Inflation and “Beyond” with eLISA
- Conclusions
GR & Gravitational Waves
In 1916 Einstein predicted the existence of **gravitational waves**, since his linearized weak-field equations had wave solutions: **transverse waves of spatial strain** that travel at speed of light, generated by time variations of the mass quadrupole moment of the source.
Why GW are important?

- Test better General Relativity
- Give information on the quantum nature of gravity
- GW Speed
- Graviton mass
- Study compact objects and their properties
- Deep understanding of the physics of the early universe
- . . . .
The spectrum of GW

Many sources => Many detectors
"Indirect" vs Direct GW detection

Tensor anisotropies on last scattering surface

Polarization of CMB photons through Thomson scattering of electron and photon

Only Tensor perturbations can source B-mode

Poor and contaminated signal:
- foregrounds
- gravitational lensing (E→B at small scales)
"Indirect" vs Direct GW detection

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GW travels freely until today

Distortion of space as GW passes detector arms

- ground-based interferometers
- space-based interferometers
- pulsar timing arrays
GW sources and eLISA scientific goals

Astrophysics

- MBHBs
- EMRIs
- Compact WD
GW sources and eLISA scientific goals

**Astrophysics**
- MBHBs
- EMRIs
- Compact WD

**Cosmology**
- EW Phase Transition BSM
- Other first-order PT
- Topological Defects
- Inflation and Beyond
- Standard sirens
Primordial GWs are out of equilibrium since the Planck scale (photons at 0.3 eV) so they carry information about the universe at really high energies.
GW are represented by tensor perturbation $h_{ij}$ of the FRW metric

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$$

- Period of accelerated (exponential) expansion driven by a scalar field (inflaton) that rolls down on its flat potential

Solve Standard Big-Bang shortcomings
Generation of perturbations
Stretches the microphysics scales to super-horizon sizes

**GW are represented by tensor perturbation $h_{ij}$ of the FRW metric**

Transverse and traceless $\partial_i h_{ij} = h_{ii} = 0 \rightarrow 2 \text{ D.O.F}$ (2 polarizations)
Dynamics governed by linearized Einstein eq:

\[ \ddot{h}_{ij}(k, t) + \left( k^2 - \frac{a''}{a} \right) \dot{h}_{ij}(k, t) = 0 \]

\[ \sim a^2 H^2 \]

**Solutions**

Sub-Horizon => \( k \gg aH \) : \( h_{ij} \sim \cos(\omega \tau) / a \)

Super-Horizon => \( k \ll aH \) : \( h_{ij} \sim \text{const} \)

**Observational quantity on the CMB**

\[ \langle h(k) h^*(k') \rangle = \frac{2\pi^2}{k^3} \delta^{(3)}(k + k') P_h(k) \]

**Single field slow-roll**

**Tensor Power Spectrum**

\[ P_T(k) = \frac{8}{M_{Pl}^2} \left( \frac{H}{2\pi} \right)^2 \left( \frac{k}{aH} \right)^{n_T} \]

\[ n_T = -2\epsilon \]

\[ \epsilon \equiv \frac{M_{Pl}^2}{2} \left( \frac{V}{V'} \right)^2 \]
Consistency relations

\[ P_{S,T} = A_{S,T} \left( \frac{k}{k_*} \right)^{(n_s-1,n_t)+...} \]

\[ A_S = \left( \frac{H^2}{2\pi \dot{\varphi}} \right)^2 \]

\[ A_T = \frac{8}{M_{Pl}^2} \left( \frac{H}{2\pi} \right)^2 \]

\[ r \equiv \frac{A_T(k_*)}{A_S(k_*)} = 16\epsilon = -8n_T \]

Importance of measuring the Tensor PS (at different scales)

Importance of measuring Violation of CR

\[ r_{0.05} < 0.07 \ (95\% CL) \]

Bicep2/Keck +95GHz

\[ \frac{\Delta \varphi}{M_{Pl}} \gtrsim O(1) \left( \frac{r}{0.01} \right)^{1/2} \]

Lyth Bound

\[ V = (1.88 \times 10^{16} \text{GeV})^4 \frac{r}{0.1} \]
\[ P_{S,T} = A_{S,T} \left( \frac{k}{k_*} \right)^{(n_s-1,n_t)+...} \]

\[ A_S = \left( \frac{H^2}{2\pi\varphi} \right)^2 \]

\[ A_T = \frac{8}{M_{Pl}^2} \left( \frac{H}{2\pi} \right)^2 \]

\[ n_t = A \log_{10} \left( \frac{r}{0.11} \right) + B, \]

<table>
<thead>
<tr>
<th>Experiment</th>
<th>(A)</th>
<th>(B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMB + PPTA</td>
<td>-0.13</td>
<td>0.68</td>
</tr>
<tr>
<td>CMB + PPTA + LIGO</td>
<td>-0.06</td>
<td>0.54</td>
</tr>
<tr>
<td>CMB + PPTA + LIGO + indirect</td>
<td>-0.04</td>
<td>0.36</td>
</tr>
<tr>
<td>CMB + PPTA(2020) + aLIGO</td>
<td>-0.06</td>
<td>0.34</td>
</tr>
</tbody>
</table>
Inflationary GWs generated by the amplification of the vacuum fluctuations, have an amplitude OUT of eLISA range

\[ h_0^2 \Omega_{gw}(f) \approx 5 \cdot 10^{-16} \left( \frac{H}{H_{\text{max}}} \right)^2 \]

\[ H_{\text{max}} \approx 8.8 \times 10^{13} \text{GeV} \]

current upper bound on energy scale of inflation

Planck 2015 results. XX
Current constraints on GW energy density

\[ S_{\text{GW}}(f) \]

\[ \text{temperature (GeV)} \]

\[ \text{frequency (Hz)} \]

Indirect

LIGO and Virgo

PTA

aLIGO

CMB

[ P. D. Lasky et al., (1511.05994)]
\( \tilde{h}''_{ij}(k, t) + \left( k^2 - \frac{a''}{a} \right) \tilde{h}_{ij}(k, t) = \)
Inflationary GWs and eLISA

\[ \tilde{h}_{ij}''(k, t) + \left( k^2 - \frac{a''}{a} \right) \tilde{h}_{ij}(k, t) = 16\pi G a \Pi^{TT}_{ij}(k, t) \]

\( \Pi_{ij}^{TT} \) transverse-traceless part of the anisotropic stress

The processes that give rise to a non-zero tensor anisotropic stress in the Early Universe can directly source GW potentially detectable by eLISA
Inflationary GWs and eLISA

\[ \dddot{h}_{ij}(k, t) + \left( k^2 - \frac{a''}{a} \right) \dddot{h}_{ij}(k, t) = 16\pi G a \Pi^{TT}_{ij}(k, t) \]

\( \Pi^{TT}_{ij} \) transverse–traceless part of the anisotropic stress

The processes that give rise to a non-zero tensor anisotropic stress in the Early Universe can directly source GW potentially detectable by eLISA

GW Energy Density

\[ \rho_{gw} = \frac{\langle \dot{h}_{ij} \dot{h}_{ij} \rangle}{32\pi G} = \int \frac{df}{f} \frac{d\rho_{gw}}{d \log f} \]

Present-day GW frequency

\[ f = \frac{k}{2\pi} \frac{a}{a_0} \]

Energy density per log frequency interval

\[ h^2 \Omega_{GW}(f) = \frac{h^2}{\rho_c} \frac{d\rho_{gw}}{d \log f} \]
**Inflationary setup**

- Second order GWs
- Particle production during inflation
  - (see M. Pieroni’s talk)
- Spectator fields
- EFT of broken space diff
- Inflationary PT

**Post-Inflationary setup**

- GW from (p)reheating
- Thermal background
- Kination-domination
- Merging of primordial BHs
  - (see J. G. Bellido’s talk)
- Alternatives to Inflation
  - String Cosmology
  - Pre-Big-Bang models
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**Inflationary PT**

**String Cosmology**

**Pre-Big-Bang models**
The rolling inflaton excites, through the coupling, quanta of EM field

\[ A_+ \text{ is exponentially amplified as } \xi \text{ becomes large } (>0), \text{ while } A_- \text{ has no amplification by the rolling field } \varphi \]

\[ (A_+ \propto e^{\pi \xi}) \]

The production of gauge quanta prolongs inflation because it sources inflaton perturbations through the inverse decay \( \delta A + \delta A \rightarrow \varphi \)

EM field sources also tensor fluctuations (GW) \( \delta A + \delta A \rightarrow \delta g \)
Chiral Gravitational Waves signal

High scalar non-Gaussian contribution

Useful to distinguish origin of the signal

Limits from the CMB $\xi < 2.5$
Since there is no direct coupling between the inflaton and the axion the inflaton perturbations, also sourced by the gauge field, are negligible with the gravity wave production.

Tensor contribution amplified by $\xi$

$$\delta A + \delta A \rightarrow \delta \chi$$

$$\delta A + \delta A \rightarrow \delta \varphi \quad \text{and} \quad \delta A + \delta A \rightarrow \delta h$$

Since there is no direct coupling between $\chi$ and $\varphi$

$$\delta A + \delta A \rightarrow \delta \varphi \sim \text{negligible}$$

The model produces:

- Large gravitational wave signal (observable B modes)
- Sufficiently small scalar perturbations (in agreement with CMB constraints)

\[
\mathcal{L} = -\frac{1}{2} (\partial \varphi)^2 - V(\varphi) - \frac{1}{2} (\partial \chi)^2 - U(\chi) - \frac{1}{4} F^2 - \alpha_2 \frac{\chi}{4 f} F_{\mu \nu} \tilde{F}^{\mu \nu}
\]

$\varphi$ inflaton, $\chi$ pseudo-scalar field

$\xi \equiv \frac{\alpha_2 \dot{\chi}}{2 f H}$
Very model dependent

Possibility to test the inflaton field at the latest stage of inflation, for which we have very poor information

Possibility to test the inflaton coupling(s)
Parity violating signal

\[ \Delta \chi = \frac{P_T^+ - P_T^-}{P_T^+ + P_T^-} \propto \xi \]

High tensor CMB non-Gaussian signal

What about measuring PARITY VIOLATION and NON-GAUSSIANITY with eLISA?

[S. G. Crowder et al. (arXiv:1212.4165)]
[N. Barnaby et al. (arXiv:1206.6117)]
[N. Bartolo et al. (arXiv:1505.02193)]
Spectator Field

\[ \mathcal{L} = \frac{1}{2} M_{Pl}^2 R - \frac{1}{2} (\partial \varphi)^2 - V(\varphi) + P(X, \sigma) \]

spectator responsible only for perturbations

\[ c_s \equiv \frac{P_X}{(P_X + P_{XX} \dot{\sigma}^2_0)} \neq 0 \]

\[ s \equiv \frac{H \dot{c}_s}{c_s} \neq 0 \]

\[ P_R = P_R^{(v)} + P_R^{(\sigma)} \simeq \frac{H^2}{4\epsilon M_{pl}^2} + \frac{1}{32\pi c_s^7 M_{pl}^4} \frac{H^4}{M_{pl}^4} \]

\[ P_t = P_t^{(v)} + P_t^{(\sigma)} \simeq \frac{2H^2}{M_{pl}^2} + \frac{8}{15\pi c_s^3 M_{pl}^4} \frac{H^4}{M_{pl}^4} \]

\[ n_T^{(\sigma)} \simeq -4\epsilon - 3s \]

\[ n_S^{(\sigma)} \simeq -4\epsilon - 7s \]

\[ X = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma \]

[M. Biagetti et al., (1305.7241)]
[M. Biagetti et al., (1411.3029)]
[T. Fujita et al., (1411.3658)]
From a **NON DETECTION** of primordial GW bg by eLISA we can put a limit on the spectral index of the sourced GW, for a given value of the sourced GW amplitude on CMB scales.

\[ A_{0.05}^{(S)} = 2.21 \times 10^{-9} \quad (65\% \; CL) \]

\[ \epsilon = 0.0068 \quad (95\% \; CL) \quad \text{[PlanckTT + lowP]} \]

\[ r_{0.05} < 0.09 \quad (95\% \; CL) \quad \text{[BICEP2 / Keck Array VI]} \]

**PRELIMINARY ANALYSIS**
EFT of broken space diffeo

General Relativity $\Rightarrow$ invariance under $x^\mu \rightarrow x^\mu + \xi^\mu (x^\nu)$

During inflation $t \rightarrow t + \xi(x^\mu)$ is broken

What happens if $x^i \rightarrow x'^i(t, x^j)$ is broken? ($\phi = \phi(x^i)$)

If space-diffeo are broken the graviton can acquire an effective mass and an effective sound speed during inflation

$$S_h = \frac{M_{Pl}^2}{4} \int d\eta d^3x a^2(\eta) \left\{ (h'_{ij})^2 - c_T^2 (\partial_t h_{ij})^2 - m^2 h_{ij}^2 \right\}.$$ 

Tensor Power Spectrum

$$P_T = \frac{2 H^2}{\pi^2 M_{Pl}^2 c_T} \left( \frac{k}{k_*} \right)^{n_T}$$

Spectral index

$$n_T = -2\epsilon + \frac{2}{3} \frac{m^2}{H^2} \left( 1 + \frac{4}{3} \epsilon \right)$$

We can generate a blue tensor spectrum w/o violating Null Energy Condition
A “sufficiently” **blue tensor spectrum** can be detectable by eLISA

\[ m_g \leq 1.2 \times 10^{-22} \text{eV}/c^2 \] (90% CL)

[LIGO & Virgo Scientific Coll. (arXiv:1602.03841)]
GW from post-inflationary processes

(p)reheating through parametric effects

Resonance parameter

\[ q \sim \frac{g^2 \Phi_*^2}{\omega^2} \]

\( g \) (coupling constant)

\( \bar{\Phi}_* \) (initial amplitude of the inflaton)

\( \omega^2 \equiv V'' \) (frequency of the oscillations)

\[ g^2 \phi^2 \chi^2 \]

(calar)

\[ g^2 \phi^2 A_\mu A^\mu \]

(vector)

\[ g \phi \bar{\psi} \psi \]

(fermion)
GW from post-inflationary processes

(p)reheating through parametric effects

Resonance parameter

\[ q \sim \frac{g^2 \Phi_*^2}{\omega^2} \]

- \( g \) (coupling constant)
- \( \Phi_* \) (initial amplitude of the inflaton)
- \( \omega^2 \equiv V'' \) (frequency of the oscillations)

Amplitude

\[ g^2 \phi^2 \chi^2 \]

(scalar)

\[ g^2 \phi^2 A_\mu A^\mu \]

(vector)

\[ g \phi \bar{\psi} \psi \]

(fermion)

Amplitude

\[ \Omega_{GW} \propto q^p \propto g^{2p} \]

Peak frequency

\[ f \propto g^{1/2} \]

These scenarios predict a bg of GW with very large amplitude peaked at very high frequencies

\[ h^2 \Omega_{GW}^{(\text{peak})} \lesssim 10^{-11} \]

\[ f \gtrsim 10^8 \] (OUT OF eLISA RANGE)
(p)reheating through spinodal instabilities

Peak frequency and Amplitude are decoupled

The vacuum energy of the waterfall field $v_\ast$ controls the amplitude

**Amplitude**  $\Omega_{GW} \propto v_\ast$

**Peak frequency**  $f \propto \lambda^{1/2}$  \hspace{1cm} $\lambda$ self-coupling of the waterfall field
(p)reheating through spinodal instabilities

Peak frequency and Amplitude are decoupled

The vacuum energy of the waterfall field $\nu_*$ controls the amplitude

Amplitude

$\Omega_{GW} \propto \nu_*$

Peak frequency

$f \propto \lambda^{1/2}$

$\lambda$ self-coupling of the waterfall field

In order to be in the eLISA sensitivity range of frequency and amplitude the coupling constant must be

$\lambda \lesssim \mathcal{O}(10^{-28})$

very unnatural
Kination-Domination

“Stiffness” period after inflation

\[ w = \frac{(K - V)}{(K + V)} \approx +1 \]

If a kination-domination period lasts sufficiently long, from the end of inflation until somewhen just before BBN, it is in principle possible that the, otherwise slightly red-tilted inflationary spectrum of GW, becomes highly blue-tilted, becoming a target for the eLISA mission.

It does not affect the modes that affect the CMB

### Summary of the inflationary scenarios

<table>
<thead>
<tr>
<th>Model</th>
<th>Effect</th>
<th>Blue Tilted</th>
<th>Single Peak</th>
<th>Other Effects</th>
<th>eLISA Detect.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum Ampl.</td>
<td></td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Second order GW</td>
<td>✓</td>
<td>×</td>
<td></td>
<td>?</td>
<td>×</td>
</tr>
<tr>
<td>Particle production (gauge fields)</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>Parity Violation</td>
<td>✓</td>
</tr>
<tr>
<td>Spectator Field</td>
<td>✓</td>
<td>×</td>
<td></td>
<td>×</td>
<td>✓</td>
</tr>
<tr>
<td>EFT of Broken Diff.</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>×</td>
<td>✓</td>
</tr>
<tr>
<td>Inflation Ph. Tr.</td>
<td>×</td>
<td>✓</td>
<td></td>
<td>Voids generation (?)</td>
<td>✓</td>
</tr>
<tr>
<td>(p)Reheating</td>
<td>×</td>
<td>✓</td>
<td></td>
<td>Anisotropies (very fine-tuned only)</td>
<td>✓</td>
</tr>
<tr>
<td>Thermal Backg.</td>
<td>×</td>
<td>✓</td>
<td></td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Kination-domination (stiff phase)</td>
<td>✓</td>
<td>×</td>
<td></td>
<td>×</td>
<td>?</td>
</tr>
<tr>
<td>PBH after Inflation</td>
<td>×</td>
<td>✓</td>
<td></td>
<td>DM candidates</td>
<td>✓</td>
</tr>
<tr>
<td>String Cosmology</td>
<td>✓</td>
<td>×</td>
<td></td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Pre-Big-Bang models</td>
<td>✓</td>
<td>×</td>
<td></td>
<td>×</td>
<td>?</td>
</tr>
</tbody>
</table>
Conclusions

The aLIGO detection officially open the decade of GWs

GWs allow to test energy scale not accessible at collider

Primordial GW gives information on the early Universe

The complementarity of CMB and direct GW measurements (eLISA) provides a powerful probe of the physics of cosmic inflation (Tensor Spectral tilt)

Consistency relations
“Inflationary” physics with eLISA:

Possibility to test latest stage of inflation and possible couplings

The NON Detection of GW constrains cosmological parameters

Possibility to test new pattern of symmetries

Possibility to give informations on the post-inflationary period
Inflation and stretching of CPs

- Comoving scales
- \((aH)^{-1}\)
- Sub-horizon
- Super-horizon
- \(\zeta \approx 0\)
- Horizon exit
  - \(k = aH\)
- Reheating
- Horizon re-entry
- CMB recombination
- Today
- Time
Polarisation of the CMB

- Thompson Scattering
- E mode (Grad)
- B mode (Curl)

- Tensor quadrupole doesn’t show axial symmetry -> B mode polarisation

Kamionkowski, Kosowsky & Stebbins 1997
Zaldarriaga & Seljak 1997
Current observational upper bounds on the amplitude of GW spectrum
Why eLISA?

\[ f_c \approx 2.6 \times 10^{-5} \text{Hz} \epsilon_*^{-1} \left( \frac{T_*}{1 \text{ TeV}} \right) \left( \frac{g_*}{100} \right)^{1/6} \]

Only for GW emitted by causal sources

\[ \lambda_* = \epsilon_* H_*^{-1} \]

Wavelength

Horizon length

\[ \epsilon_* \leq 1 \text{ param. depend. on the dynamics of the GW source} \]
Sources of Gravitational Waves

- **Supernova**: Explosion caused by the collapse of an old, burnt-out star
  - Produces a burst of gravitational radiation, *if it is non-symmetric!*

- **Neutron star**: A city-sized atomic nucleus!
  - Can spin at up to 600 cycles per second
  - Emits continuous gravitational radiation (again, if it is non-symmetric)

- **Merging compact binary**: Collision of two stellar remnants (neutron stars or black holes)
  - Produce a sweeping “chirp” as they spiral together

- **Primordial background**: Leftover radiation from the beginning of the Universe
  - Tells us about the state of the Universe at or before the Big Bang!
  - Sounds like “noise” with a characteristic spectrum

? Other sources
Strain Spectrum

\[ S(f) = \frac{3H_0^2}{10\pi^2} \frac{\Omega_{GW}(f)}{f^3} \]

\[ h(f) = 6.3 \times 10^{-22} \sqrt{\Omega_{GW}(f)} \left( \frac{100\text{Hz}}{f} \right)^{3/2} \text{Hz}^{-1/2} \]