The Δg workflow:
from measured displacements to the differential external acceleration

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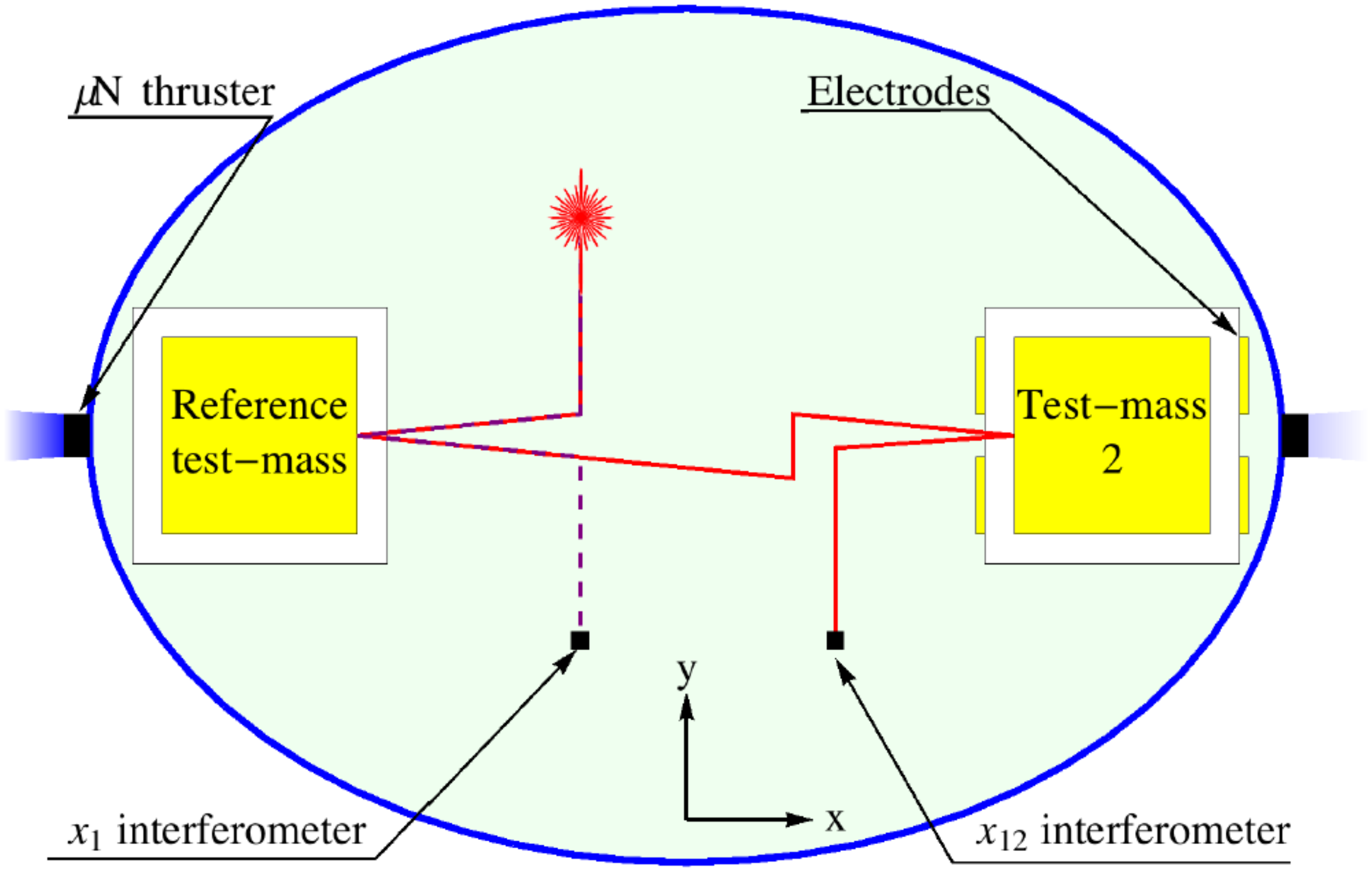
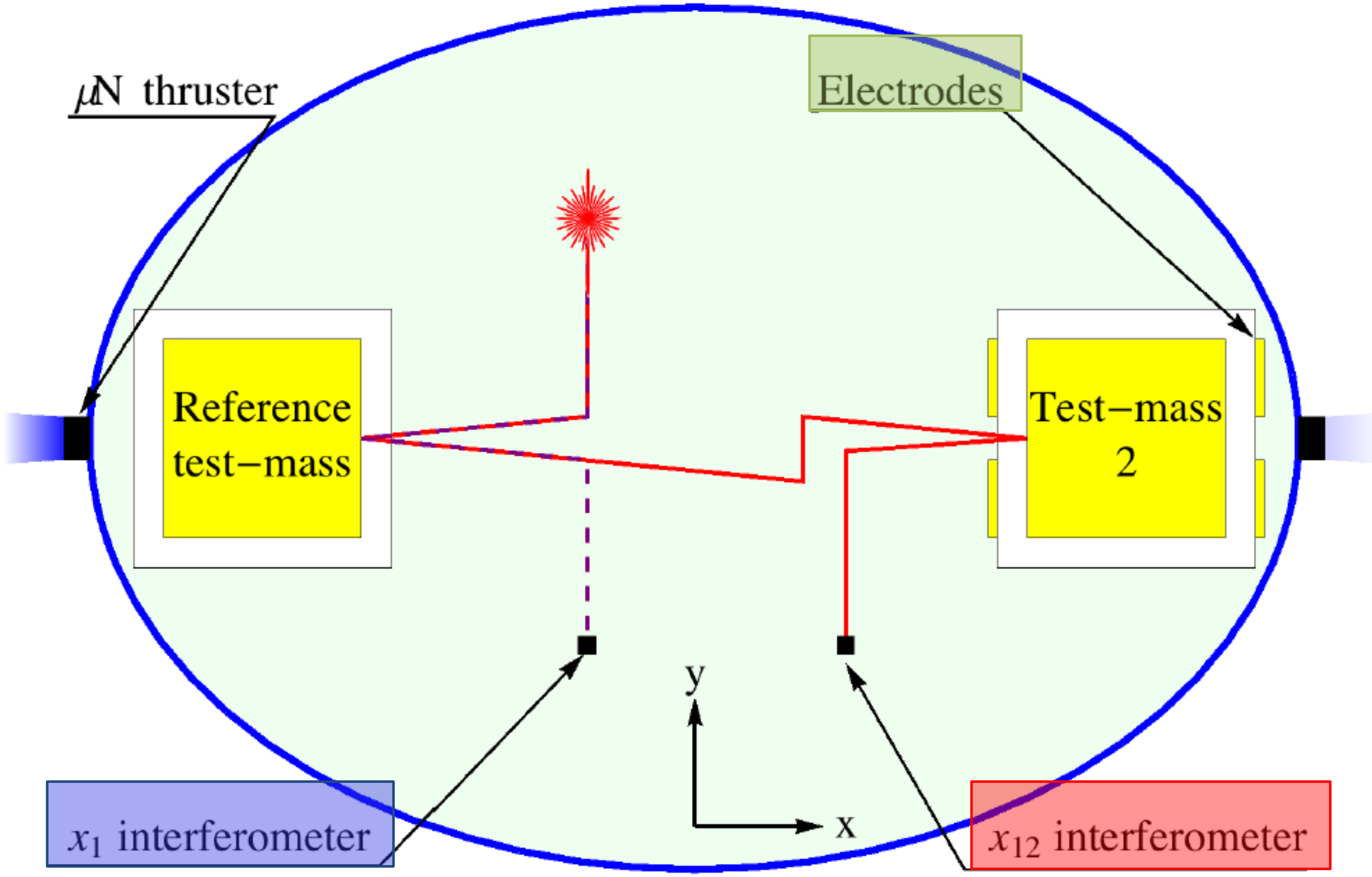
We want to measure the relative acceleration between two freely falling testmass

$$S_{\Delta g}^{1/2} \lesssim 3 \times 10^{-14} \frac{\mathrm{m}}{\mathrm{s}^2\sqrt{\mathrm{Hz}}} \times \sqrt{\left(1 + \frac{f}{3\mathrm{mHz}}\right)^4}$$

Δg is the differential external acceleration in LISA Pathfinder



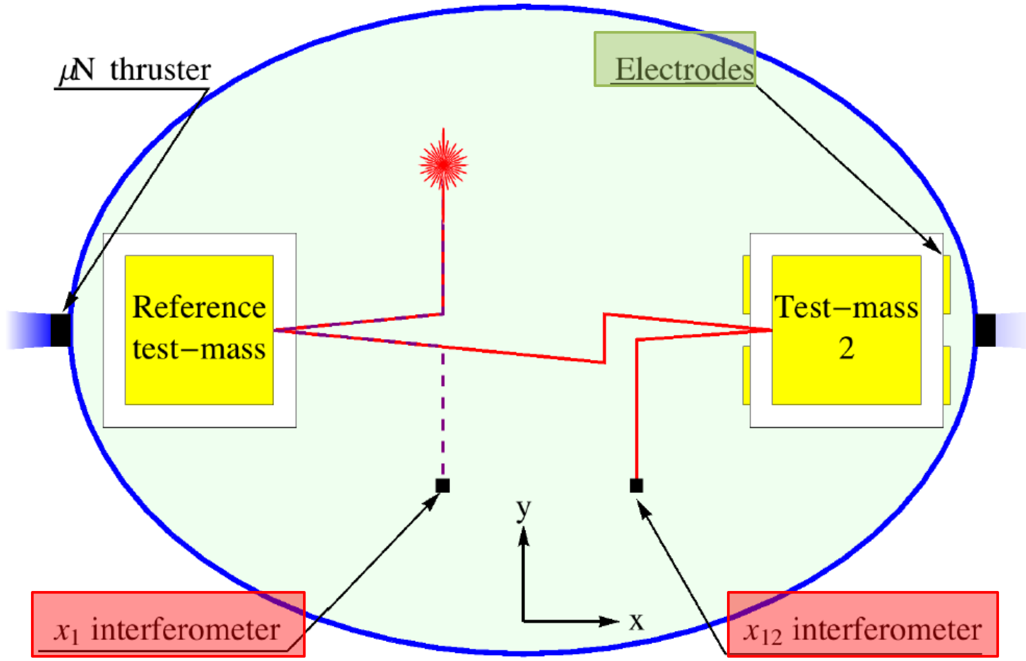
$$\Delta g(t) = \ddot{o}_{12}(t) + \omega_2^2 o_{12}(t) + \Delta\omega^2 o_1(t) - C_{sus} g_c(t) - c_1 \dot{g}_c(t)$$



$$\Delta g(t) = \ddot{o}_{12}(t) + \omega_2^2 o_{12}(t) + \Delta\omega^2 o_1(t) - C_{sus} g_c(t) - c_1 \dot{g}_c(t)$$

Stiffness (Force Gradient): Electrostatic, Gravitation, Magnetic, etc…

Electrostatic Actuator: Gain and linearized delay.



Calibration process

1. Design and plan the calibration experiments.
2. Fit the dynamics of the system to a fiducial model.
3. Convert a noise run into external differential force.
4. Compare residuals with noise.

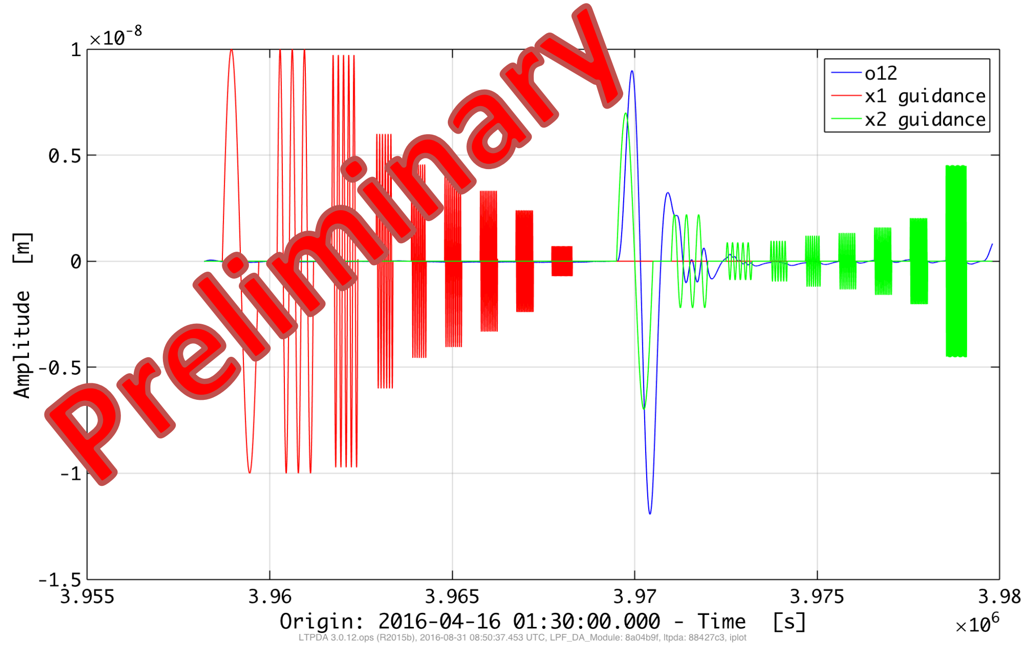
Calibration experiments

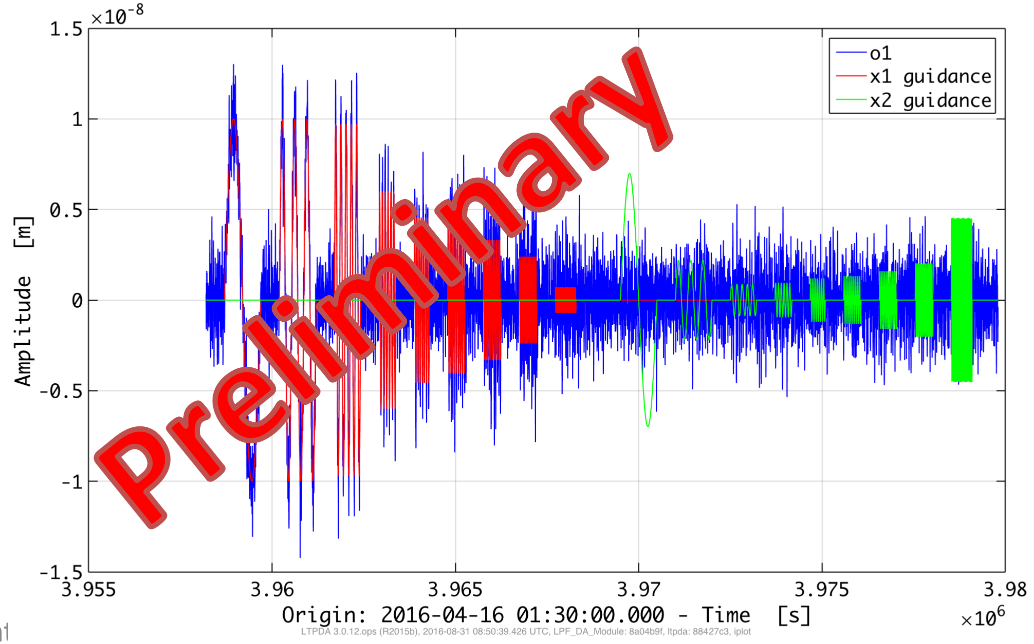
Name	Duration (hrs)	Type	Features
3045 default	6	Guidance	[1 -50 mHz] Drag Free (DF) and Suspension Loop (SL)
3045 short	3	Guidance	[1 -50 mHz] DF and SL
3045 long	9	Guidance	[0.55 -50 mHz] DF and SL
Calibration Tone 1	39	OoI force	20 fN - 7 mHz
Calibration Tone 2	3	OoI force	100 fN - 7 mHz
Calibration Tone 3	65	OoI force	100 fN - 10 mHz
IFOX1X12	1	Guidance	400 mHz – 1 nm
Low frequency calibration	16	Voltage modulation on Z	[++++] 0.25 V – 1 mHz 5 mHz 0.25 mHz

See of N. Karnesis poster

3045-like experiments

- Guidance in Drag Free and suspension loop of the order of nanometers
- Frequency range from 1 to 50 mHz
- Applied force of the order of pN





The 3045 fitting models

gain of the commanded force

delay

$$\Delta g(t) = \ddot{o}_{12}(t) + \omega_2^2 o_{12}(t) + \Delta\omega^2 o_1(t) - C_{sus} g_c(t) - c_1 \dot{g}_c(t) + \delta_1 \ddot{o}_1(t)$$

testmass 2 stiffness

differential stiffness

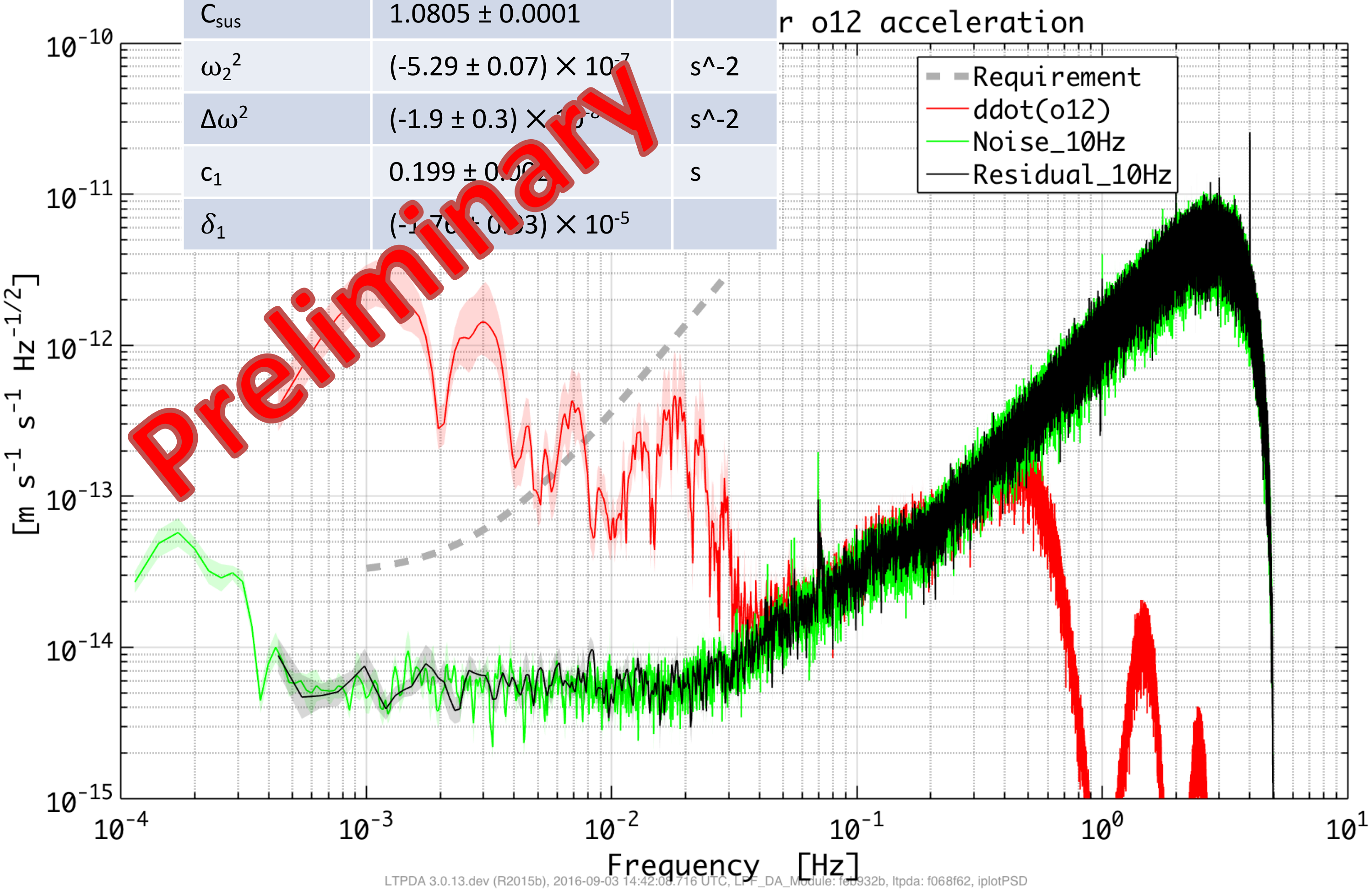
IFOX1X12

Fit with Iterative Reweighted Least square fit (IRLS)
Analysis frequency range
Default and short : [0.5 – 60] mHz
Long : [0.3 – 60] mHz

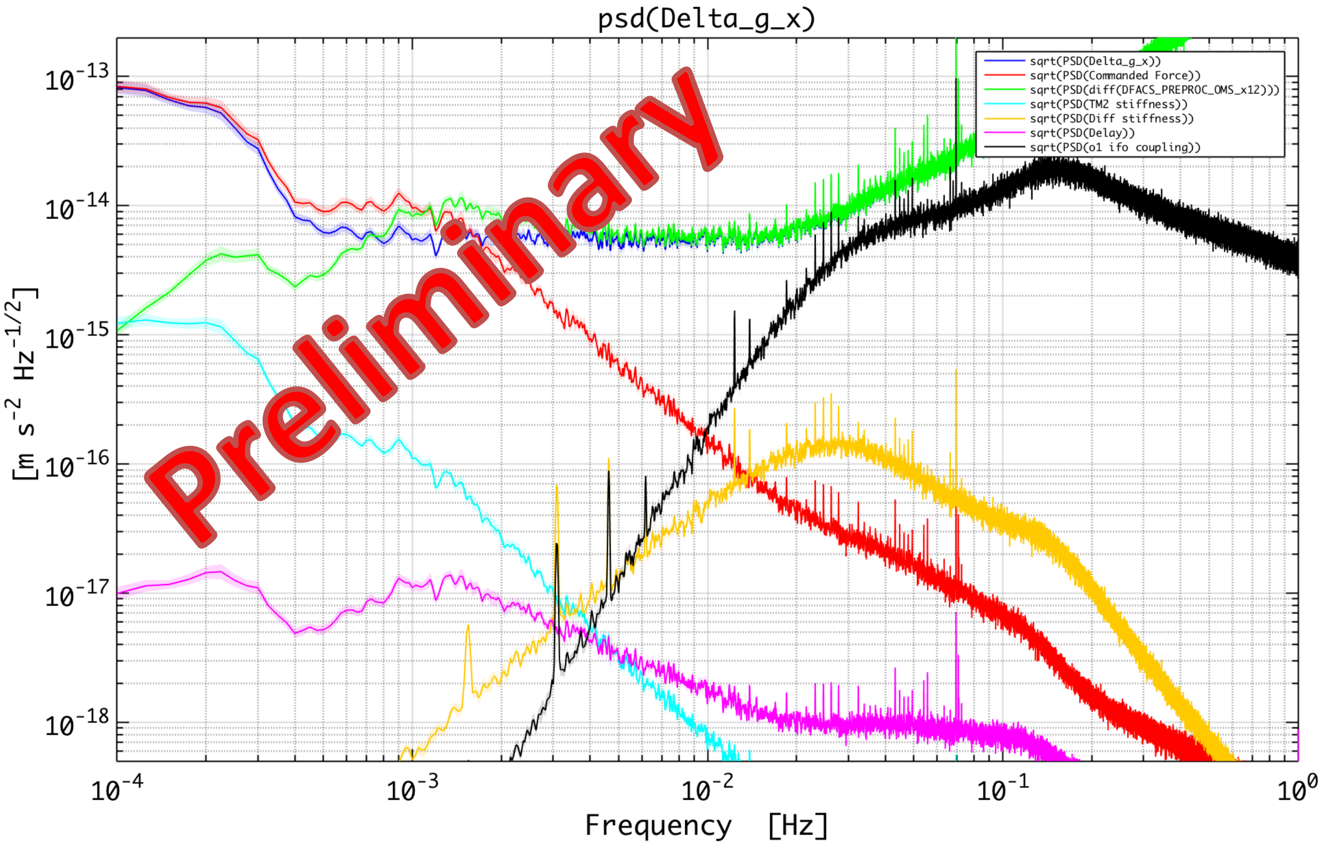
See of N. Karnesis poster

Parameters and residuals

Parameters	Value	Units
C_{sus}	1.0805 ± 0.0001	
ω_2^2	$(-5.29 \pm 0.07) \times 10^{-7}$	s^-2
$\Delta\omega^2$	$(-1.9 \pm 0.3) \times 10^{-8}$	s^-2
c_1	0.199 ± 0.002	s
δ_1	$(-1.76 \pm 0.03) \times 10^{-5}$	

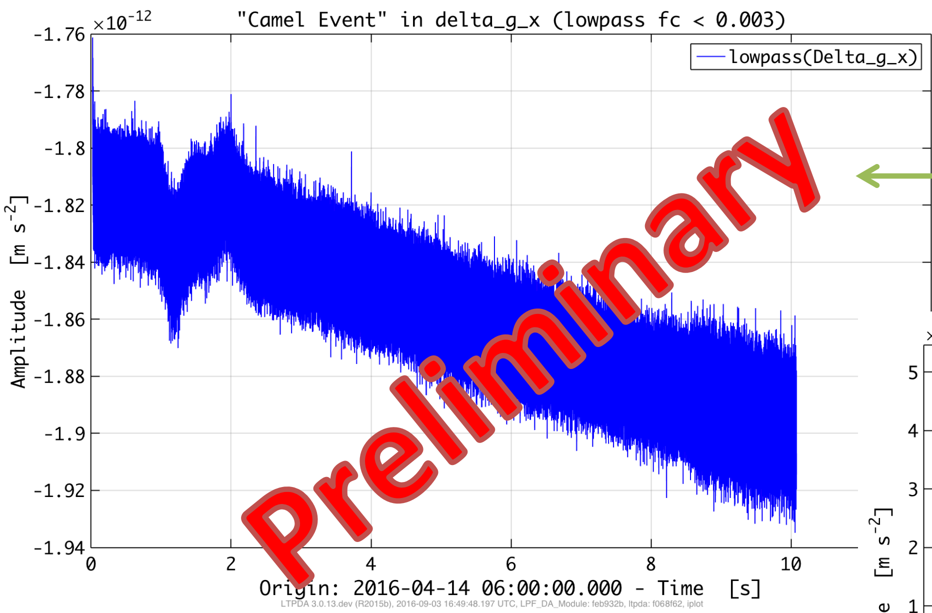


The contributions to the noise

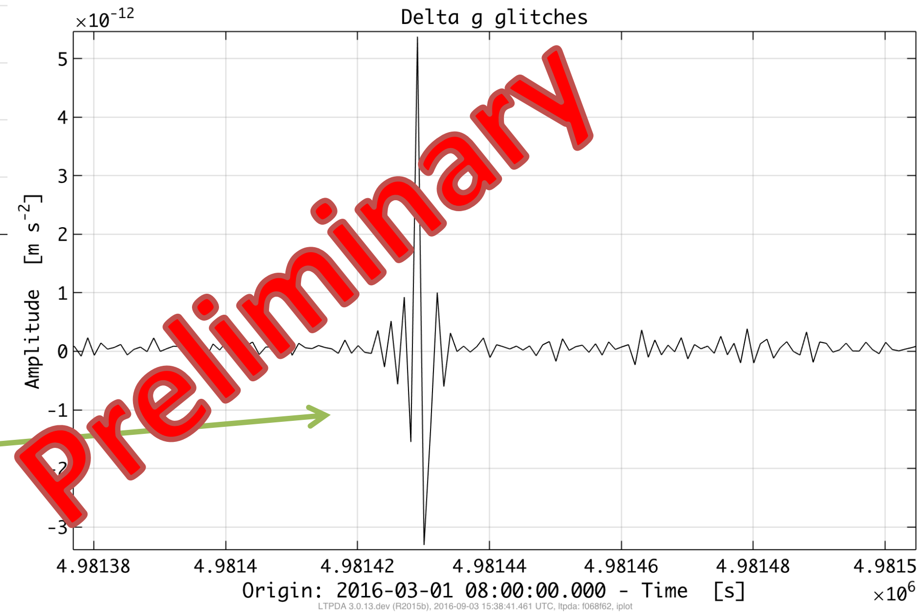


Coupling to the spacecraft motion through force gradient is almost two order of magnitude less that expected

Features of Δg(t)



Long events (rare)



Glitches (less rare)

Corrections to the residuals acceleration models

$$\Delta g(t) = \ddot{o}_{12}(t) + \omega_2^2 o_{12}(t) + \Delta\omega^2 o_1(t) - C_{sus} g_c(t) - c_1 \dot{g}_c(t)$$

$$+ \sum_k c_k[n] * s_k[n]$$

$$\ddot{\vec{r}} = \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

(centrifugal force)

$$\alpha\bar{\phi} + \beta\bar{\eta} + \gamma\bar{y} + \delta\bar{z} + \delta_1\ddot{o}_1$$

(sensing crosstalk)

IFOX1X12

crosstalk and other contributions

Subtraction of the centrifugal forces…

$$\Delta g(t) = \ddot{o}_{12}(t) + \omega_2^2 o_{12}(t) + \Delta\omega^2 o_1(t) - C_{sus} g_c(t) - c_1 \dot{g}_c(t)$$

$$+ \sum_k c_k[n] * s_k[n]$$

$$\ddot{\vec{r}} = \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

(centrifugal force)

$$\alpha\bar{\phi} + \beta\bar{\eta} + \gamma\bar{y} + \delta\bar{z} + \delta_1\ddot{o}_1$$

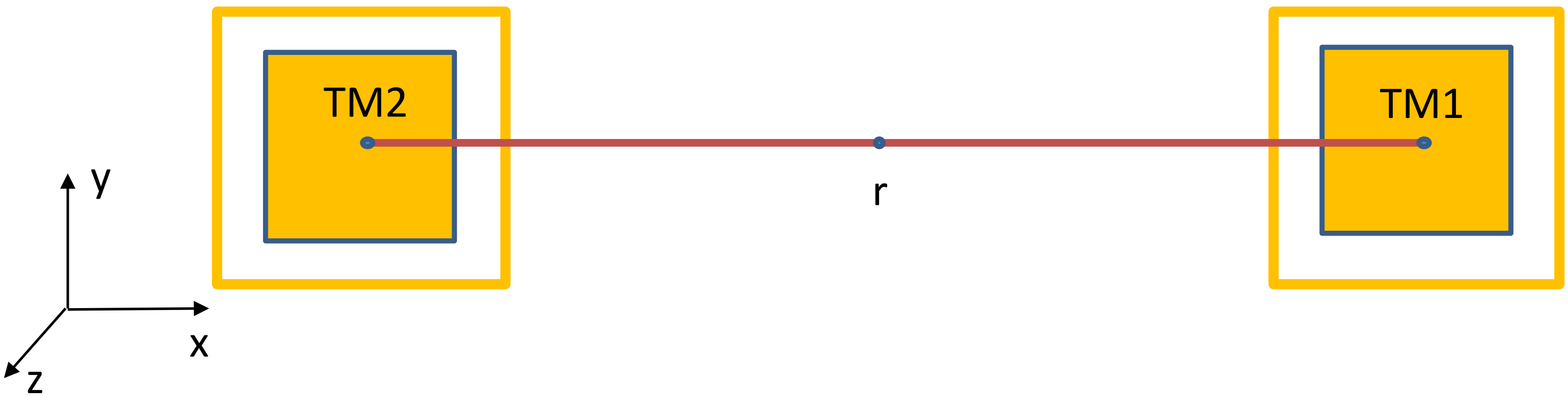
(sensing crosstalk)

IFOX1X12

crosstalk and other contributions

… a.k.a. Decentrifugation

$$\ddot{x}(t) = (\dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) \cdot \hat{x}$$



$$\ddot{x}_1 = (-\omega_\phi^2 - \omega_\eta^2)x_1 + (-\dot{\omega}_\phi + \omega_\eta\omega_\theta)y_1 + (-\dot{\omega}_\eta - \omega_\phi\omega_\theta)z_1$$

$$\ddot{x}_2 = (\omega_\phi^2 + \omega_\eta^2)x_2 + (\dot{\omega}_\phi - \omega_\eta\omega_\theta)y_2 + (\dot{\omega}_\eta + \omega_\phi\omega_\theta)z_2$$

Δg non inertial

$$\Delta g_\Omega = \ddot{x}_2 - \ddot{x}_1 = (\omega_\phi^2 + \omega_\eta^2)(x_2 + x_1) +$$
$$(\dot{\omega}_\phi - \omega_\eta \omega_\theta)(y_2 - y_1) +$$
$$(\dot{\omega}_\eta + \omega_\phi \omega_\theta)(z_2 - z_1)$$

$$\Delta g_\Omega = (\omega_\phi^2 + \omega_\eta^2) 2 x_1$$

← ***Dead-reckoning subtractions***

How do we get it?

$$\Delta g_\Omega = \ddot{x}_2 - \ddot{x}_1 = (\omega_\phi^2 + \omega_\eta^2)(x_2 + x_1) +$$

$$(\dot{\omega}_\phi - \omega_\eta \omega_\theta)(y_2 - y_1) +$$

$$(\dot{\omega}_\eta + \omega_\phi \omega_\theta)(z_2 - z_1)$$

$$\Delta g_\Omega = (\omega_\phi^2 + \omega_\eta^2) 2 x_1$$

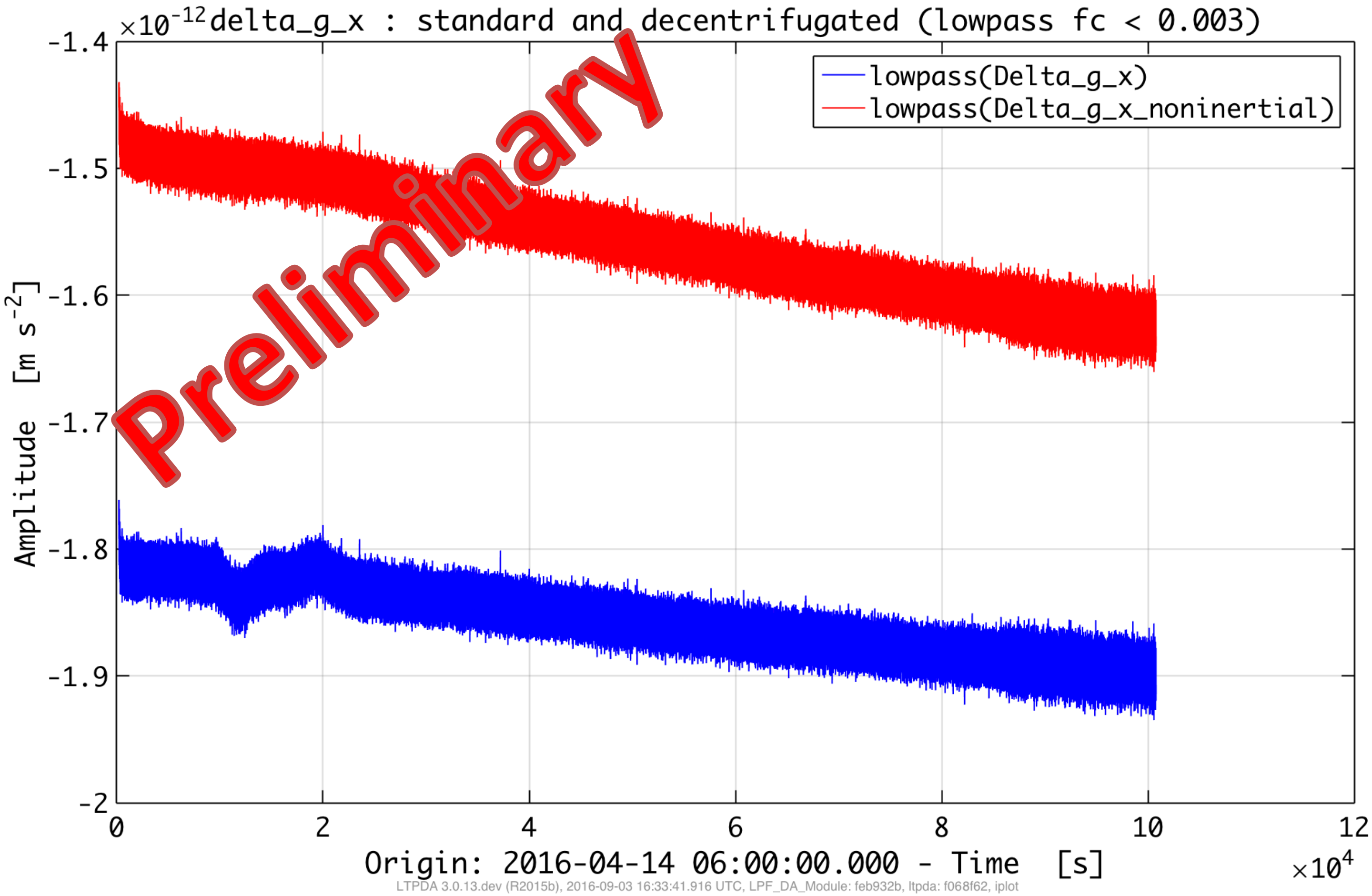
← ***Dead-reckoning subtractions***

$$\omega = \omega_{DC} + \omega_{Noise}$$

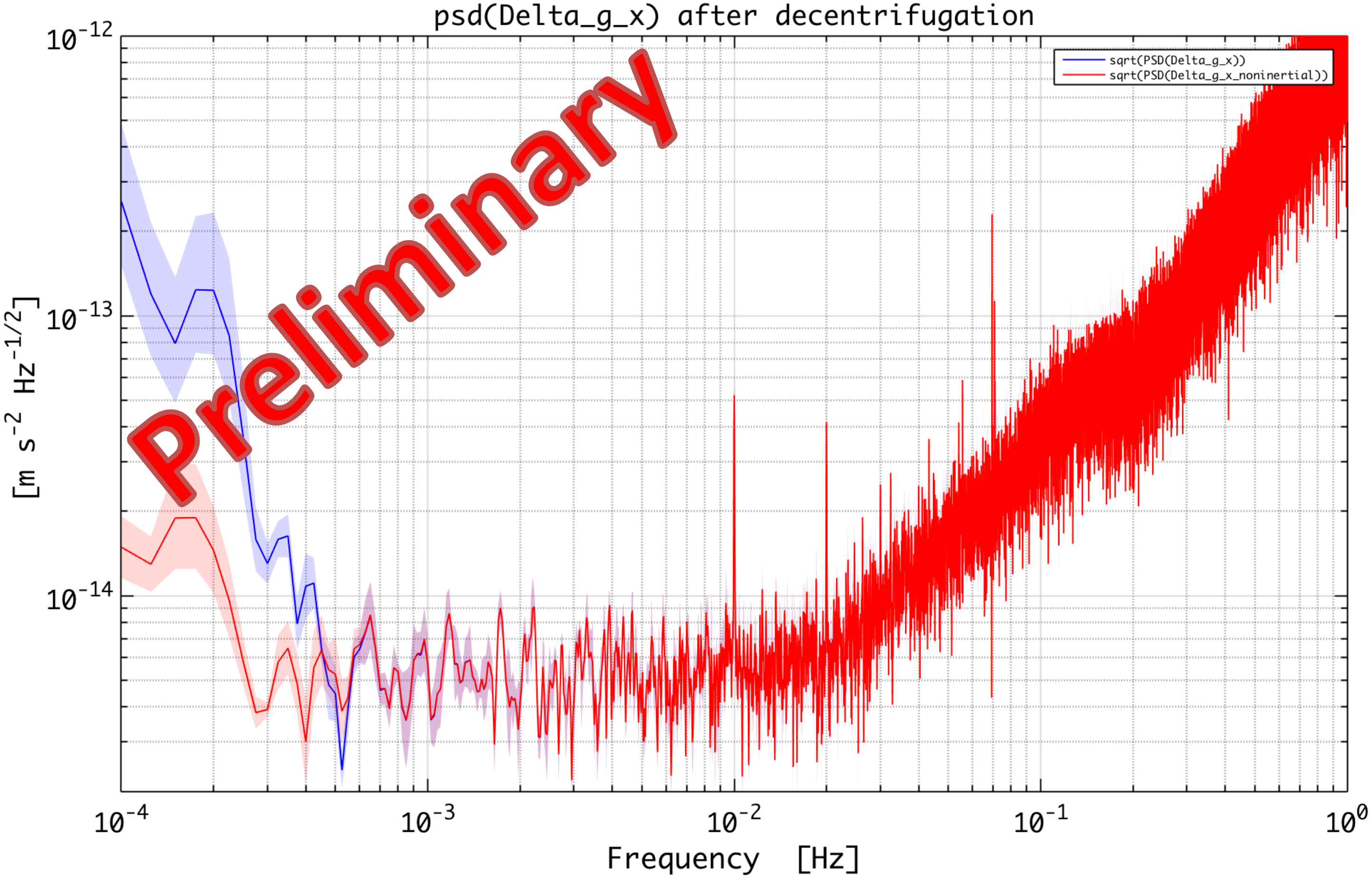
$$\omega_{DC} \simeq \omega_{ST,polyfit}$$

$$\omega_{Noise} \simeq \omega_{TM,gyro} \simeq \int N_{angles} dt$$

Some features disappear after subtracting centrifugal forces



Sometimes noise at low frequency decreases after subtracting centrifugal forces



Δg corrected (or debumped)

$$\Delta g(t) = \ddot{o}_{12}(t) + \omega_2^2 o_{12}(t) + \Delta\omega^2 o_1(t) - C_{sus} g_c(t) - c_1 \dot{g}_c(t)$$

$$+ \sum_k c_k[n] * s_k[n]$$

crosstalk and other contributions

$$\ddot{\vec{r}} = \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

(centrifugal force)

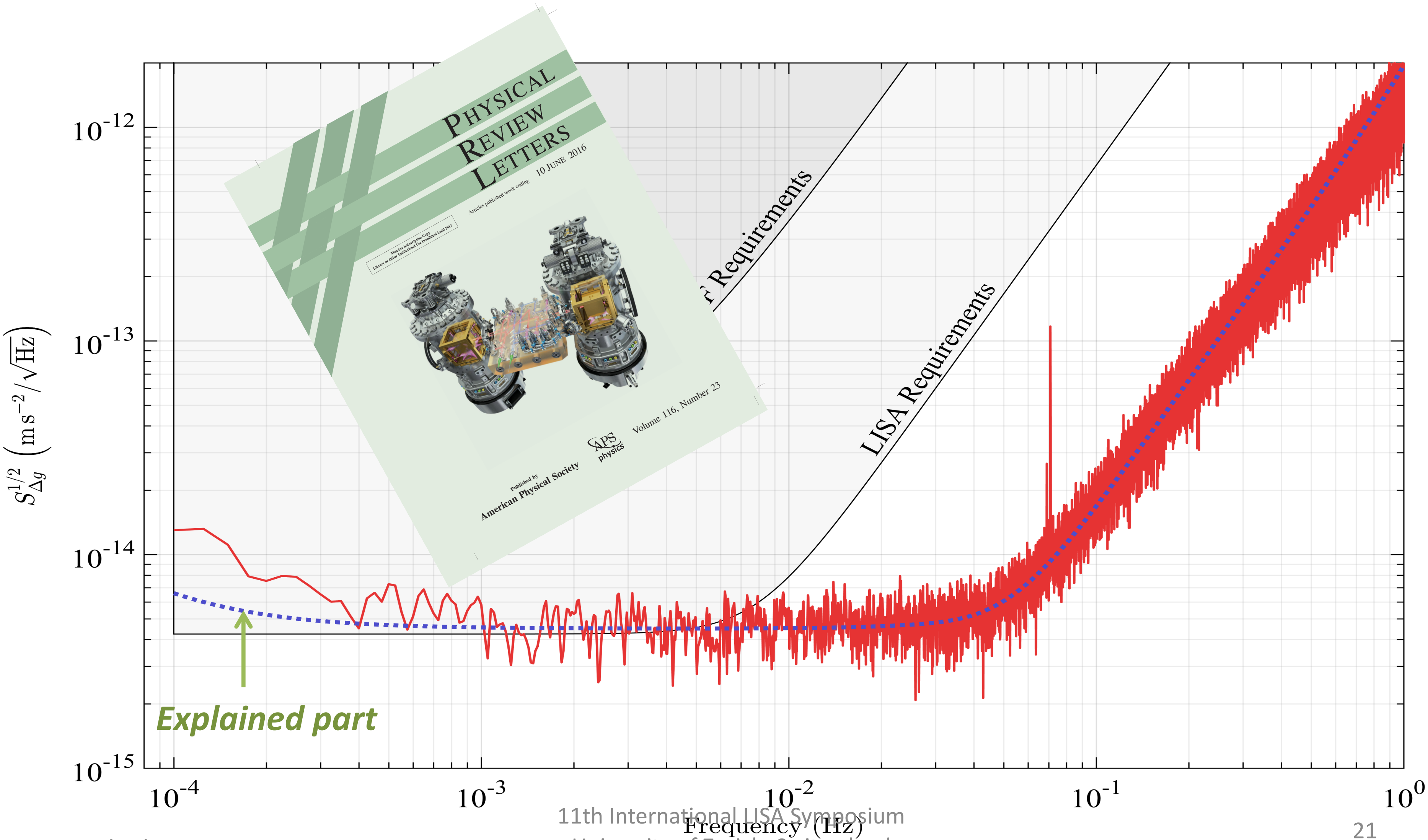
$$\alpha\bar{\phi} + \beta\bar{\eta} + \gamma\bar{y} + \delta\bar{z} + \delta_1\ddot{o}_1$$

(sensing crosstalk)

IFOX1X12

See of G. Wanner poster

The published Δg



Other correction to Δg

- Gas depletion in the tanks that makes the DC Δg to change (see V. Ferroni's poster).

- Speculative corrections:
  - Correction for a possible error in commanded force introduced by digitization.
  - Second order effects in the centrifugal forces.