# Geons in asymptotically anti-de Sitter spacetimes

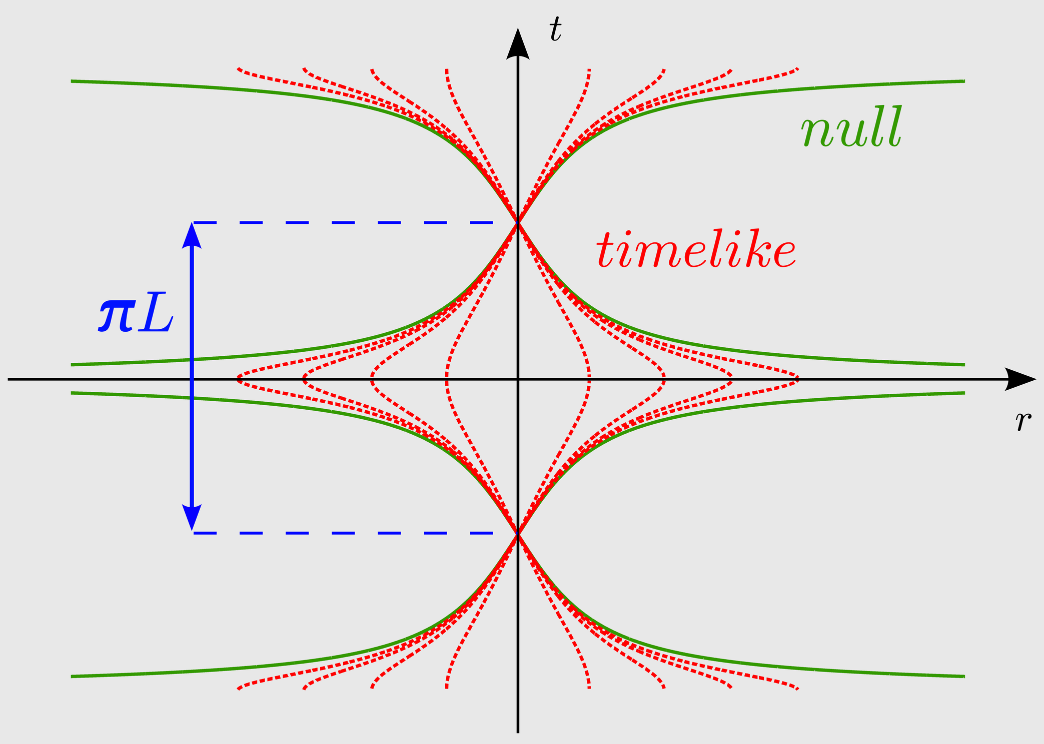
Grégoire Martinon, Gyula Fodor, Philippe Grandclément

Observatoire de Paris

Université Paris Diderot

## What is AdS spacetime ?

$$R_{\mu\nu} = \Lambda g_{\mu\nu} \qquad \Lambda = -\frac{3}{L^2}$$



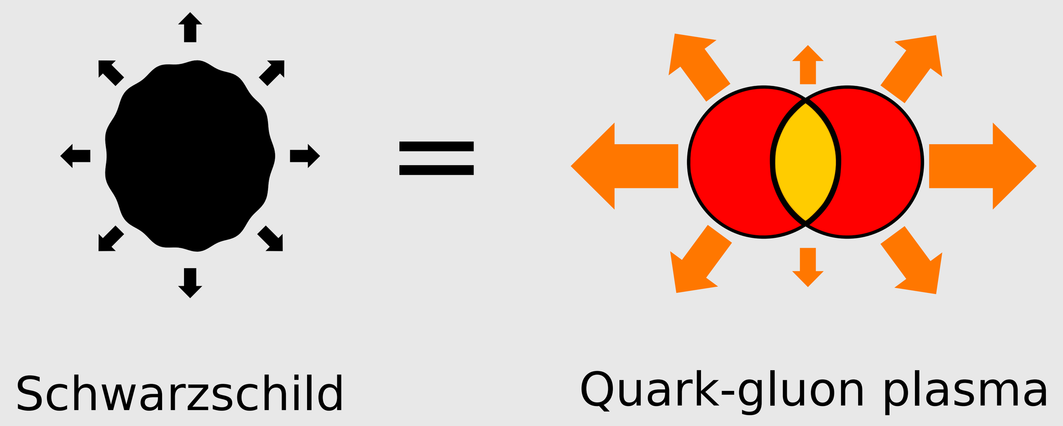

## AdS/CFT correspondance

$AdS_5 \Leftrightarrow \mathcal{N} = 4$ SYM $\Leftrightarrow$ proxy for QCD

**AdS/CFT recipe**

- Solve Einstein equation in $AdS$
- Euclidean action $\underline{S_E}$ on the AdS boundary
- GKP-Witten : $Z_{\mathrm{CFT}} \simeq exp(-\underline{S_E})$

One example (out of many) : Damping time $\Leftrightarrow$ viscosity




## Instability of AdS

$$R_{\alpha\beta} = \Lambda g_{\alpha\beta}$$

*Perturbative expansion :* $y = y_0 + \varepsilon y_1 + \varepsilon^2 y_2 + \dots$

Third order equation :

$$y_3'' + \omega_j^2 y_3 = A\cos(\omega_j t) + \dots$$

$$\Rightarrow y_3 = \frac{A}{2\omega_j} t \sin(\omega_j t) + \dots$$

Secular resonance : breakdown at $\boxed{t = O(\varepsilon^{-2})}$.

*Numerical simulations :* (Bizon and Rostworowski 2011)

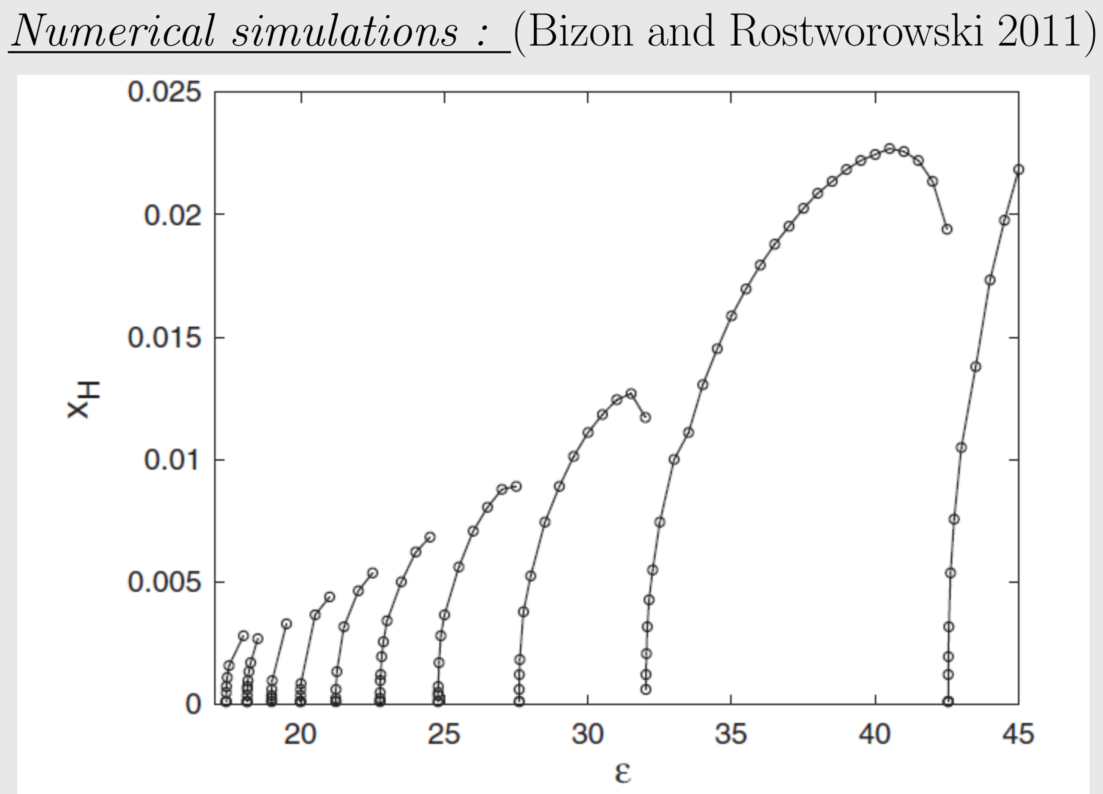



FIG. 1. Horizon radius vs amplitude for initial data (9). The number of reflections off the AdS boundary before collapse varies from zero to nine (from right to left).

Formation of Black Hole $\boxed{t = O(\varepsilon^{-2})}$.

## Geons

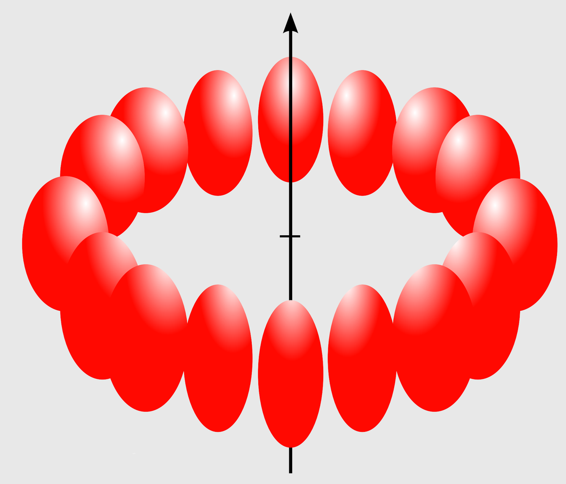
**GEON = Electro-Gravitational Entity**

(originally coined by J.A. Wheeler in 1955)

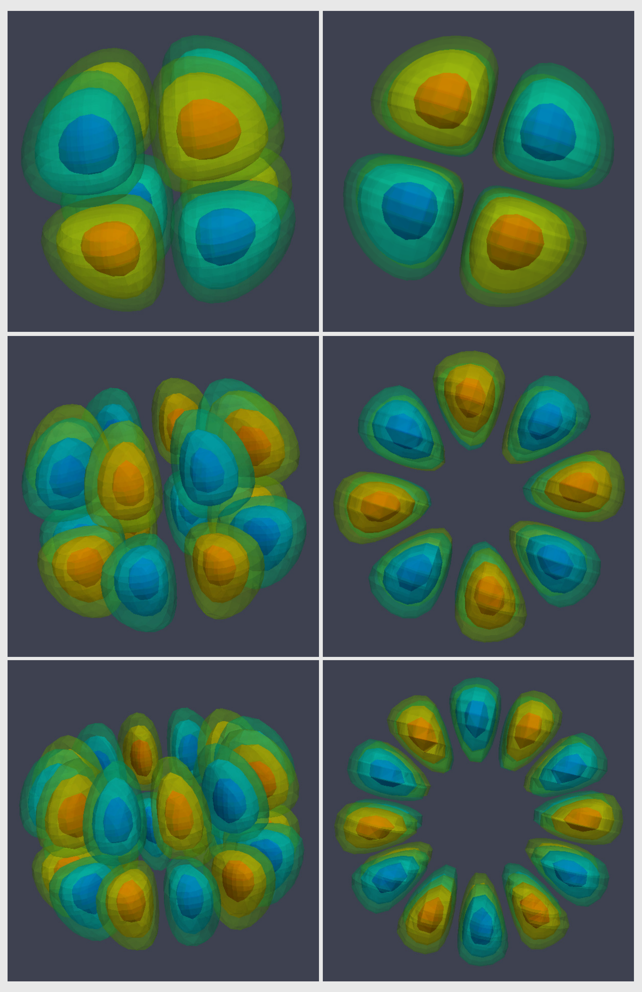
- **Massive boson:** boson stars (Buchel+ 2013)
- **Massless boson:** scalar breathers (Fodor+ 2015)
- **Massive vector:** Proca stars (Brito+ 2015)
- **Massless vector:** Maxwell solitons (Heirdeiro+ 2016)
- ***Gravitational geons :*** (Horowitz and Santos 2015)

## What is a gravitational geon ?

- Self-gravitating
- Self-rotating
- Self-interacting



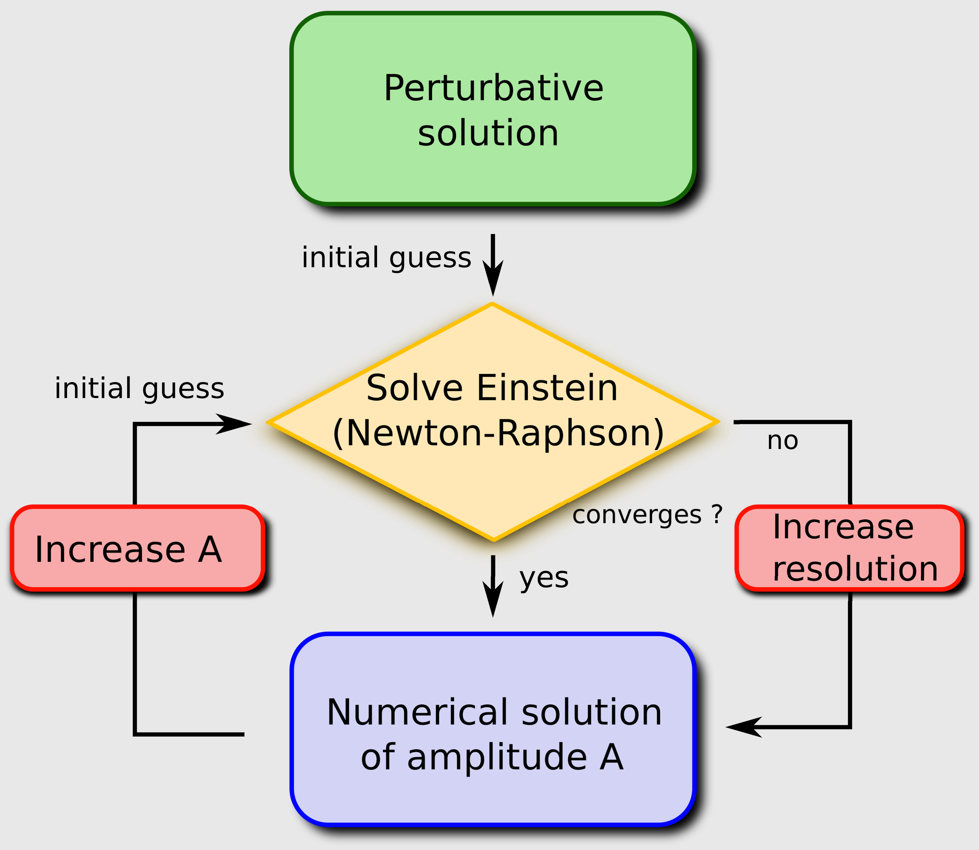
## Pictures



## How to build a geon ?

**(Dias et al 2012)**

- Look for periodic GW
- Choose the $Y_m^l$ you prefer
- Perturbative expansion
  $y = y_0 + \varepsilon y_1 + \varepsilon^2 y_2 + \dots$
  $y_3'' + \omega^2 y_3 = A\cos(\omega t) + \dots$
  $\Rightarrow$ BAD !
- Poincaré-Linstedt method :
  Promote $\omega = \omega_0 + \varepsilon^2 \omega_2 + \dots$
  $\Rightarrow$ GOOD ! (sometimes)
- Look for helical symmetry
  Killing vector $\partial_\tau = \partial_t + \omega\partial_\varphi$
  $\Rightarrow$ AWESOME !




## Gauge freedom

Consider 3+1 formalism, the 3D metric $\gamma_{ij}$ and the 3D Ricci tensor :

$$R_{ij} = -\frac{\gamma^{kl}}{2}\Big(\underbrace{\partial_k\partial_l\gamma_{ij}}_{\text{laplacian}} + \underbrace{\partial_i\partial_j\gamma_{kl} - \partial_i\partial_l\gamma_{jk} - \partial_j\partial_k\gamma_{il}}_{\text{BAD !}}\Big) + \underbrace{\gamma^{kl}\gamma_{mn}(\Gamma^l_{jk}\Gamma^k_{il} - \Gamma^m_{kl}\Gamma^n_{ij})}_{1^{st}\text{ order derivatives}}$$

But if we introduce $\boxed{V^i = \gamma^{kl}(\Gamma^i_{kl} - \bar{\Gamma}^i_{kl})}$ and $\boxed{V_{ij} = D_{(i}V_{j)}}$, then :

$$R_{ij} - V_{ij} = \text{laplacian} + 1^{st} \text{ order derivatives}$$

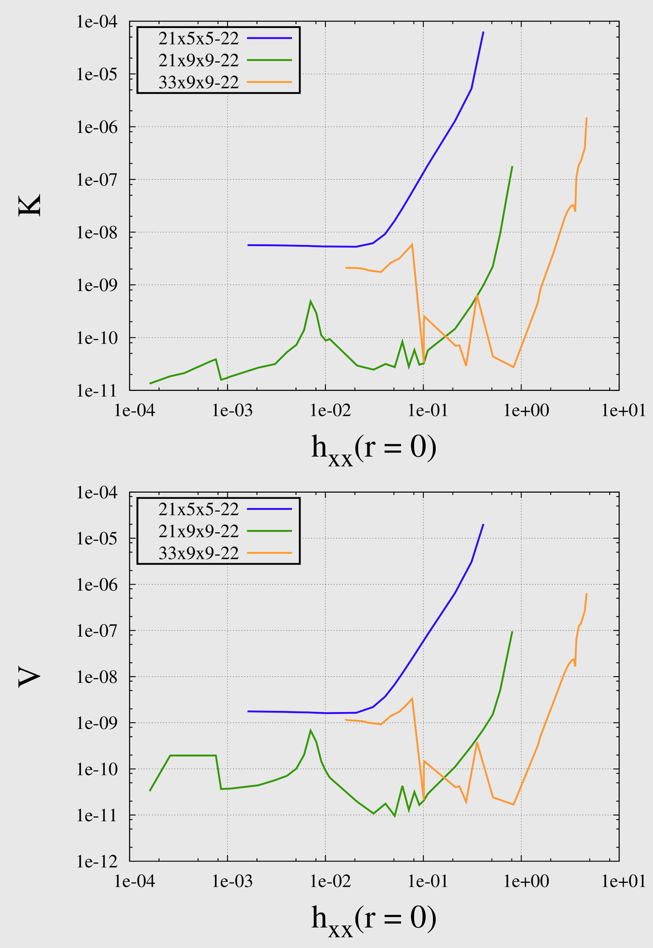
We want $K = 0$ and $V^i = 0$ (**4 gauge conditions**), so we solve :

$$R - D_i V^i + K^2 - K_{ij}K^{ij} - 2\Lambda = 0$$

$$D_j K^j_i - D_i K = 0$$

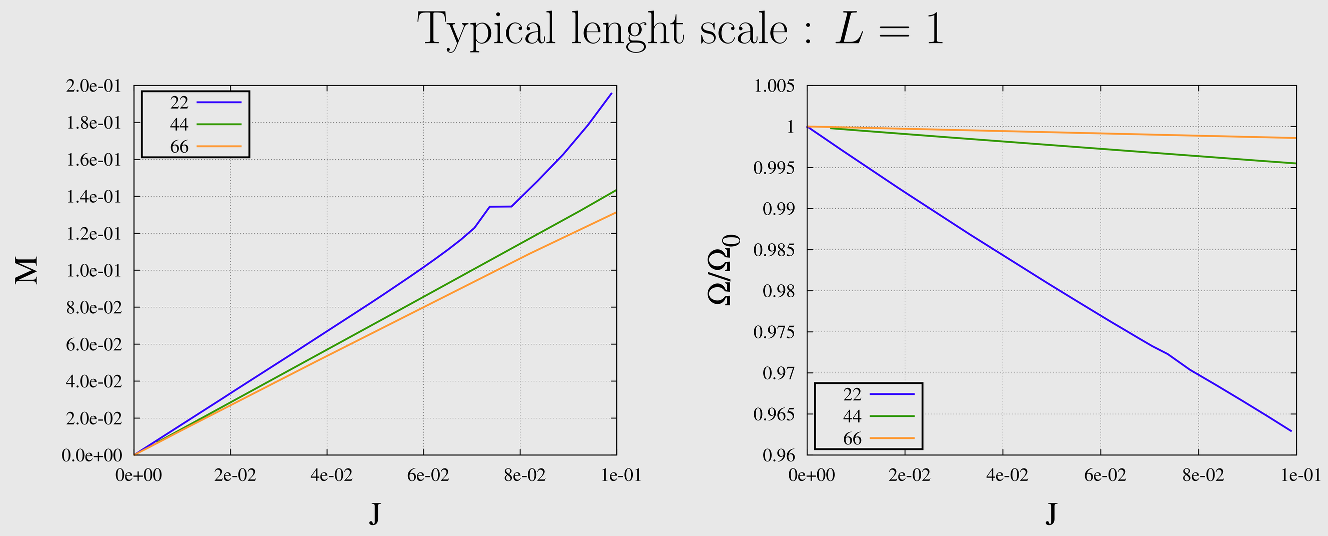
$$\mathcal{L}_\beta K_{ij} - D_i D_j N + N\left[R_{ij} - V_{ij} + K K_{ij} - 2K_{ik}K^k_j - \Lambda\gamma_{ij}\right] = 0$$

and check *a posteriori* that $K$ and $V^i$ are indeed zero.
(Andersson and Moncrief 2003)



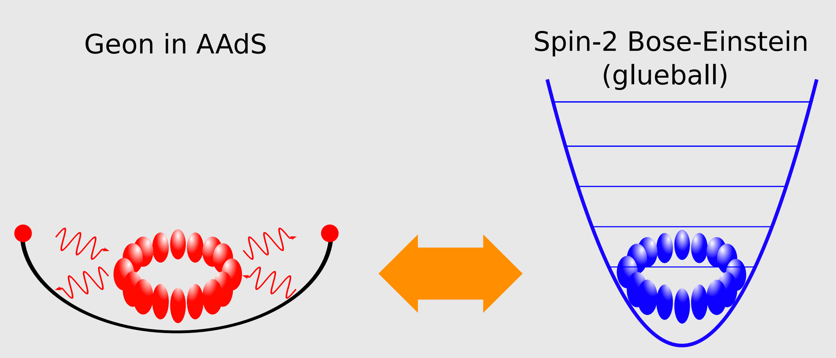

## Global quantities

Typical lenght scale : $L = 1$




## CFT dual

- Dual representation :
  No black hole $\Rightarrow$ $\boxed{T = 0}$




## References

Martinon, G., Fodor, G. & Grandclément, P., *in preparation*