Dark Attack

Global Fits - Bayesian

Roberto Ruiz de Austri

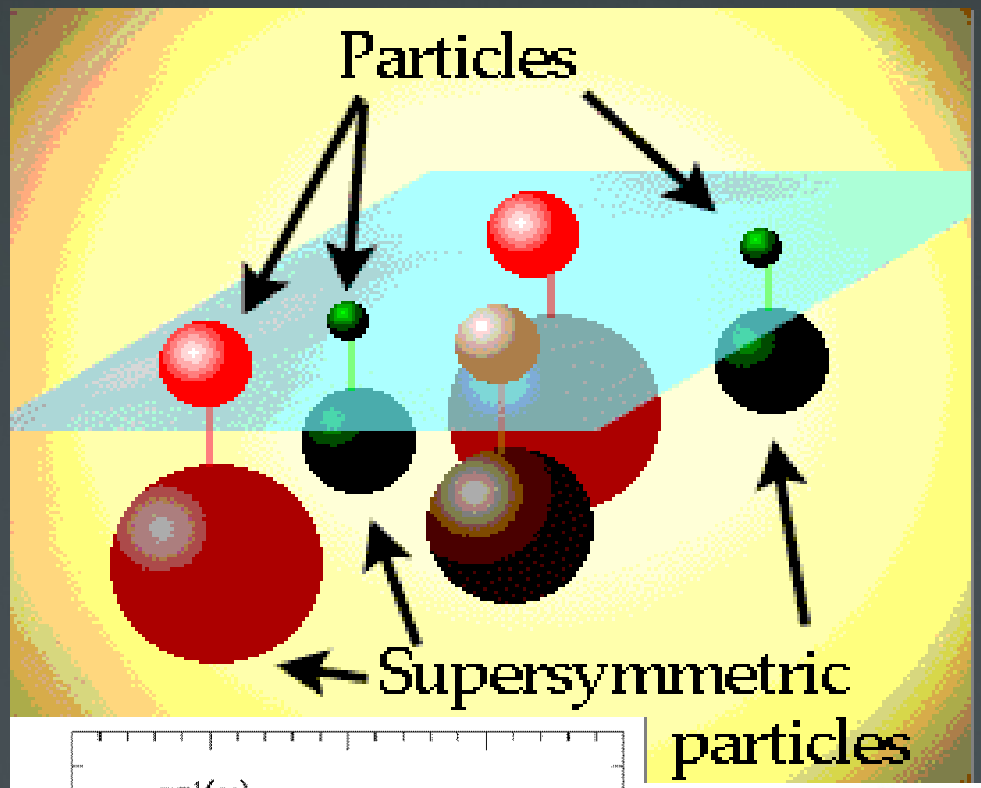
IFIC (Valencia)

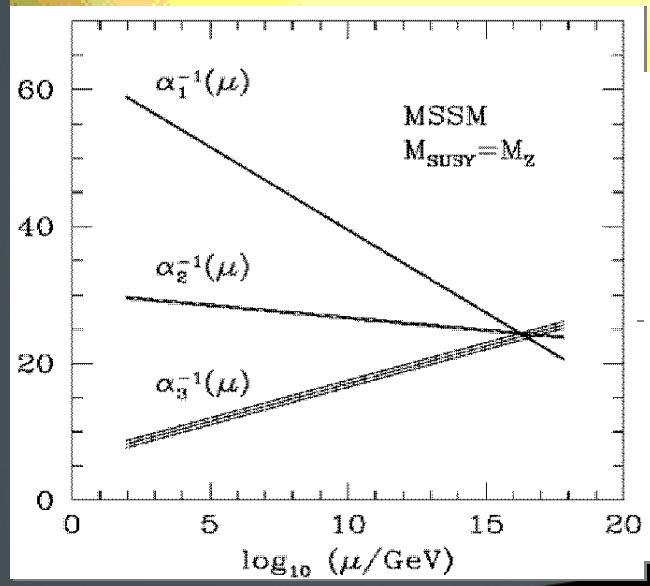
Outline

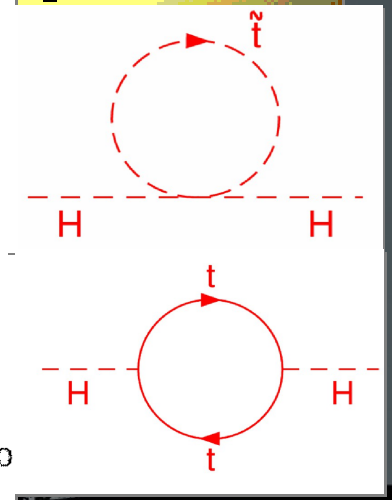
- Why SUSY inference in Bayesian sense ?
- How is this done ?
- cMSSM: case study
- Complementarity of LHC and DM Direct Detection experiments

Supersymmetry

- Perhaps the most theoretically appealing (certainly the most well studied) extension of the Standard Model
- Natural solution to hierarchy problem (stabilizes quadratic divergences to Higgs mass)
- Restores unification of couplings
- Natural theory of gravity (Supergravity)
- Vital ingredient of string theory
- Naturally provides a compelling candidate for dark matter







Minimal Supersymmetric Standard Model: MSSM

Particle Spectrum:

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\tilde{g}$	g	$(\mathbf{8},\ \mathbf{1},\ 0)$
winos, W bosons	$\tilde{W}^{\pm}\quad \tilde{W}^{0}$	$W^{\pm}\quad W^{0}$	$(\mathbf{1},\ \mathbf{3},\ 0)$
bino, B boson	$\tilde{B}^{0}$	B^{0}	$(\mathbf{1},\ \mathbf{1},\ 0)$

Names		spin 0	spin 1/2	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
squarks, quarks	Q	$(\tilde{u}_L\ \ \tilde{d}_L)$	$(u_L\ \ d_L)$	$(\mathbf{3},\ \mathbf{2},\ \frac{1}{6})$
(×3 families)	$\overline{u}$	$\tilde{u}_R^*$	$u_R^\dagger$	$(\overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	$\overline{d}$	$\tilde{d}_R^*$	$d_R^\dagger$	$(\overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	L	$(\tilde{\nu}\ \ \tilde{e}_L)$	$(\nu\ \ e_L)$	$(\mathbf{1},\ \mathbf{2},\ -\frac{1}{2})$
(×3 families)	$\overline{e}$	$\tilde{e}_R^*$	$e_R^\dagger$	$(\mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u	$(H_u^+\ \ H_u^0)$	$(\tilde{H}_u^+\ \ \tilde{H}_u^0)$	$(\mathbf{1},\ \mathbf{2},\ +\frac{1}{2})$
	H_d	$(H_d^0\ \ H_d^-)$	$(\tilde{H}_d^0\ \ \tilde{H}_d^-)$	$(\mathbf{1},\ \mathbf{2},\ -\frac{1}{2})$

The partners of the gauge and Higgs bosons mix to form the Neutralinos and Charginos.

$$\begin{pmatrix} M_1 & 0 & -m_Z\cos\beta\sin\theta_W & m_Z\sin\beta\sin\theta_W \\ 0 & M_2 & m_Z\cos\beta\cos\theta_W & -m_Z\sin\beta\cos\theta_W \\ -m_Z\cos\beta\sin\theta_W & m_Z\cos\beta\cos\theta_W & 0 & -\mu \\ m_Z\sin\beta\sin\theta_W & -m_Z\sin\beta\cos\theta_W & -\mu & 0 \end{pmatrix}$$

$$\chi^0 = N_{11}\tilde{B} + N_{12}\tilde{W}^3 + N_{13}\tilde{H}_1 + N_{14}\tilde{H}_2$$

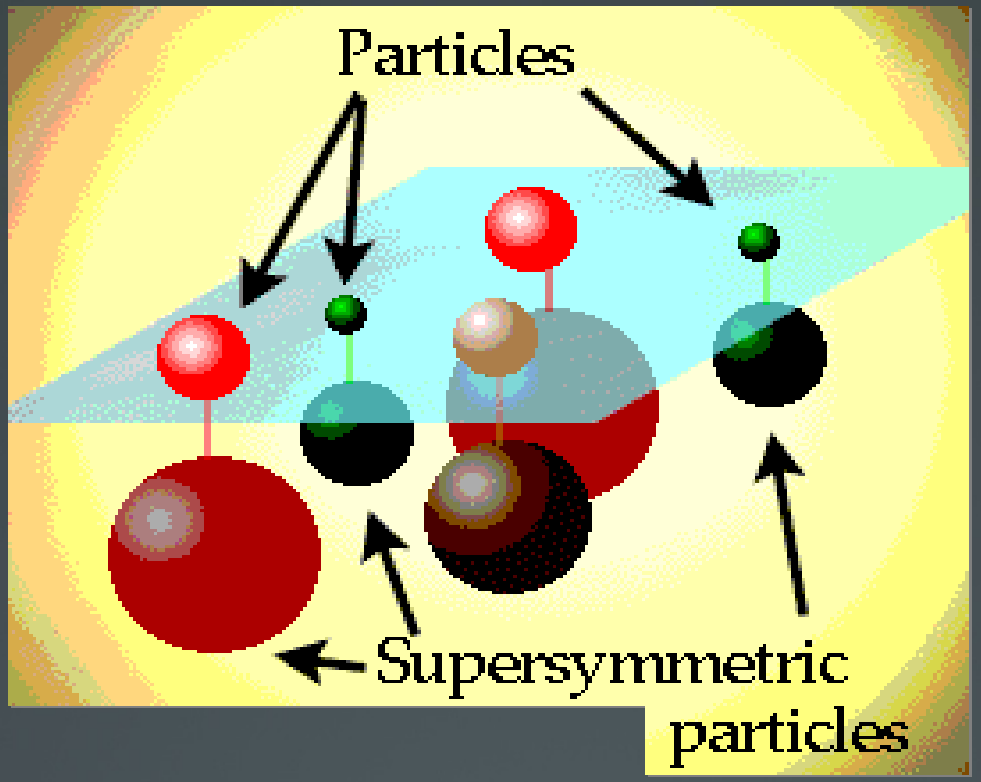
Also left- and right-chiral Sfermions mix

124 parameters !!!

All couplings for SUSY particles are the same as for SM particles!

Supersymmetric Dark Matter

- R-parity must be introduced in supersymmetry to prevent rapid proton decay
- Another consequence of R-parity is that superpartners can only be created and destroyed in pairs, making the lightest supersymmetric particle (LSP) stable
- Possible WIMP candidates from supersymmetry include:



γ, Z, h, H ← 4 Neutralinos

ν ← 3 Sneutrinos

Excluded by DM DD

Weakly Interacting Massive Particles (WIMPs)

- As a result of the thermal freeze-out process, a relic density of WIMPs is left behind:

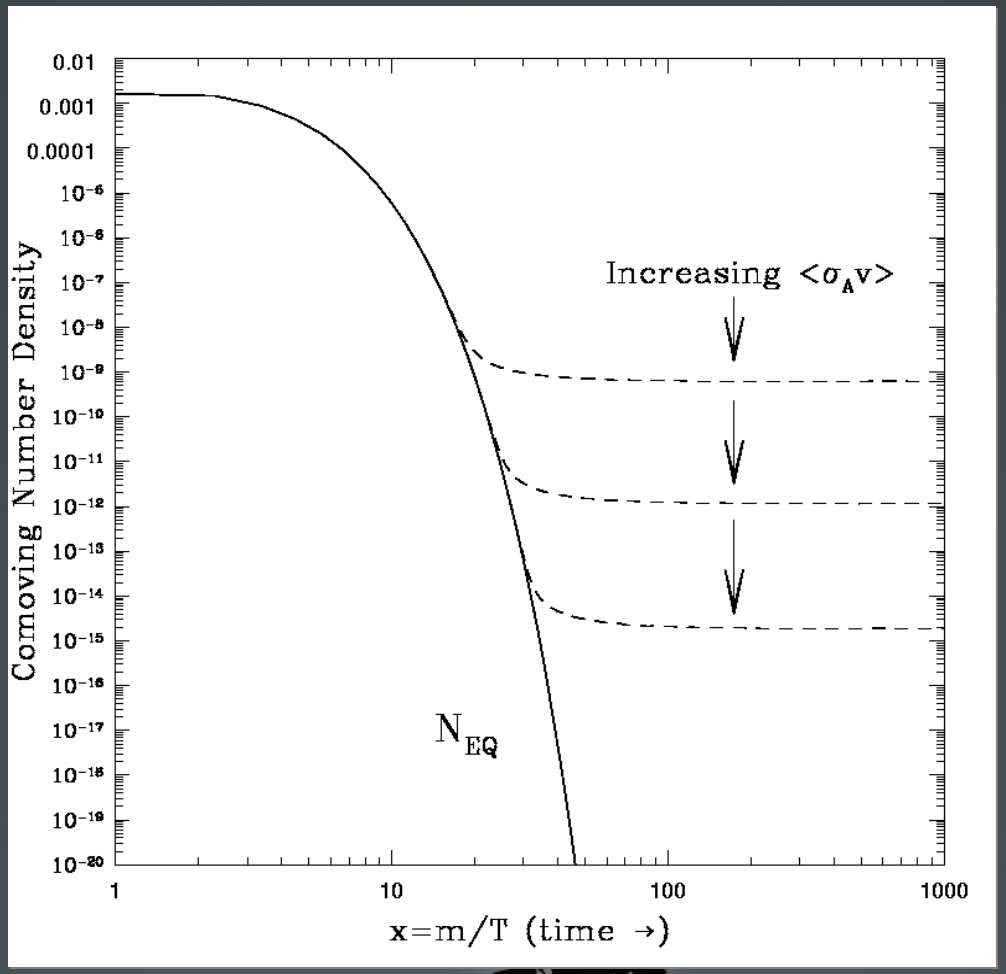
$$\Omega h^2 \sim x_F / \langle \sigma v \rangle$$

- For a particle with a GeV-TeV mass, to obtain a thermal abundance equal to the observed dark matter density, we need an annihilation cross section of

$$\langle \sigma v \rangle \sim \text{pb}$$

- Generic weak interaction yields:

$$\langle \sigma v \rangle \sim \alpha^2 (100\ \text{GeV})^{-2} \sim \text{pb}$$



Numerical coincidence? Or an indication that dark matter originates from EW physics?

Constrained Minimal Supersymmetric Standard Model: CMSSM

- 5 parameters (MSSM has 124) => suitable for pheno analyses

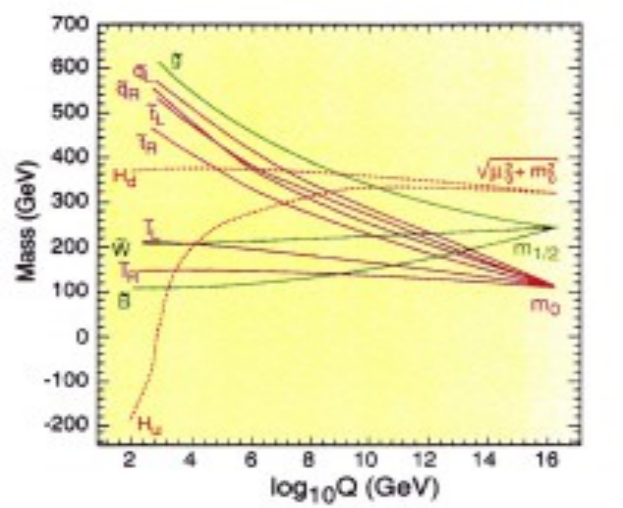
0. Common Gaugino mass: m1/2
1. Common Scalar mass: m0
2. Common trilinear coupling: A0
3. Ratio of the Higgs vacuum expectation values: tanβ
4. Sign of the Higgsino mass parameter μ

- First 3 parameters are for some high unification scale Mx.

Supersymmetry is spontaneously broken by gravitational couplings with a hidden sector on the Planck scale

∑ Fields get vevs

∑ Analog to Higgs mechanism



Goal: inference of the model parameters from collider and DM data but it is difficult problem

Is SUSY exact ?

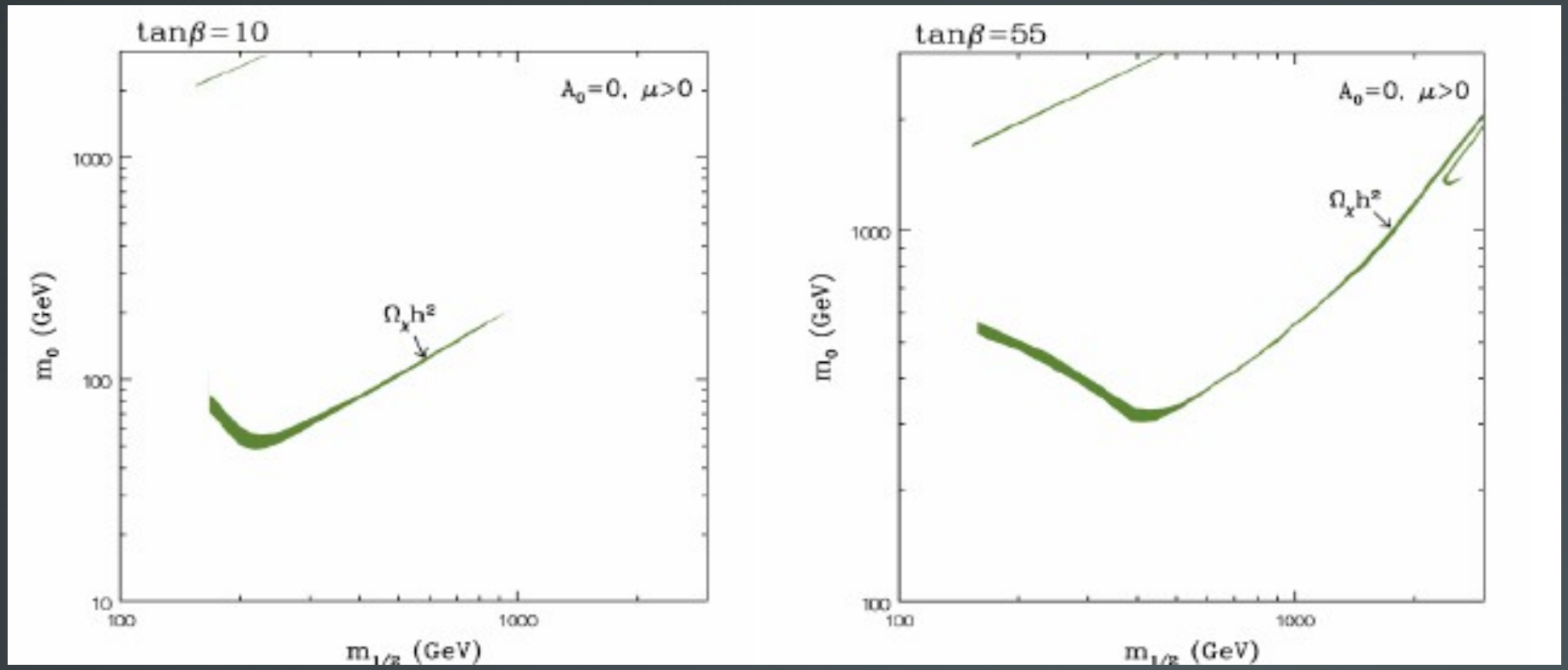
- SUSY theories have to be broken since we have not seen SUSY particles
- To solve the hierarchy problem SUSY has to be broken at the TeV scale ($mF - mB \sim 1$ TeV)
- Different mechanisms to generate the mass spectrum:

- Gravity mediated, gravity interactions with hidden sector fields, e.g. minimal supergravity (mSUGRA) -> CMSSM
- Gauge mediated, gauge interactions with messenger sector, GMSB
- Others: Anomaly mediated …

Fixed Grid Scans

$\tan\beta \lesssim 45$

$\tan\beta \gtrsim 45$

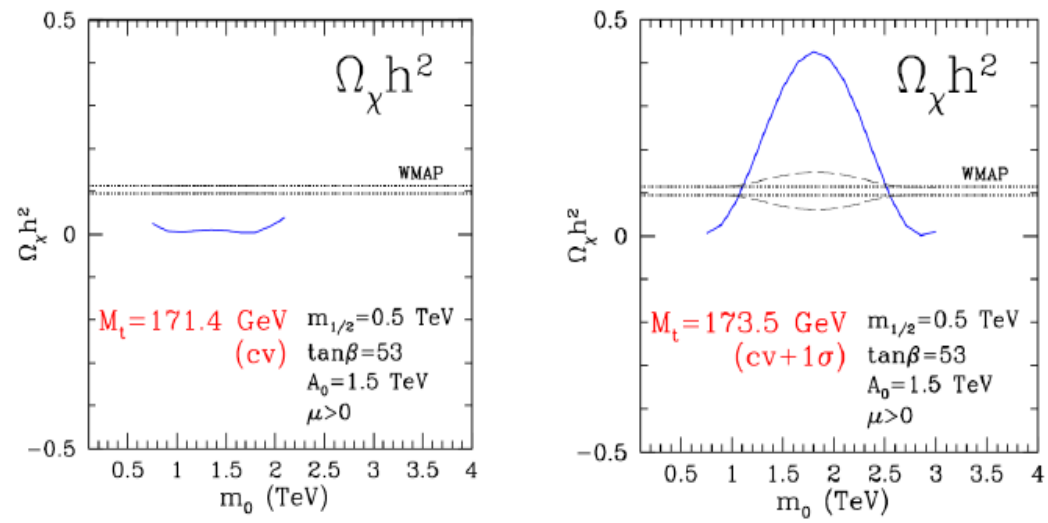


- green: consistent with WMAP-7yr (at 2σ)
- all the rest excluded by LEP, BR(B → Xs γ), Ωχ h2 , EWSB, charged LSP,...

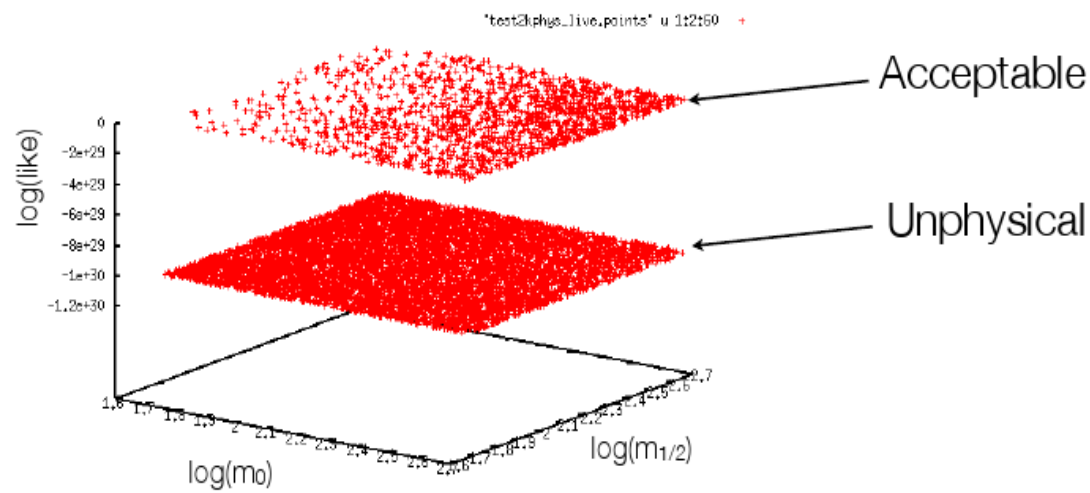
Why is a difficult problem ?

- Inherently 8-dimensional: reducing the dimensionality over-simplifies the problem. Nuisance parameters (in particular mt) cannot be fixed!

- Likelihood discontinuous and multi-modal due to physicality conditions



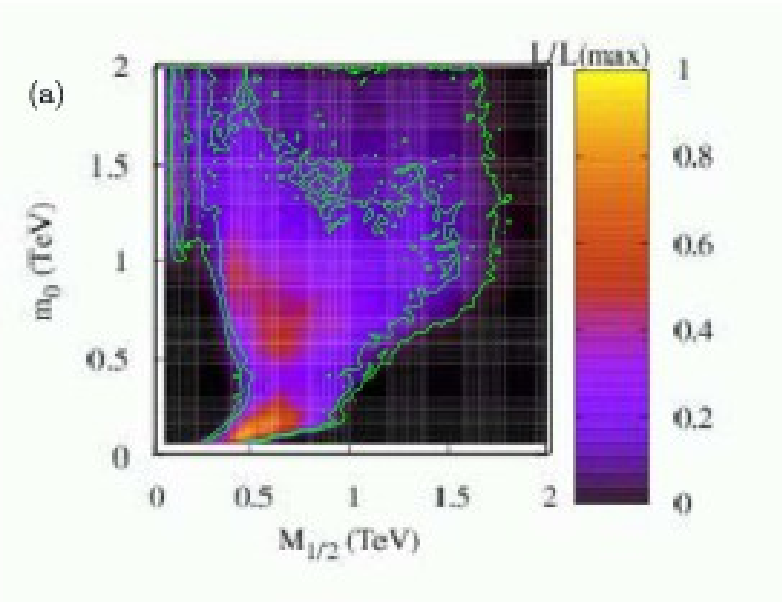
Roszkowki et al (2007)



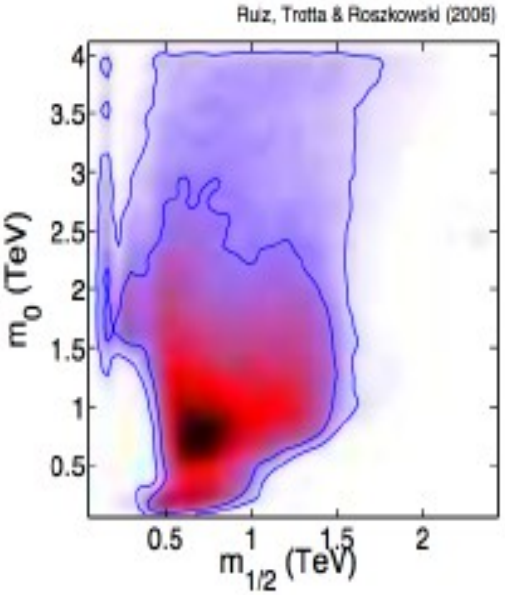
Courtesy F. Feroz & M. Bridges

The Bayesian Approach

- Bayesian approach led by two groups (early work by Baltz & Gondolo, 2004)
- Ben Allanach (DAMPT) et al (Allanach & Lester, 2006 onwards, Cranmer, and others)
- RdA, Roszkowski & Roberto Trotta (2006 onwards) SuperBayeS public code (available from: superbayes.org) + Feroz & Hobson (MultiNest), + Silk (indirect detection), + de los Heros (IceCube), + Casas et al. (Naturalness) + Bertone et al. (pmssm)



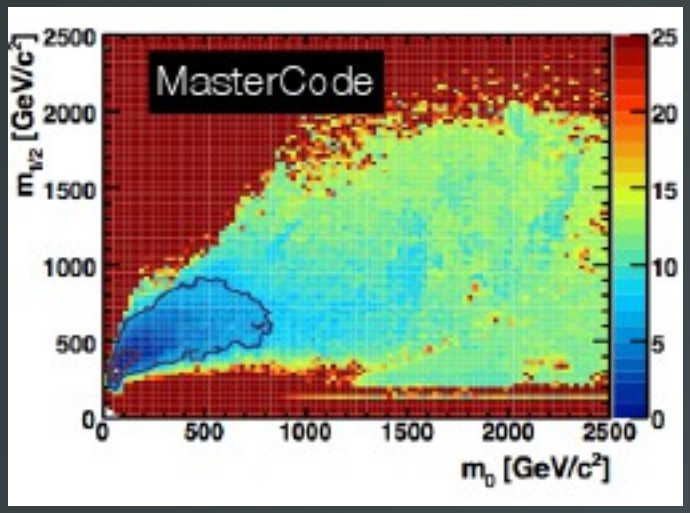
Allanach & Lester (2006)



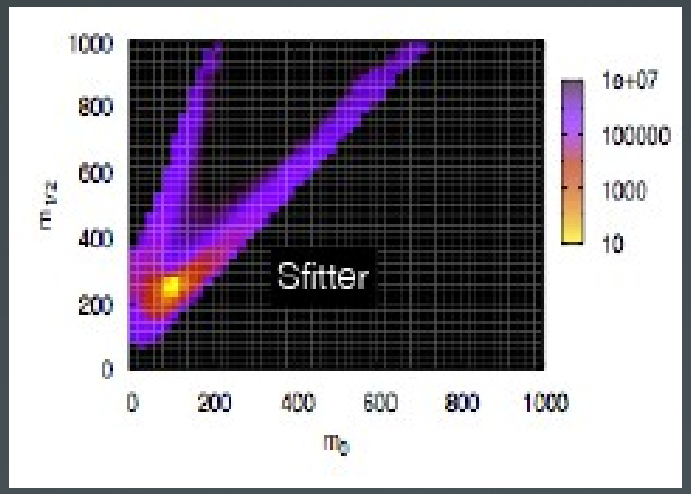
Ruiz de Austri, Roszkowski & RT (2006)

The Likelihood-based Approach

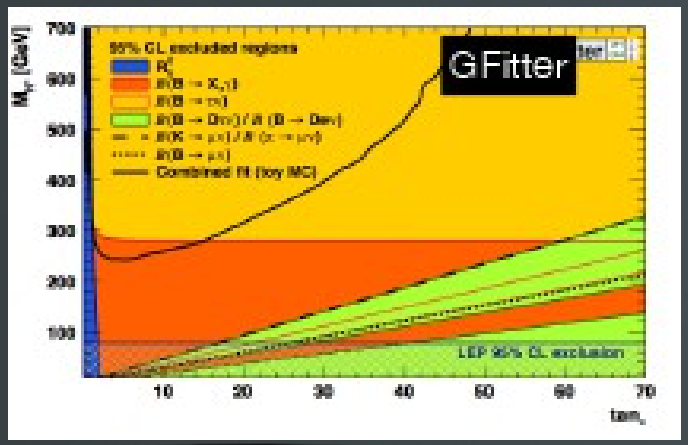
O. Buchmueller, R. Cavanaugh, A. De Roeck,
J.R. Ellis, H.Flacher, S. Heinemeyer, G. Isidori,
K.A. Olive, F.J. Ronga, G. Weiglein



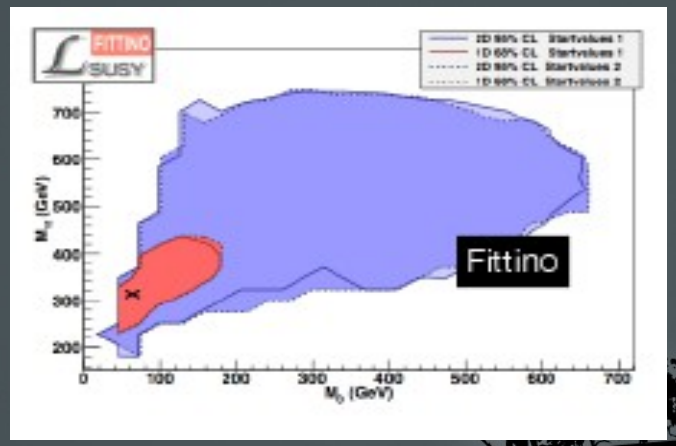
R. Lafaye , M. Rauch, T. Plehn, D. Zerwas



H. Flächer, M. Goebel, J. Haller,
A. Höcker, K. Mönig, J. Stelzer



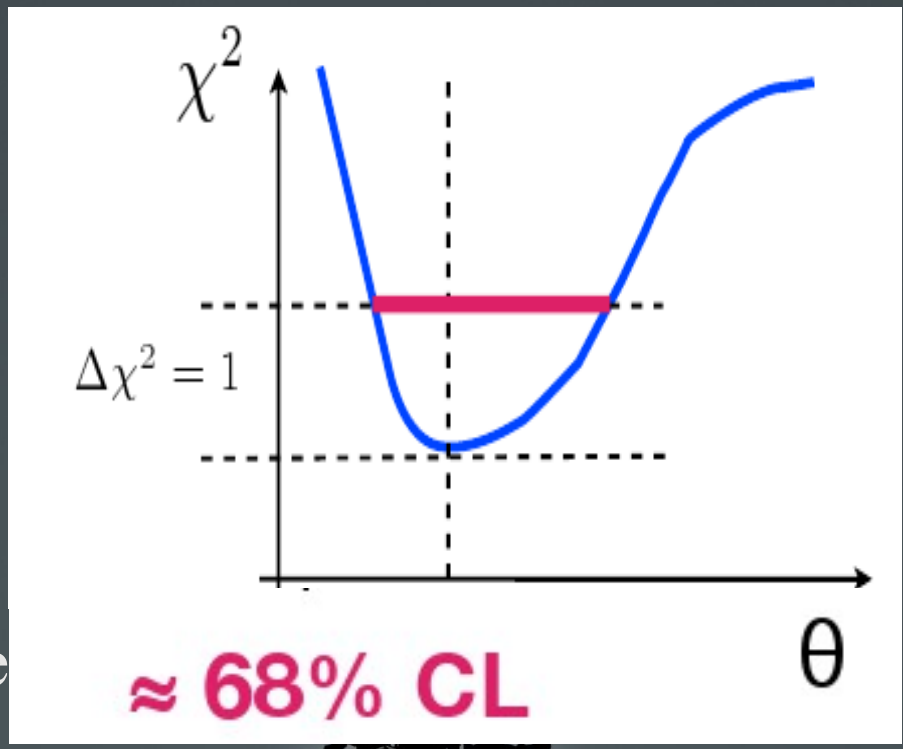
P. Bechtle, K. Desch, M. Uhlenbrock, P. Wienemann



Favoured regions: Likelihood-based Approach

- Due to the weak nature of constraints, different scanning techniques and statistical methods will generally give different answers
- Likelihood-based methods: determine the best fit parameters by finding the minimum of -2 Log(Likelihood) = chi-squared

1. Markov Chain Monte Carlo and Minuit as "afterburner"
2. Simulated annealing
3. Genetic algorithms



- Determine approximate confidence intervals: Local Δ(chi-squared) method
- Profile likelihood: way to treat nuisance

L(x,y) => PL(x) = max. L(x,y) for fixed x in y

Bayes Theorem

posterior

likelihood

prior

$$P(H|d,I) = \frac{P(d|H,I)P(H|I)}{P(d|I)}$$

evidence

- H: hypothesis
- D: data
- I: external information

- Prior: what we know about H (given information I) before seeing the data
- Likelihood: the probability of obtaining data d if hypothesis H is true
- Posterior: the probability of obtaining data d if hypothesis H is true
- Evidence: normalization constant (independent of H), crucial for model comparison

Priors

- Ignoring the prior and identifying

$$p(\theta_i|\text{data}) \equiv p(\text{data}|\theta_i)$$

- implicitly amounts to

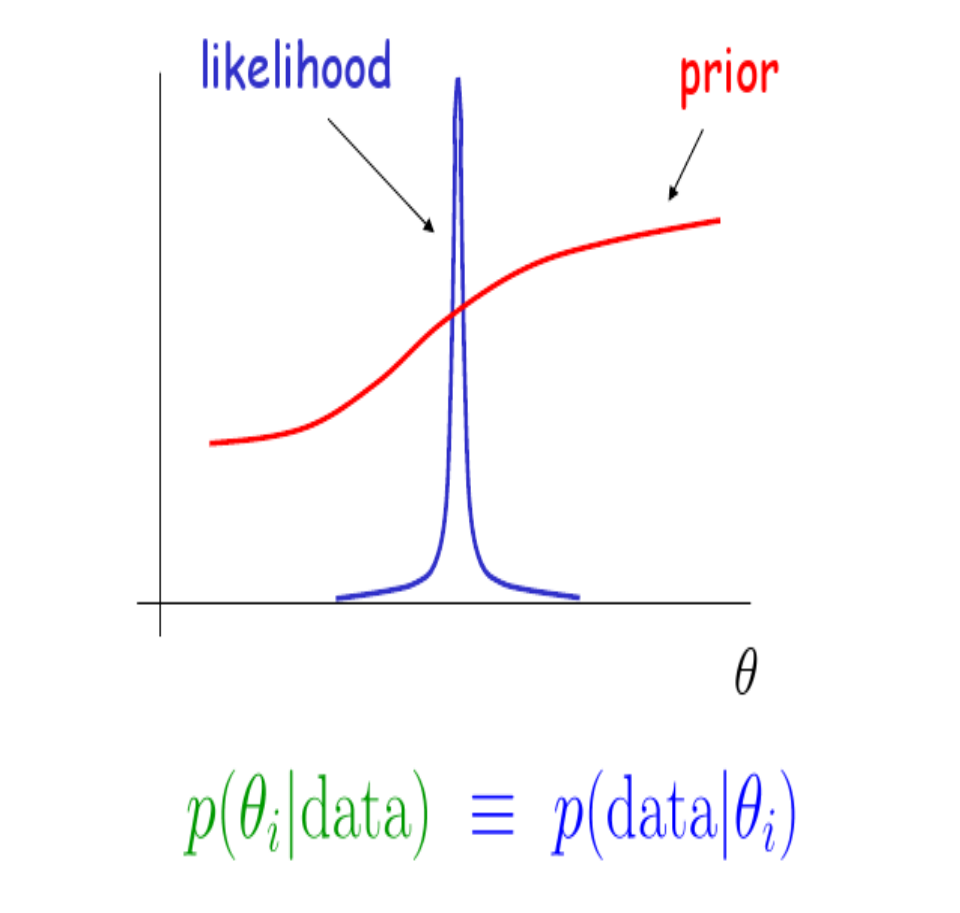
$$p(\theta_i) = \text{const.} \equiv \text{“}flat\text{”}$$

- But e. g.

$$\theta_i \longrightarrow \theta_i^2$$

$$\text{“}flat\text{”} \longrightarrow \text{“}non\text{-}flat\text{”}$$

- There is a vast literature on priors: Jeffreys’, conjugate, non-informative, ignorance, etc



If data are good enough to select a small region of {θ} then the prior p(θ) becomes irrelevant

Key adventages

- Efficiency: computational effort scales ~ N rather than k^N as in grid-scanning methods. Orders of magnitude improvement over previously used techniques.
- Marginalisation: integration over hidden dimensions comes for free. Suppose we have θ_i and are interested in $p(\theta_1|\text{data})$

$$p(\theta_1|\text{data}) = \int d\theta_2 \cdots d\theta_N \; p(\theta_i|\text{data})$$

- Inclusion of nuisance parameters: simply include them in the scan and marginalise over them. Notice: nuisance parameters in this context must be well constrained using independent data.
- Derived quantities: probabilities distributions can be derived for any function of the input variables (crucial for DD/ID/LHC predictions).

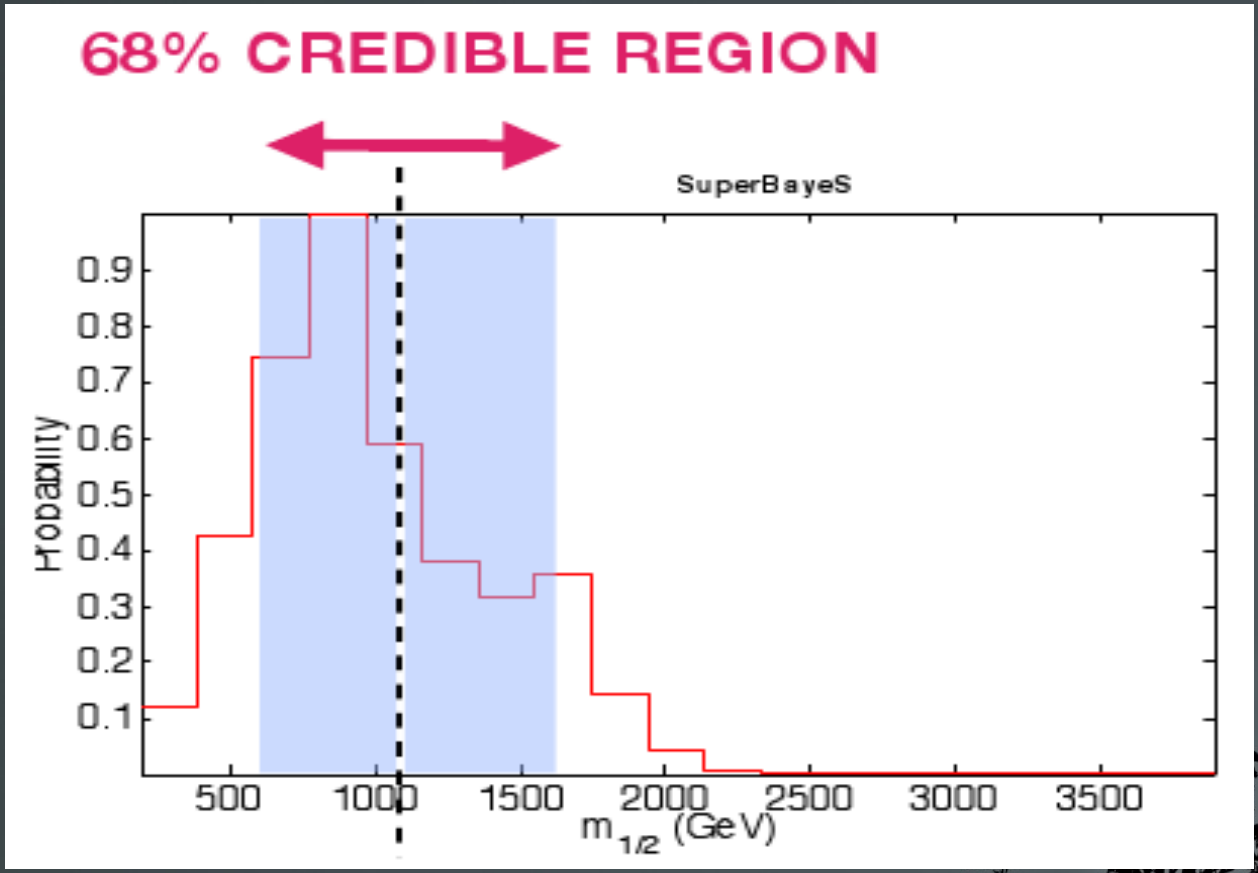
Posterior Samples

- MCMC: A Markov Chain is a list of samples θ1, θ2, θ3,... whose density reflects the (unnormalized) value of the posterior
- Crucial property: a Markov Chain converges to a stationary distribution, i.e. one that does not change with time. In our case, the posterior
- Different algorithms: MH, Gibbs... all need a proposal distribution => difficult to find a good one in complex problems
- Nested: New technique for efficient evidence evaluation (and posterior samples) (Skilling 2004)
- MultiNest: Also an extremely efficient sampler for multi-modal likelihoods !

  Feroz & Hobson (2007), RT et al (2008), Feroz et al (2008)

Favoured regions: Bayesian-based Approach

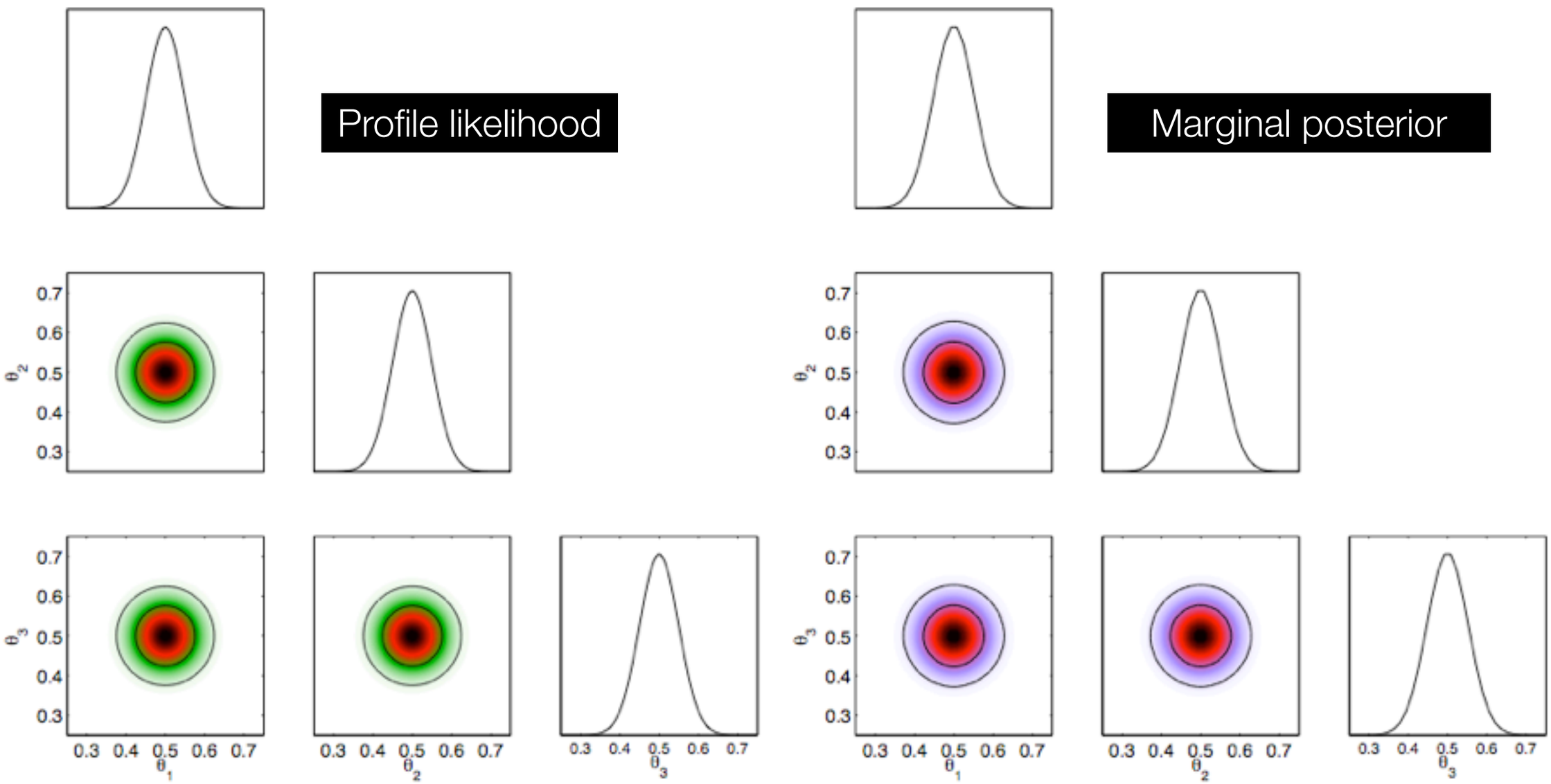
- Bayesian methods: the best-fit has no special status. Focus on region of large posterior probability mass instead.
- Determine posterior credible regions: e.g. symmetric interval around the mean containing 68% of samples



The Gaussian case

Imperial College London

- Life is easy (and boring) in Gaussianland:



Marginalization vs profiling (maximising)

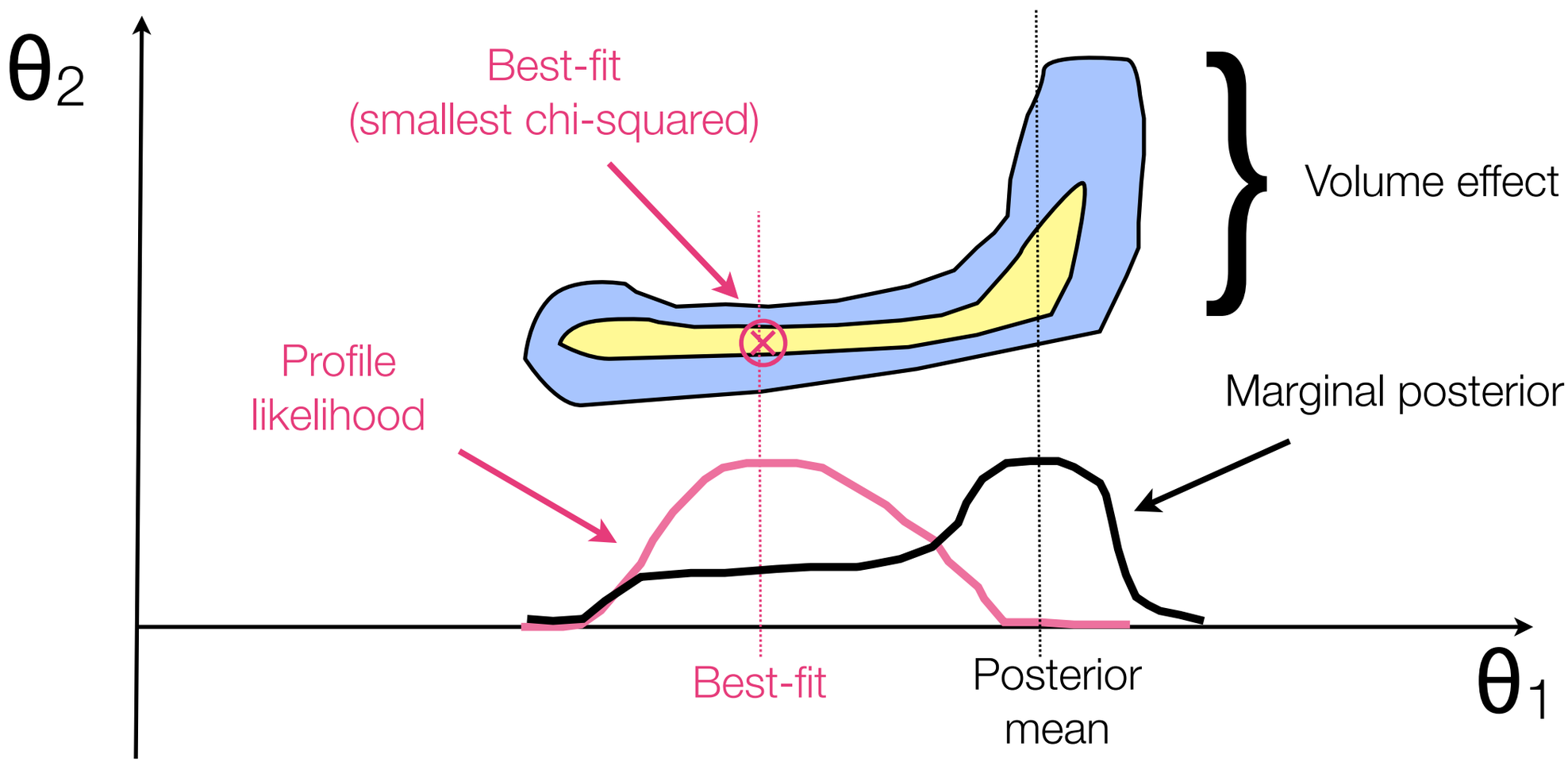
Imperial College London

Physical analogy: (thanks to Tom Loredo)

Likelihood = hottest hypothesis
Posterior = hypothesis with most heat

Heat: $Q = \int c_V(x) T(x) dV$

Posterior: $P \propto \int p(\theta) L(\theta) d\theta$



(2D plot depicts likelihood contours - prior assumed flat over wide range)

MCMC

- MCMC: Markov Chain Monte Carlo with Metropolis-Hastings

(1) select a random point in the parameter space, $\theta 0$

Compute $P(\theta 1) = \text{Like} \times \text{Prior}$

(2) propose a new point, $\theta 1$ with transition probability T, satisfying

$$T(\theta_0, \theta_1) = T(\theta_1, \theta_0)$$

(3) evaluate $P(\theta 1) = \text{Like} \times \text{Prior}$

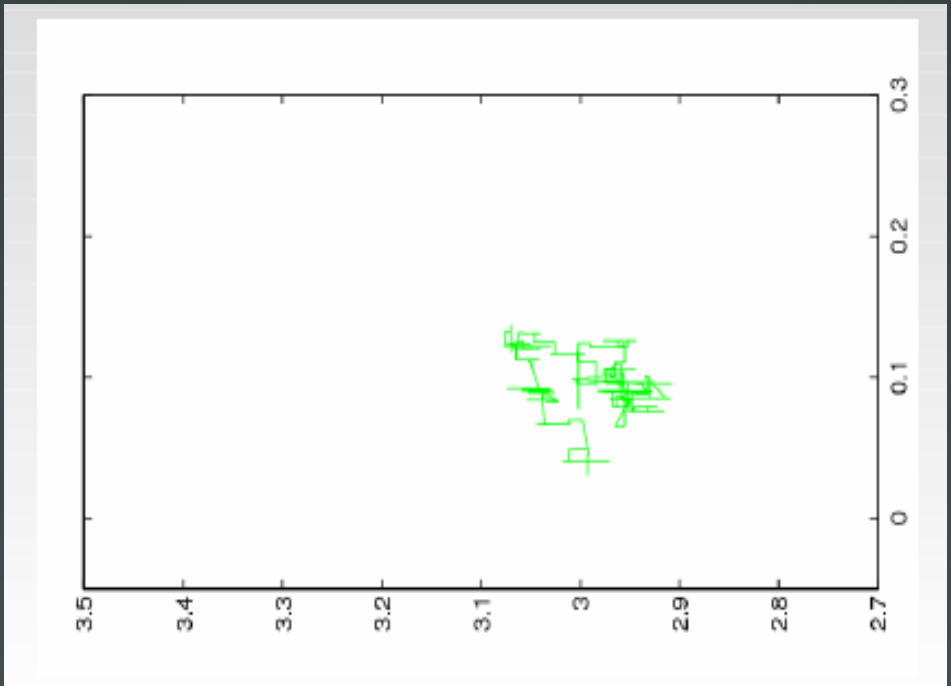
(4) if $P(\theta 1) > P(\theta 0)$ move to $\theta 1$
else
move to $\theta 1$ with probability $P(\theta 1)/P(\theta 0)$
otherwise stays in $\theta 0$

=> obtain a Markov Chain: θi , $i = 1, 2, ..., t$

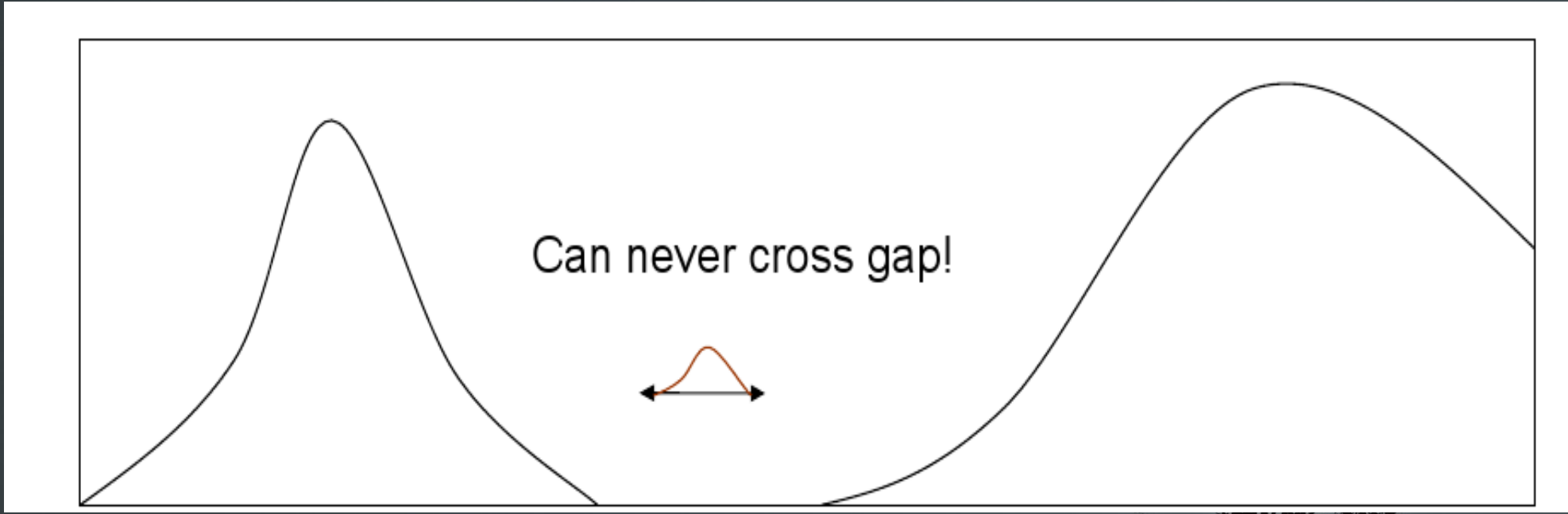
$t \rightarrow \infty$ chain $\Rightarrow$ target distribution, $P(\theta)$

- Proposal width low or directions bad: slow random walk through space

- Proposal width too large: low probability of proposing similar likelihood, low acceptance rate



Proposal distribution must be able to reach all points in parameter space



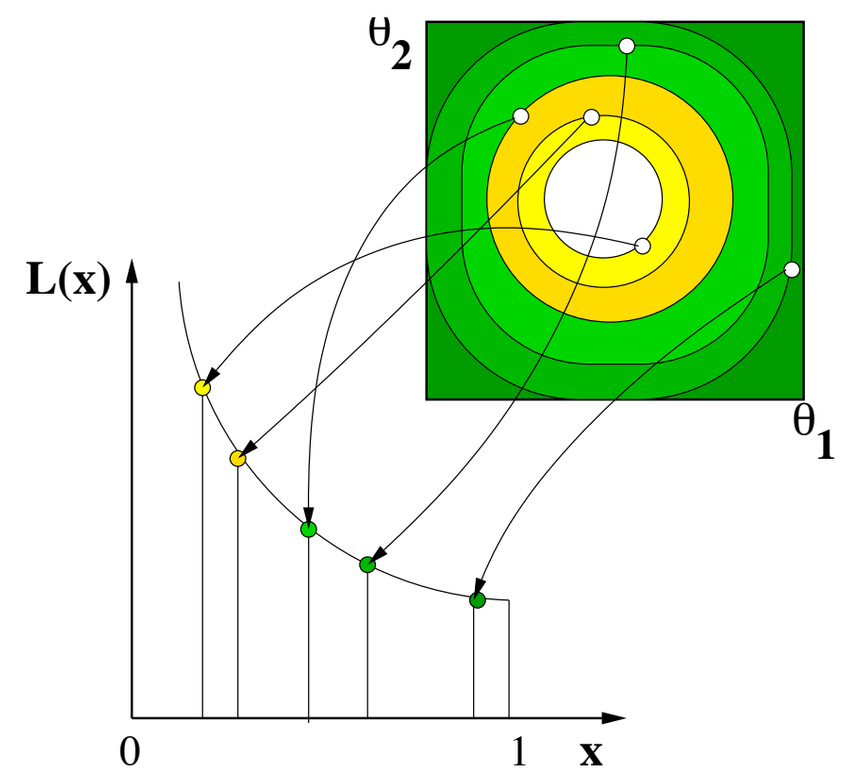
The Nested Sampling algorithm

Imperial College London

- Skilling (2006) introduced Nested Sampling as an algorithm originally aimed at the efficient computation of the model likelihood (Skilling, 2006).
- The idea is to map a multi-dimensional integral onto a 1D integral which is easy to compute numerical
- **The method requires to sample uniformly from the fraction of the prior volume X(μ) above the iso-likelihood level μ**

$$X(\mu) = \int_{L(\theta)>\mu} P(\theta) d\theta$$

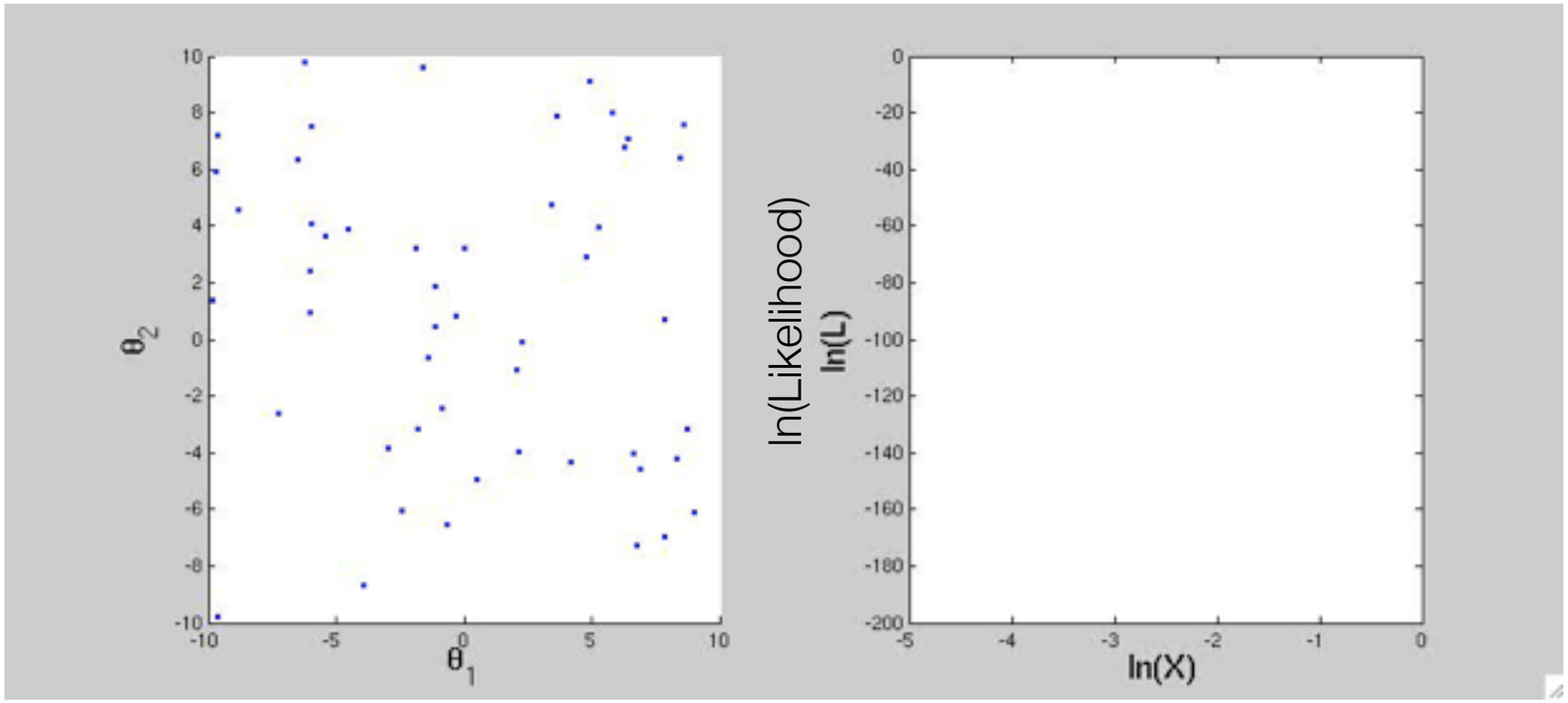
$$P(d) = \int d\theta L(\theta) P(\theta) = \int_0^1 X(\mu) d\mu$$



Feroz et al (2008), *arxiv: 0807.4512*
Trotta et al (2008), *arxiv: 0809.3792*

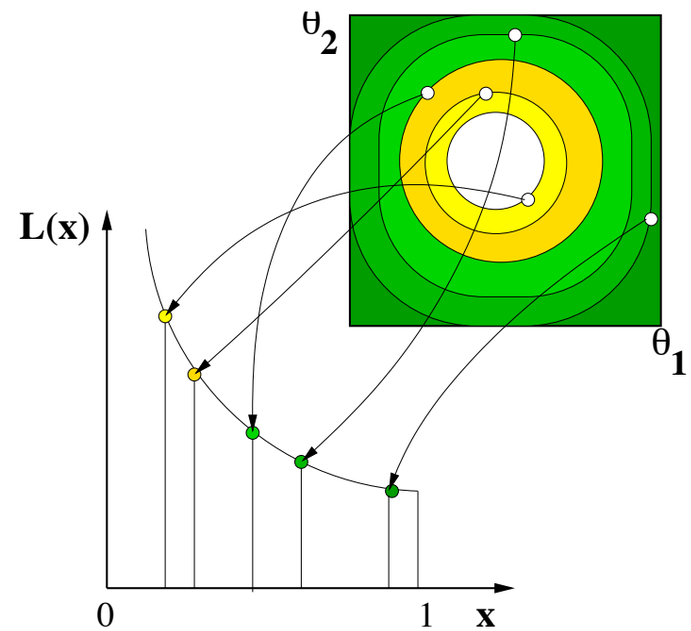
Nested Sampling in action

Imperial College London



Prior volume

(Animation courtesy of David Parkinson)



Prior volume

Nested sampling pseudo-code

Initialization

- Draw N "live points" from the prior (typically, N ~ 2000)
- Compute the likelihood for each live point

[Loop beings]

  - Select the live point with the lowest likelihood value, μ
  - Replace it with a new live point θ drawn from the prior with the constraint $L(\theta) > \mu$
  - Save the previous live point, together with μ and the prior volume fraction $X(\mu)$
  - If L_{max} X < tolerance, exit

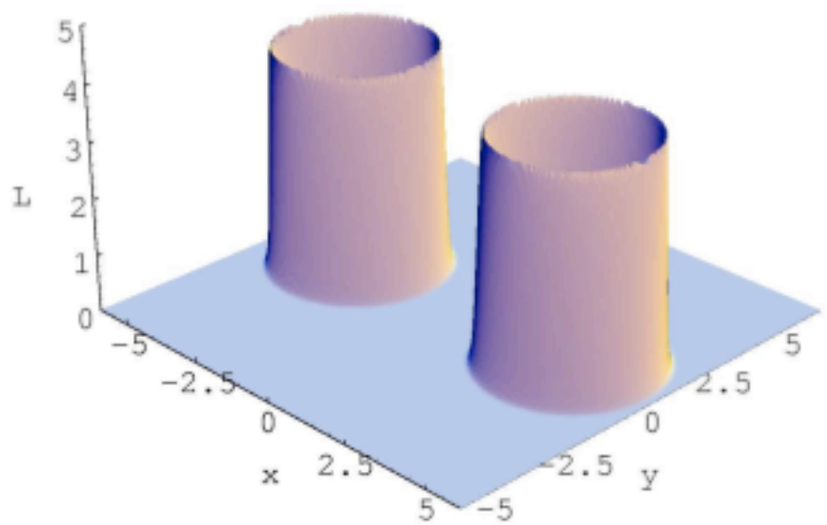
[Loop ends]

Sampling of multi-modal likelihoods

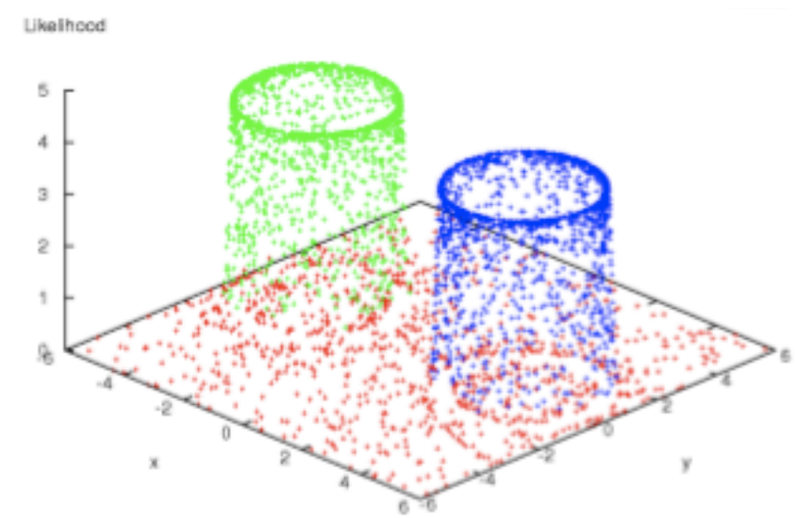
Imperial College London

- Thanks to the multi-modal ellipsoidal decomposition, MultiNest is an extremely efficient sampler for highly challenging multi-modal likelihoods!
  Feroz & Hobson (2007), RT et al (2008), Feroz et al (2008)

Target likelihood (D=2)



Sampled likelihood with MultiNest



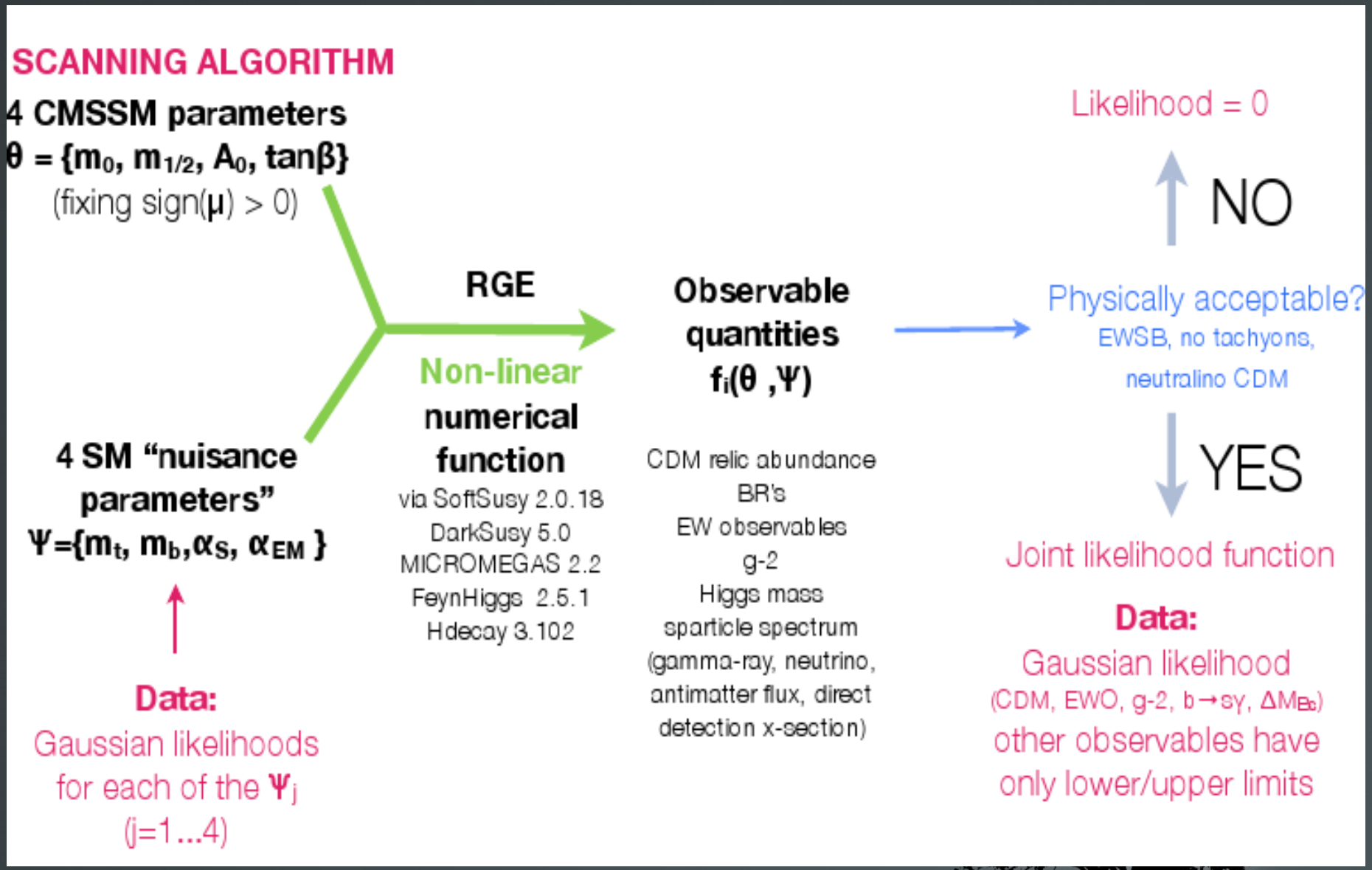
Dimensionality (D)	Likelihood evaluations (N)	Efficiency	$N^{1/D}$
2	7,000	70%	83
10	53,000	34%	3
20	255,000	15%	1.8
30	753,000	8%	1.6

The SuperBayeS package (superbayes.org)

- Supersymmetry Parameters Extraction Routines for Bayesian Statistics
- Implements the CMSSM, but can be easily extended to the general MSSM
- Currently linked to SoftSusy 2.0.18, DarkSusy 4.1, MICROMEGAS 2.2, FeynHiggs 2.5.1, Hdecay 3.102. **New release (v 1.5.1)**
- Includes up-to-date constraints from all observables
- Bayesian MCMC, MULTI-MODAL NESTED SAMPLING or grid scan mode.
- MULTI-MODAL NESTED SAMPLING (Feroz & Hobson 2008), efficiency increased by a factor 200. **A full 8D scan now takes 3 days on a single CPU** (previously: 6 weeks on 10 CPUs)
- Fully parallelized, MPI-ready, user-friendly interface a la cosmomc (thanks Sarah Bridle & Antony Lewis),
- SuperEGO: SuperBayeS Enhanced Graphical Output as a MATLAB graphical user interface for statistical analysis and plotting

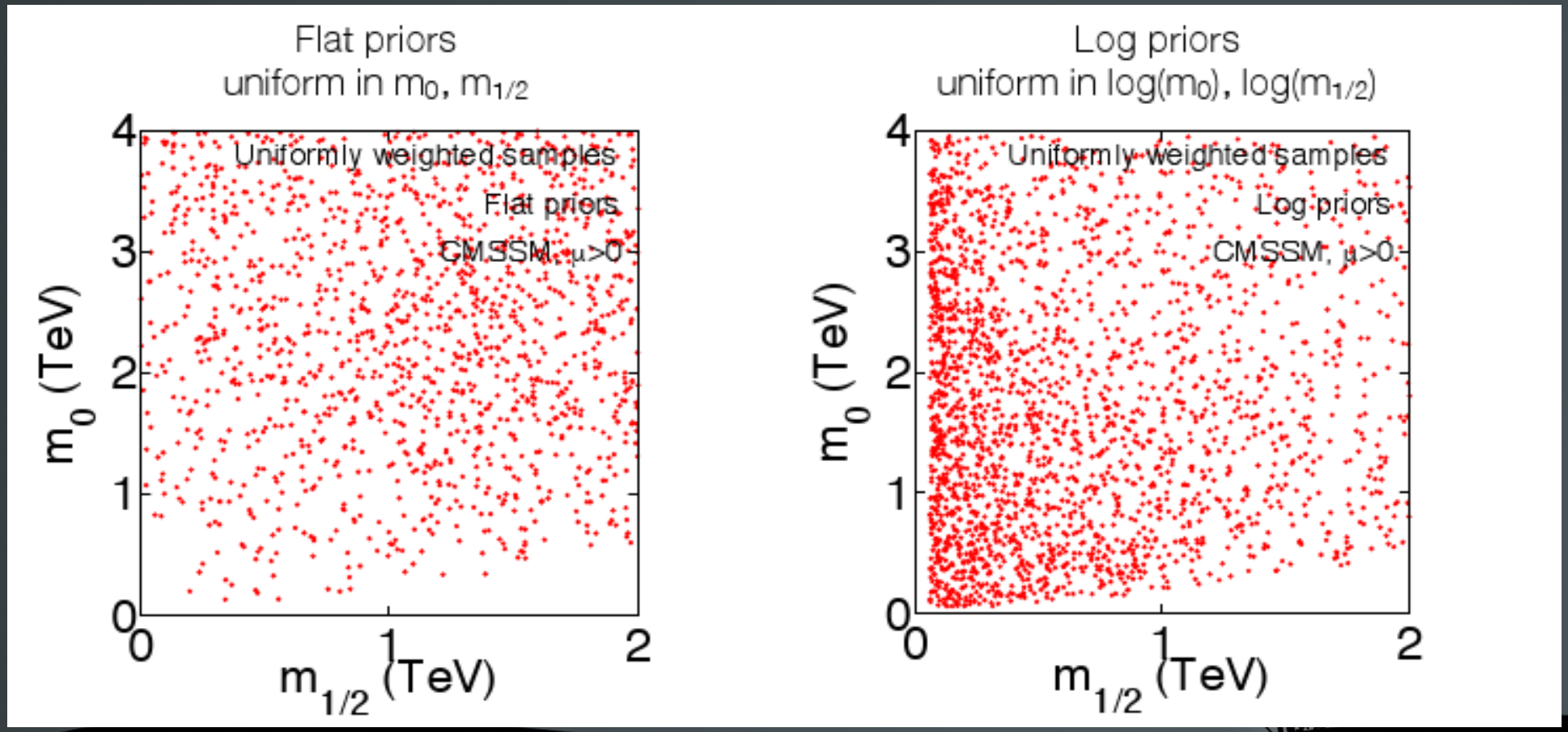
Global CMSSM fits

Analysis Pipeline



Samples from priors only

- No data in the likelihood, non-physical points discarded priors
- $p(\mathcal{F}) = p(m)\left|\frac{\mathrm{d}m}{\mathrm{d}\mathcal{F}}\right|$ => flat priors on log means $p(m) \propto m^{-1}$



Analysis

Prior ranges

flat priors: CMSSM parameters
$50\ \text{GeV} < m_0 < 4\ \text{TeV}$
$50\ \text{GeV} < m_{1/2} < 4\ \text{TeV}$
$\lvert A_0 \rvert < 7\ \text{TeV}$
$2 < \tan\beta < 62$

Likelihood function

$$\mathcal{L} = p(\sigma, c|\xi(m)) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{\chi^2}{2}\right]$$

$$\chi^2 = \frac{[\xi(m) - c]^2}{\sigma^2} \quad \sigma \to s = \sqrt{\sigma^2 + \tau^2}$$

Data: Indirect observables

Observable	Mean value μ	Uncertainties σ (exper.)	τ (theor.)	ref.
M_W	80.398 GeV	25 MeV	15 MeV	[30]
$\sin^2\theta_{\text{eff}}$	0.23153	16×10^{-5}	15×10^{-5}	[30]
$\delta a_\mu^{\text{SUSY}} \times 10^{10}$	29.5	8.8	1.0	[31]
$BR(\overline{B} \to X_s\gamma) \times 10^4$	3.55	0.26	0.21	[32]
ΔM_{B_s}	17.77 ps^{-1}	0.12 ps^{-1}	2.4 ps^{-1}	[33]
$BR(\overline{B}_u \to \tau\nu) \times 10^4$	1.32	0.49	0.38	[32]
$\Omega_\chi h^2$	0.1099	0.0062	$0.1\,\Omega_\chi h^2$	[34]
	Limit (95% CL)		τ (theor.)	ref.
$BR(\overline{B}_s \to \mu^+\mu^-)$	$< 5.8 \times 10^{-8}$		14%	[35]
m_h	> 114.4 GeV (SM-like Higgs)		3 GeV	[36]
ζ_h^2	$f(m_h)$ (see text)		negligible	[36]
$m_{\tilde{q}}$	> 375 GeV		5%	[25]
$m_{\tilde{g}}$	> 289 GeV		5%	[25]
other sparticle masses	As in table 4 of ref. [6].			

SM parameters

SM (nuisance) parameter	Mean value μ	Uncertainty σ (exper.)	Ref.
M_t	172.6 GeV	1.4 GeV	[24]
$m_b(m_b)^{\overline{MS}}$	4.20 GeV	0.07 GeV	[25]
$\alpha_s(M_Z)^{\overline{MS}}$	0.1176	0.002	[25]
$1/\alpha_{\text{em}}(M_Z)^{\overline{MS}}$	127.955	0.03	[26]

CMSSM in 2011: Frequentist fits

Imperial College London

SuperBayeS: profile likelihood

(~41M samples)

Best fit

incl. ATLAS 35 fb^{-1} and WMAP-7:

m_0 ~ 80..100 GeV

m_{12} ~ 360..370 GeV

A_0 ~ -280..-400 GeV

tanβ ~ 11..14

MasterCode: profile likelihood

(~30M samples)

Best fit

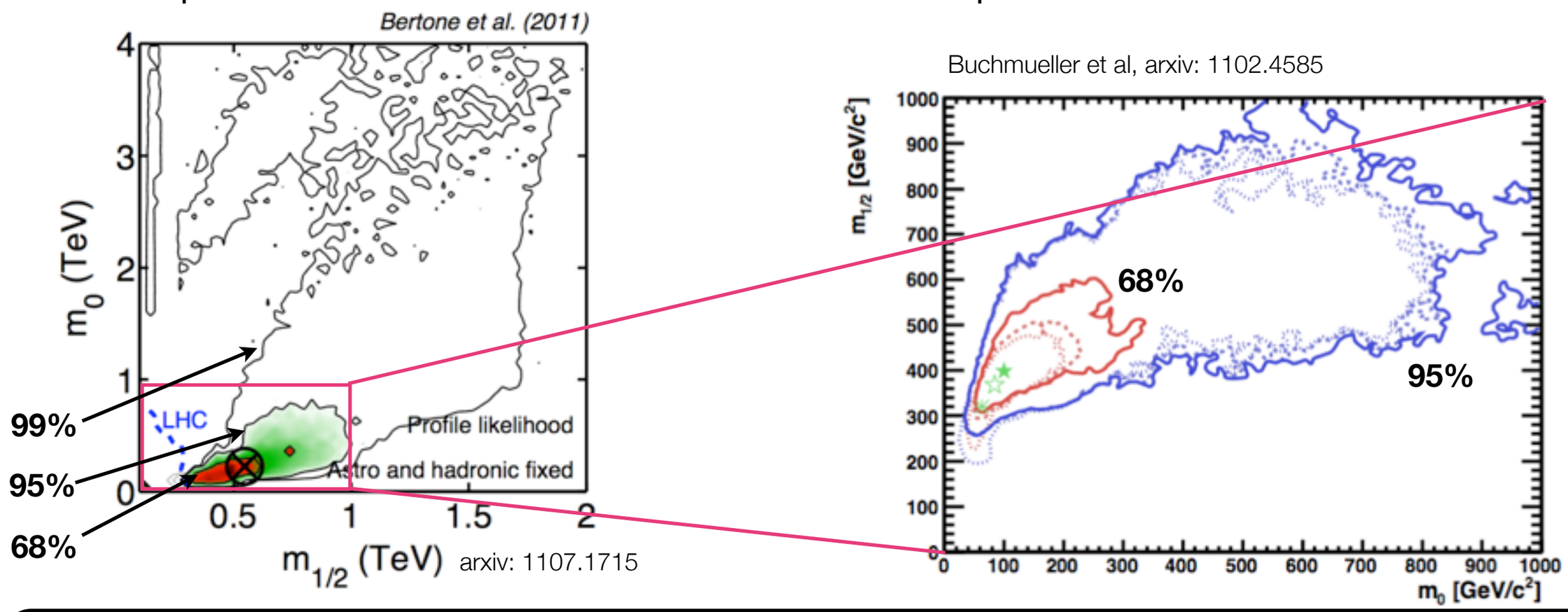
incl. ATLAS/CMS 35 fb^{-1} and WMAP:

m_0 ~ 80..100 GeV

m_{12} ~ 370..400 GeV

A_0 ~ -300..-400 GeV

tanβ ~ 14..16

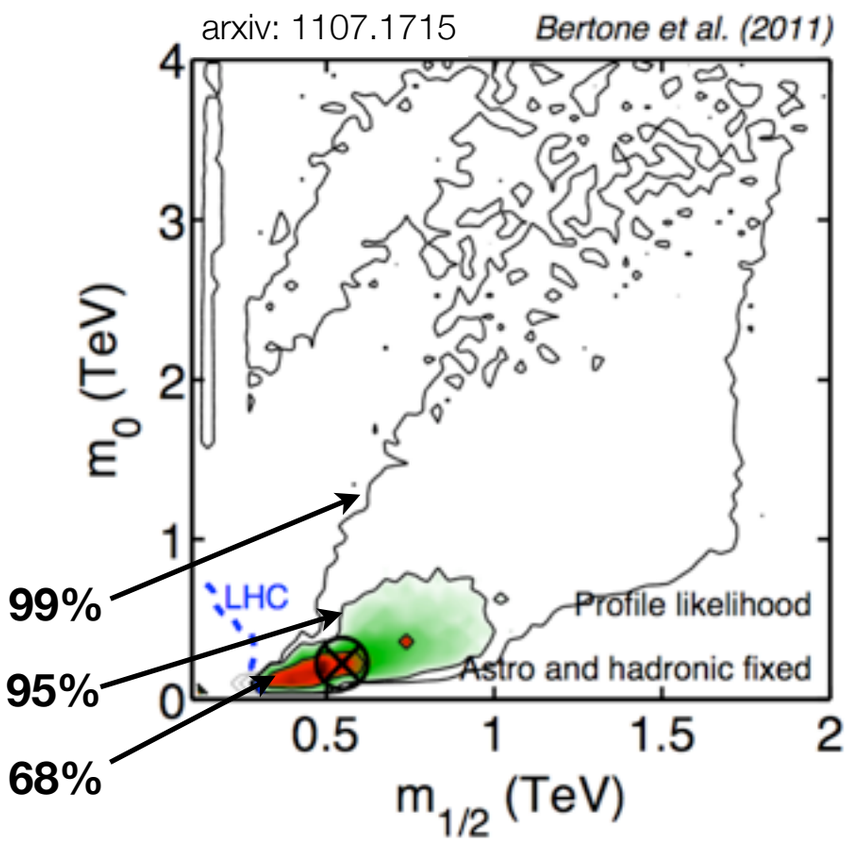


Low-energy SUSY preferred though Focus Point region cannot be ruled out at 99%

CMSSM in 2011: Bayesian pdf

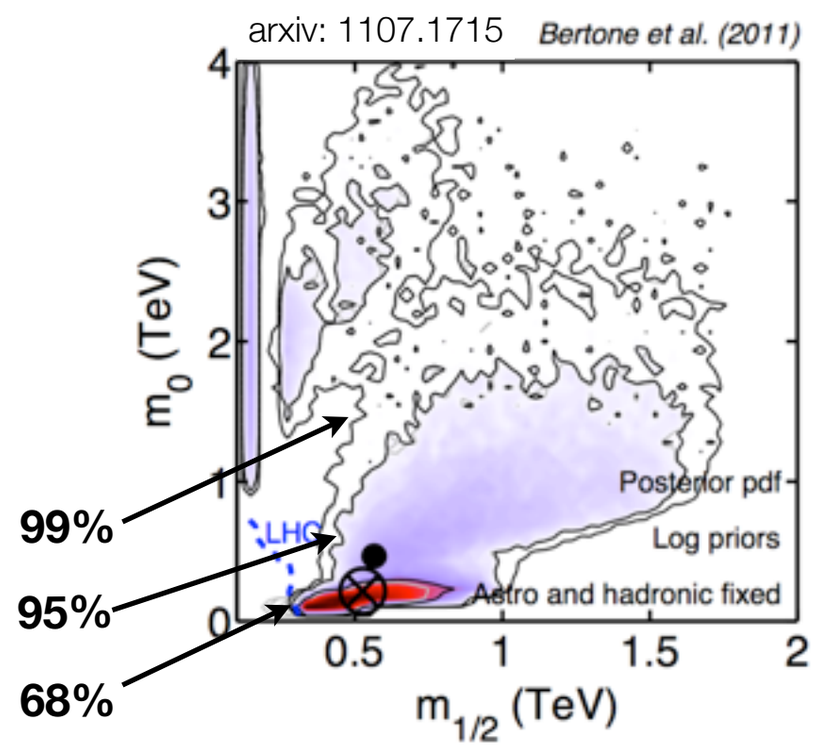
Imperial College London

SuperBayeS: profile likelihood

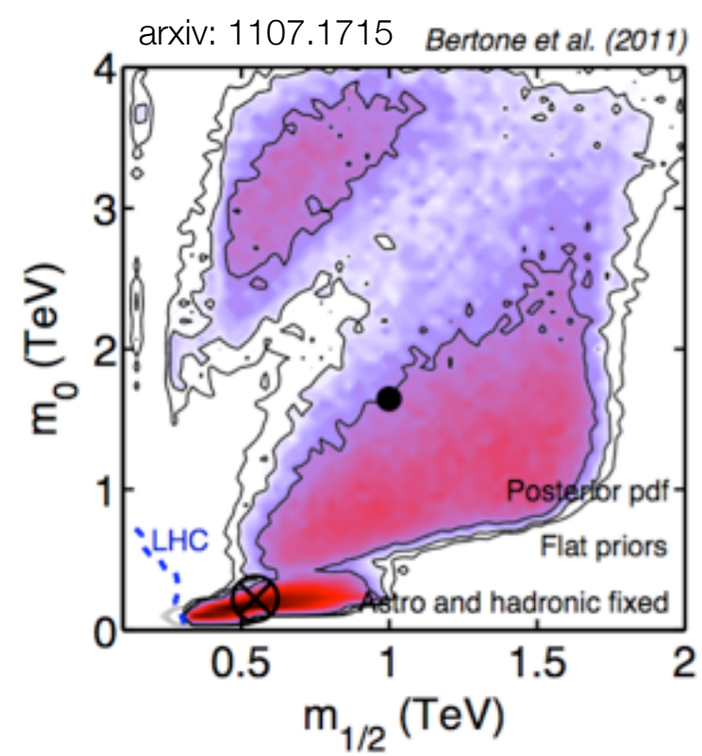


SuperBayeS: Bayesian pdf

"log" priors



"flat" priors

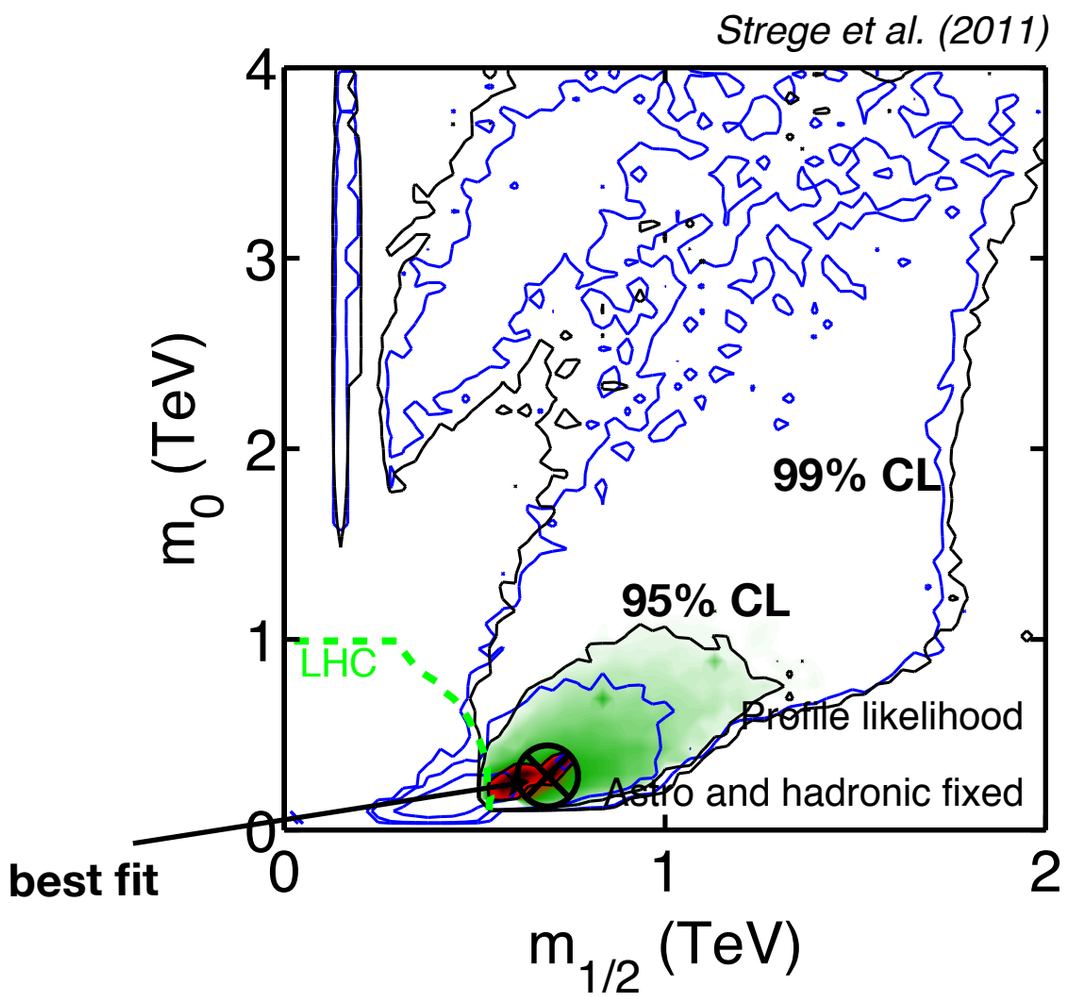


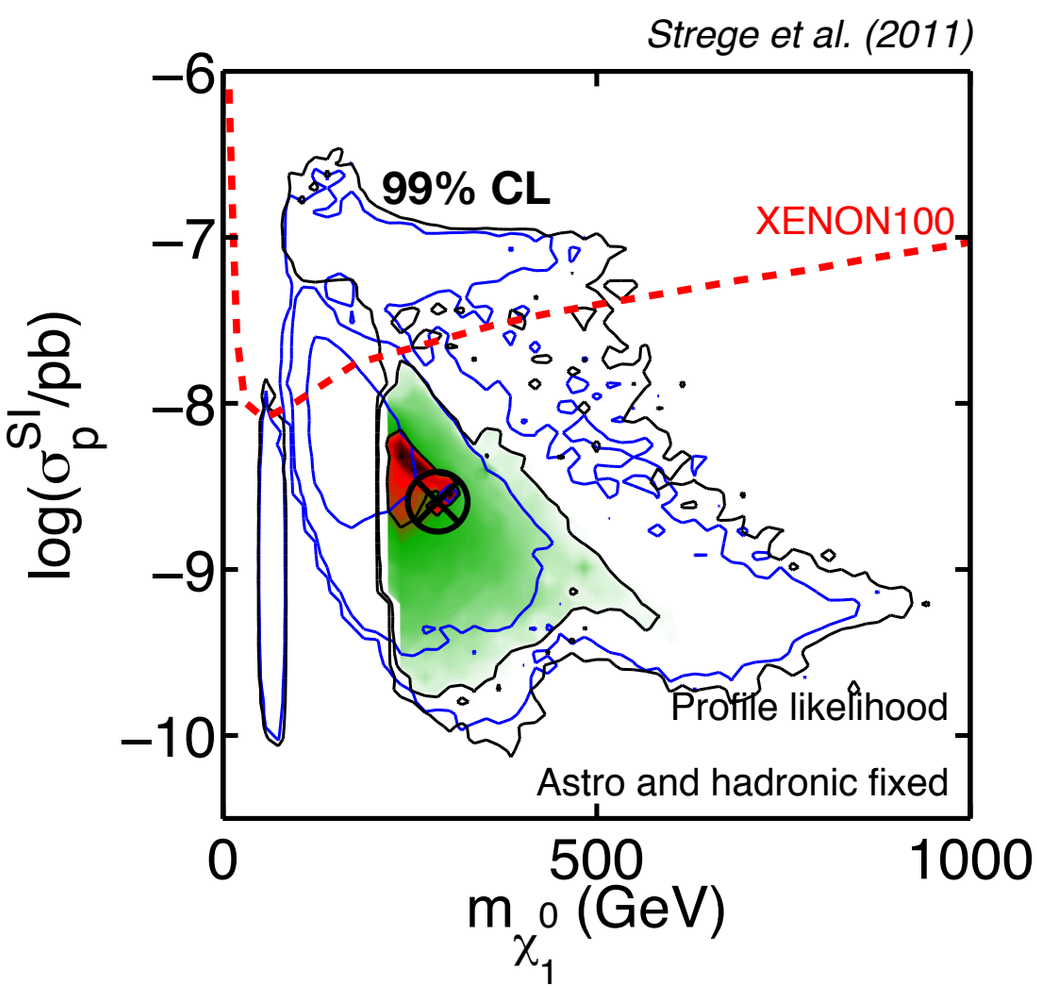
Low-energy SUSY remains favoured in both approaches.
Tentative convergence between Frequentist and Bayesian scans, although prior effects remain present (in particular, the relative viability of the Focus Point region cannot be robustly established with LHC constraints)

The cMSSM at the end of 2011

Imperial College London

Global fits of the Constrained MSSM (4+4 parameters) including relic abundance data, rare processes, g-2, EW precision observables and constraints from 1/fb ATLAS data and Higgs limits from 5 1/fb



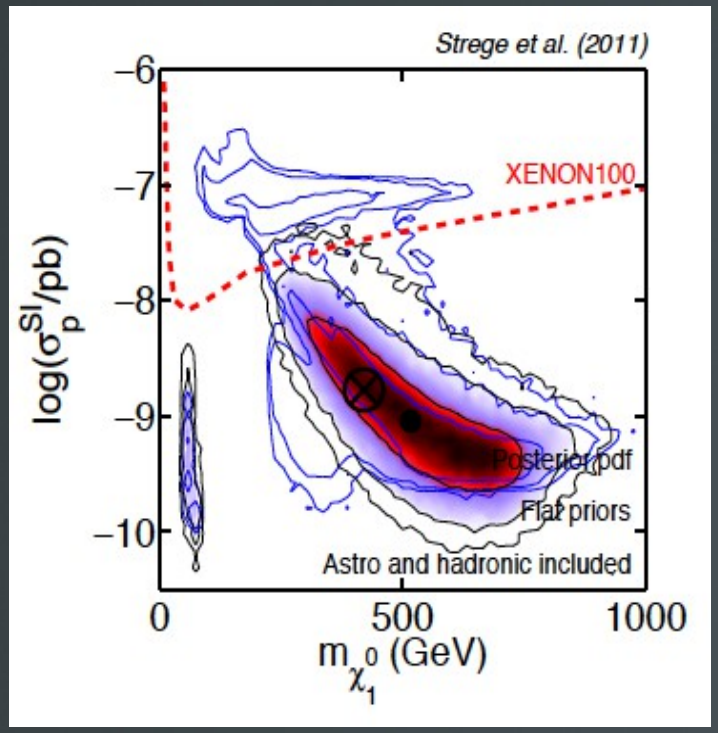


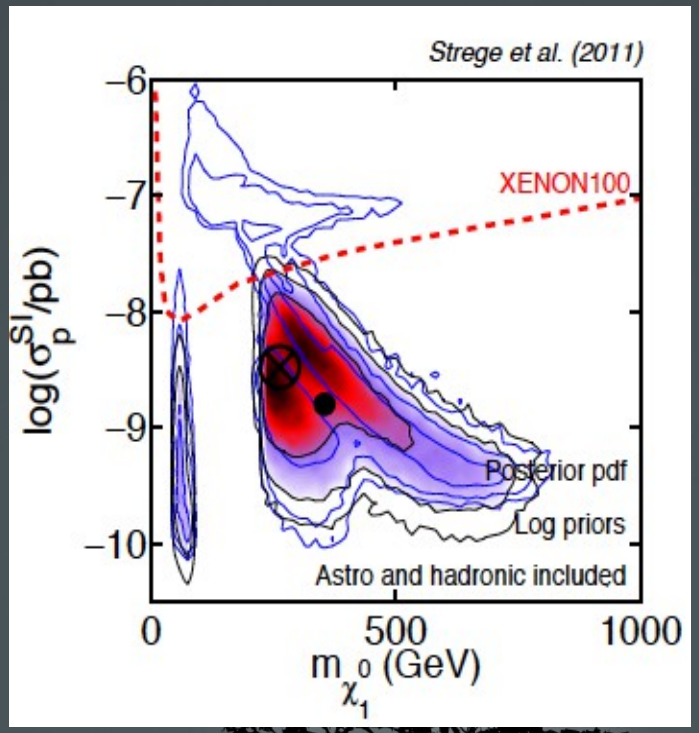
Higher luminosity LHC data are putting pressure on the best fit point

Xenon100 Data

- 48 Kg in 100.9 days of exposure
- 3 events observed and bkg = 0.8 ∓ 0.6
- Likelihood: $\mathcal{L}_{\mathrm{Xe100}}(\Theta) \propto p(\hat{N}|\lambda) = L_0 \frac{\lambda^{\hat{N}}}{\hat{N}!} e^{-\lambda}$

Posterior

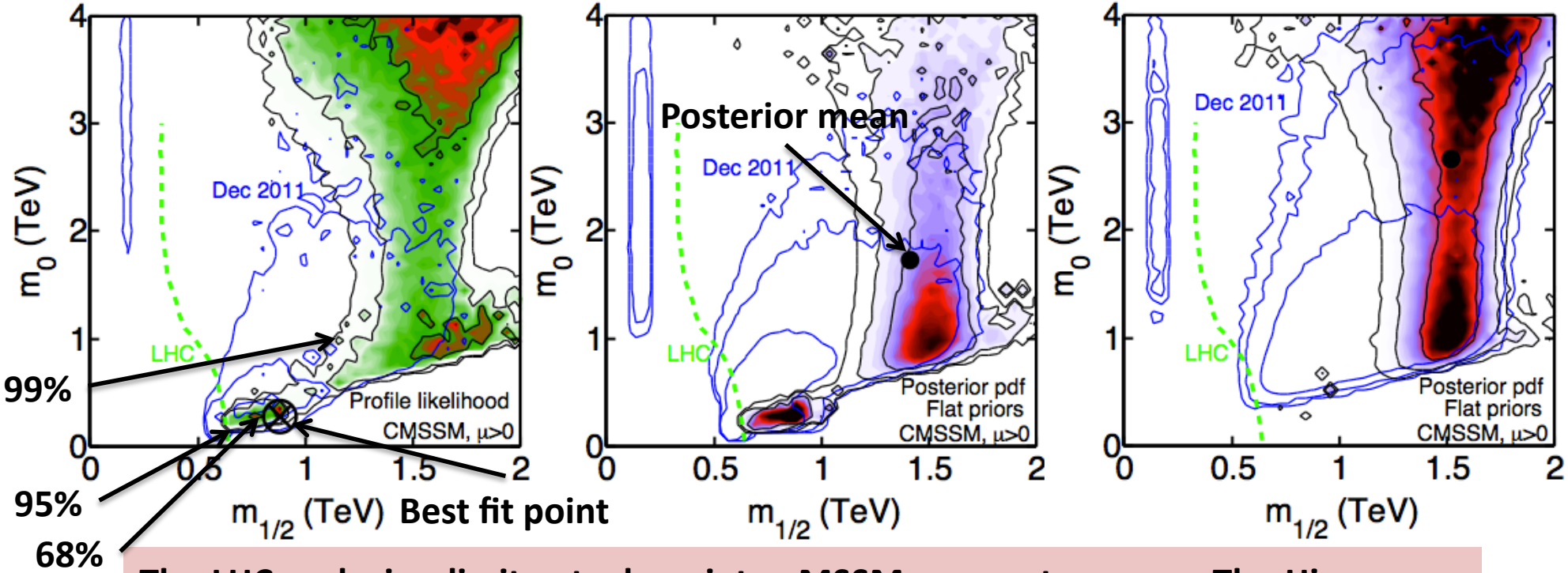




Impact on the cMSSM

Imperial College London

Profile likelihood | **Bayesian posterior Log priors** | **Bayesian posterior Flat priors**



The LHC exclusion limit cuts deep into cMSSM parameter space. The Higgs mass constraint pushes contours towards significantly larger values of $m_{1/2}$. The best-fit point remains at low masses. Posterior pdfs become less dependent on the choice of priors and are in good agreement with PL results.

Profile likelihood results: comparison

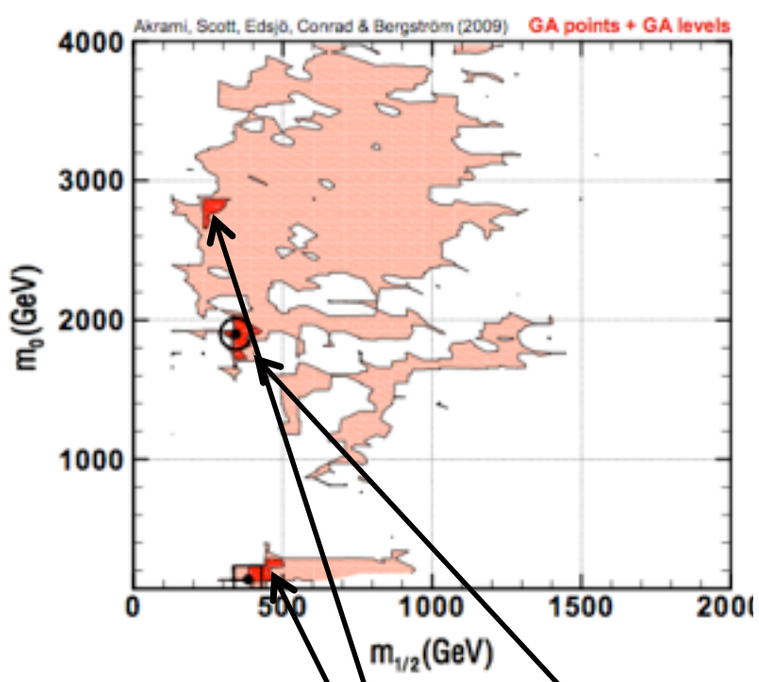
Imperial College London

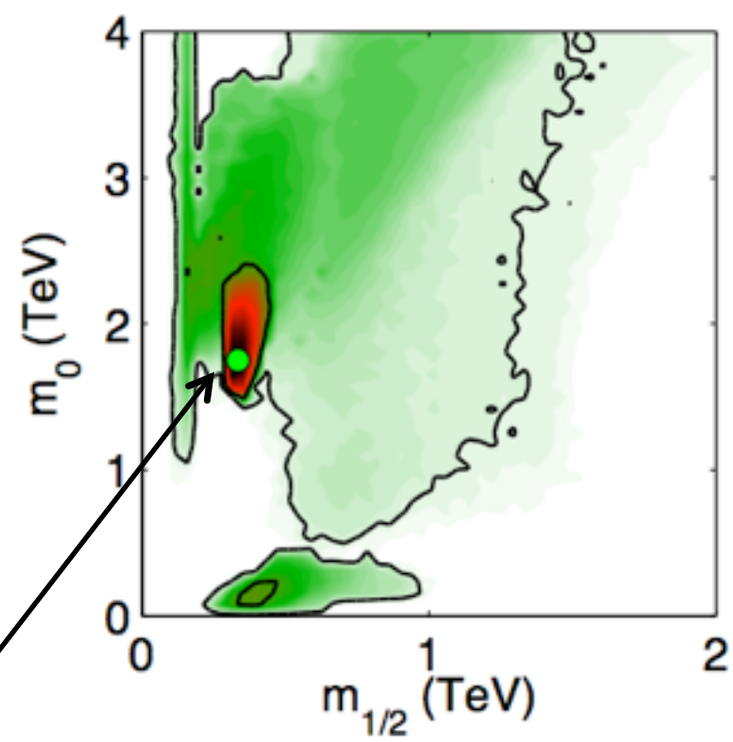
- Akrami et al (0910.3950) adopted a genetic algorithm (GA) to map out the profile likelihood.
- This allows to find isolated spikes in the likelihood in the focus point region: **is this something other frequentist fits might have missed?**

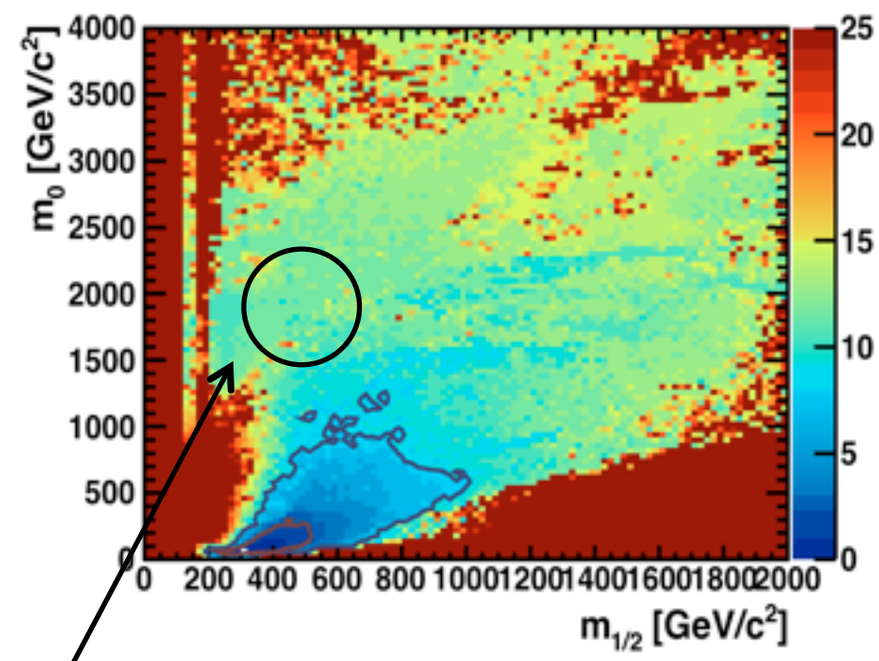
Genetic Algorithm profile likelihood

MultiNest profile likelihood

MasterCode profile likelihood







isolated local maxima

overall best-fit

excluded at ~ 3σ

Challenges of profile likelihood evaluation

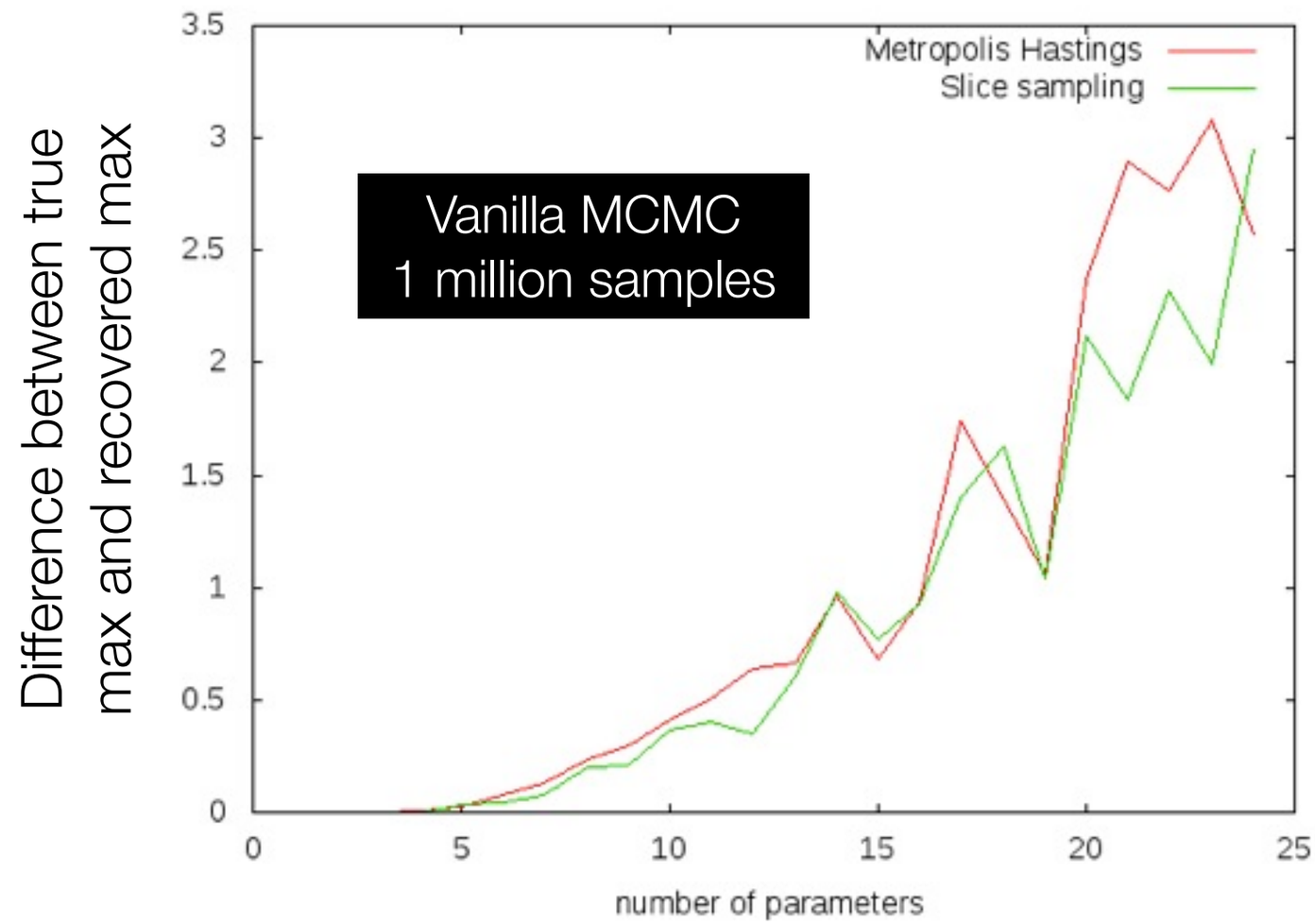
Imperial College London

- MCMC/MultiNest are not designed to find the best-fit point. Bayesian algorithms are designed to map out regions of significant posterior probability mass

- Even for a simple Gaussian toy model, this becomes difficult to do as the number of dimensions of the parameter space increases

- **Profiling with vanilla MCMC or MultiNest scans has to be done with caution!**

Toy multinormal likelihood



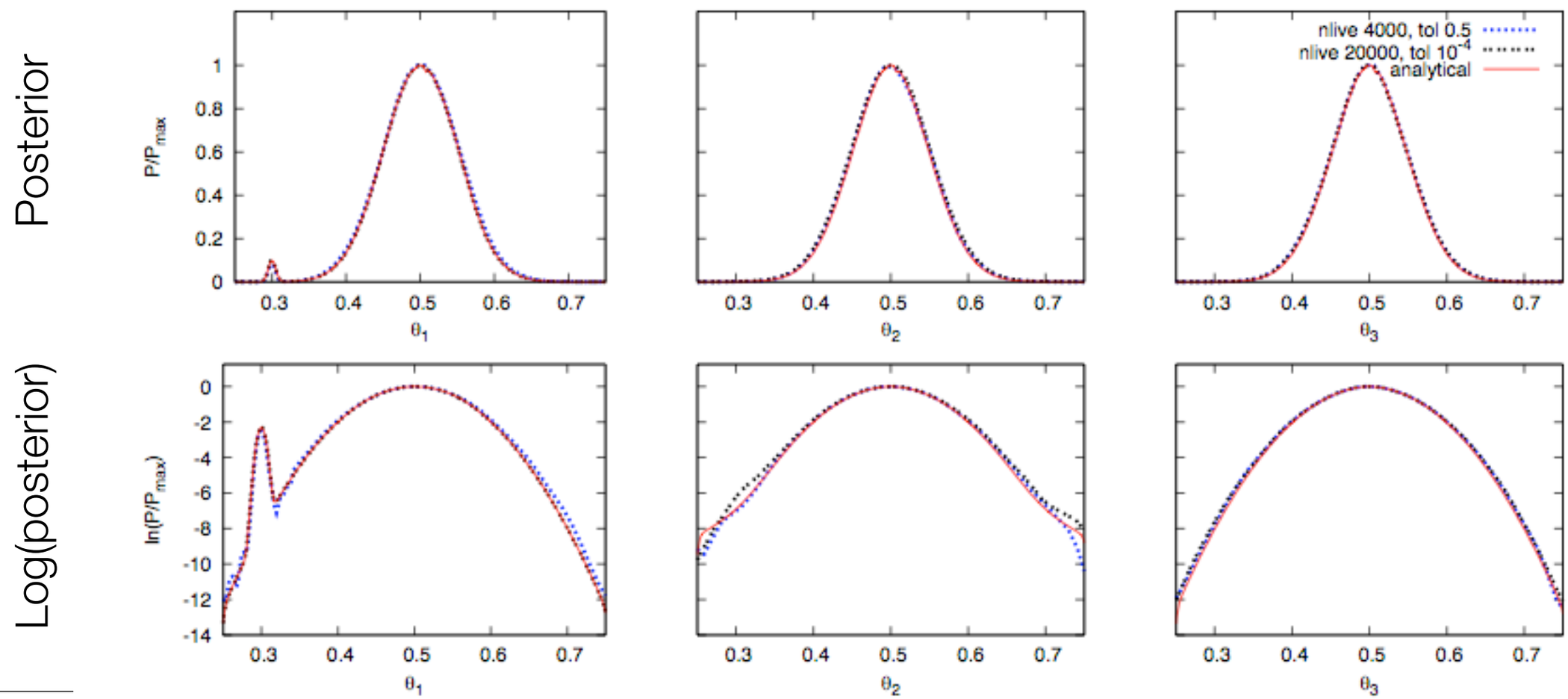
Posterior pdf from MultiNest scans

Imperial College London

- MultiNest is primarily aimed at evaluation of the posterior pdf. It does an excellent job even for multi-modal problems. 8D toy case (Feroz, KC, RT et al, in prep)

- The tolerance parameter (**tol**) determines the stopping criterium (based on the incremental change of the value of the local evidence). Lower tol gives a finer exploration around the peak, important for profile likelihood evaluation

red: analytical **blue: MN, tol=0.5** **black: MN, tol=0.0001**

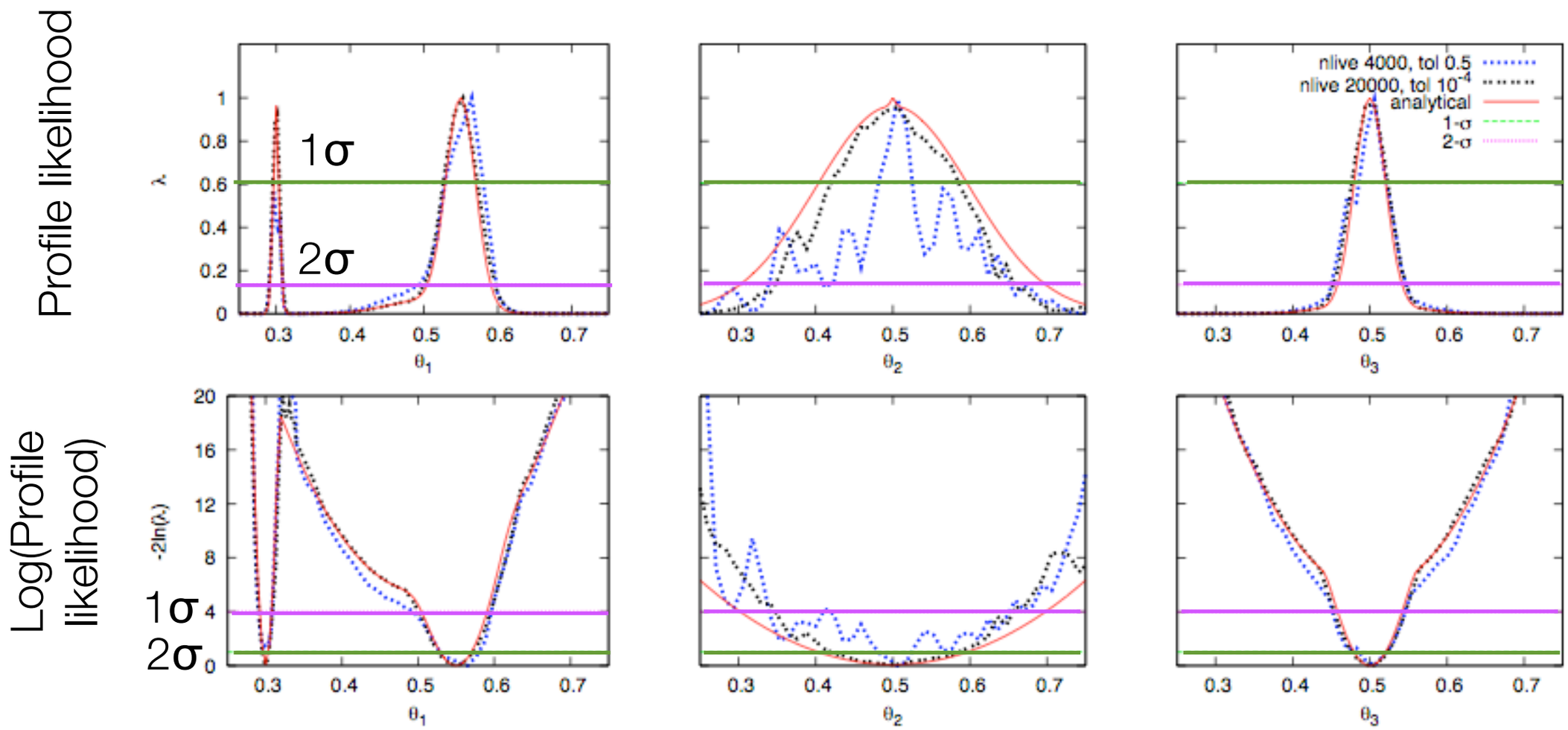


Profile likelihood from MultiNest scans

Imperial College London

- A fairly accurate the profile likelihood can be obtained with MultiNest by tuning the tolerance (lower, **tol=0.0001**) and the number of live points (higher, **n_{live}=20,000**) (Feroz, KC, RT et al, in prep), even for highly multi-modal distributions. 8D toy:

red: analytical **blue: MN, tol=0.5** **black: MN, tol=0.0001**

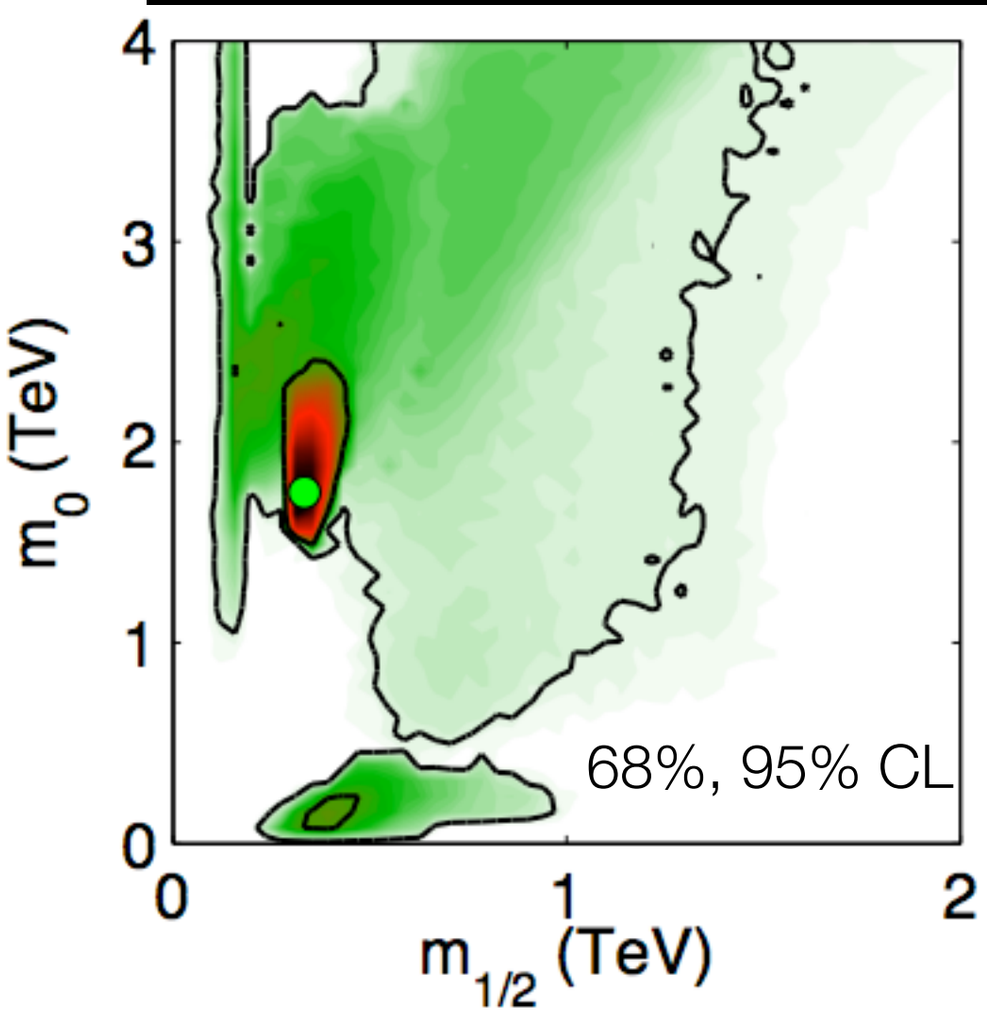


Profile likelihood from MultiNest

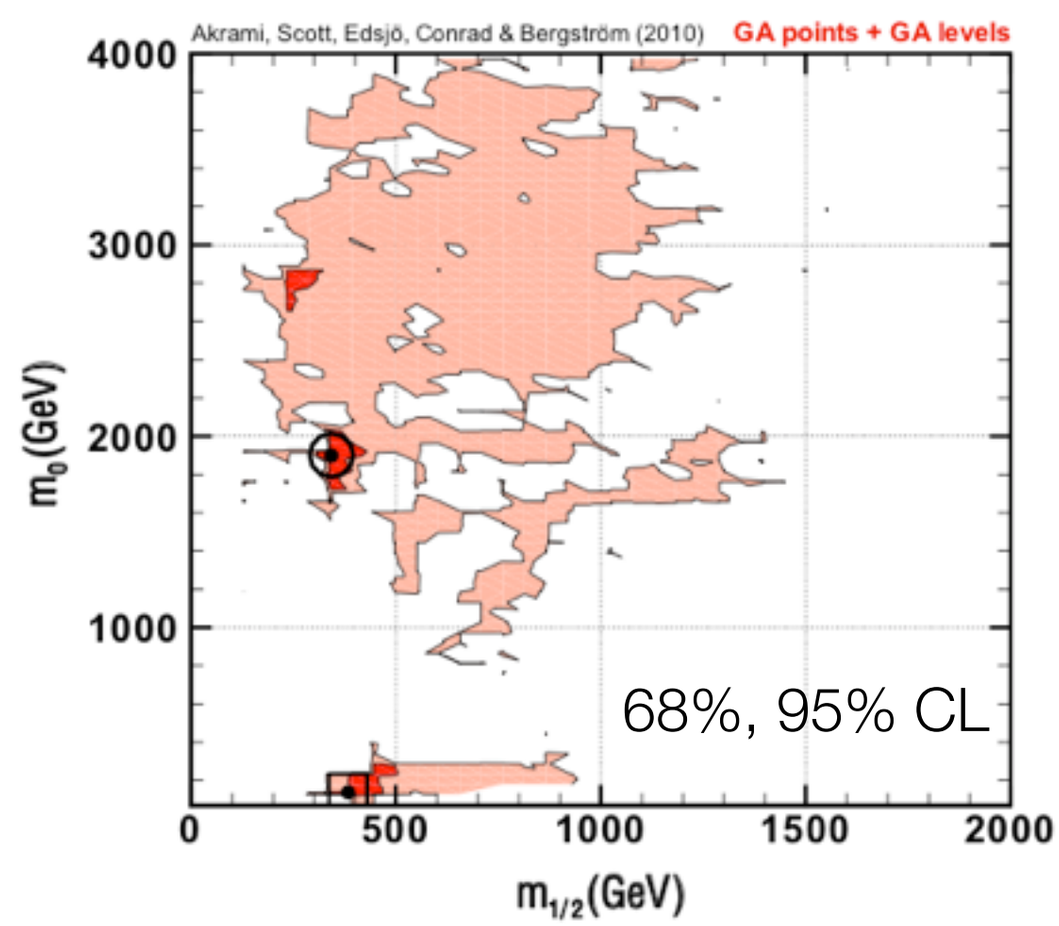
Imperial College London

- MultiNest scan with 20,000 live points (usually: 4,000) and tolerance 0.0001 (usually: 0.5) results in 5.5 million likelihood evaluations (Akrami et al, GA: 3 million), and best-fit chi-square = 9.26 (Akrami et al, GA: 9.35).
  MultiNest finds a better best-fit + smoother contours than GA.

Profile likelihood MultiNest, tol=10^{-4}
Merged log and flat priors scans



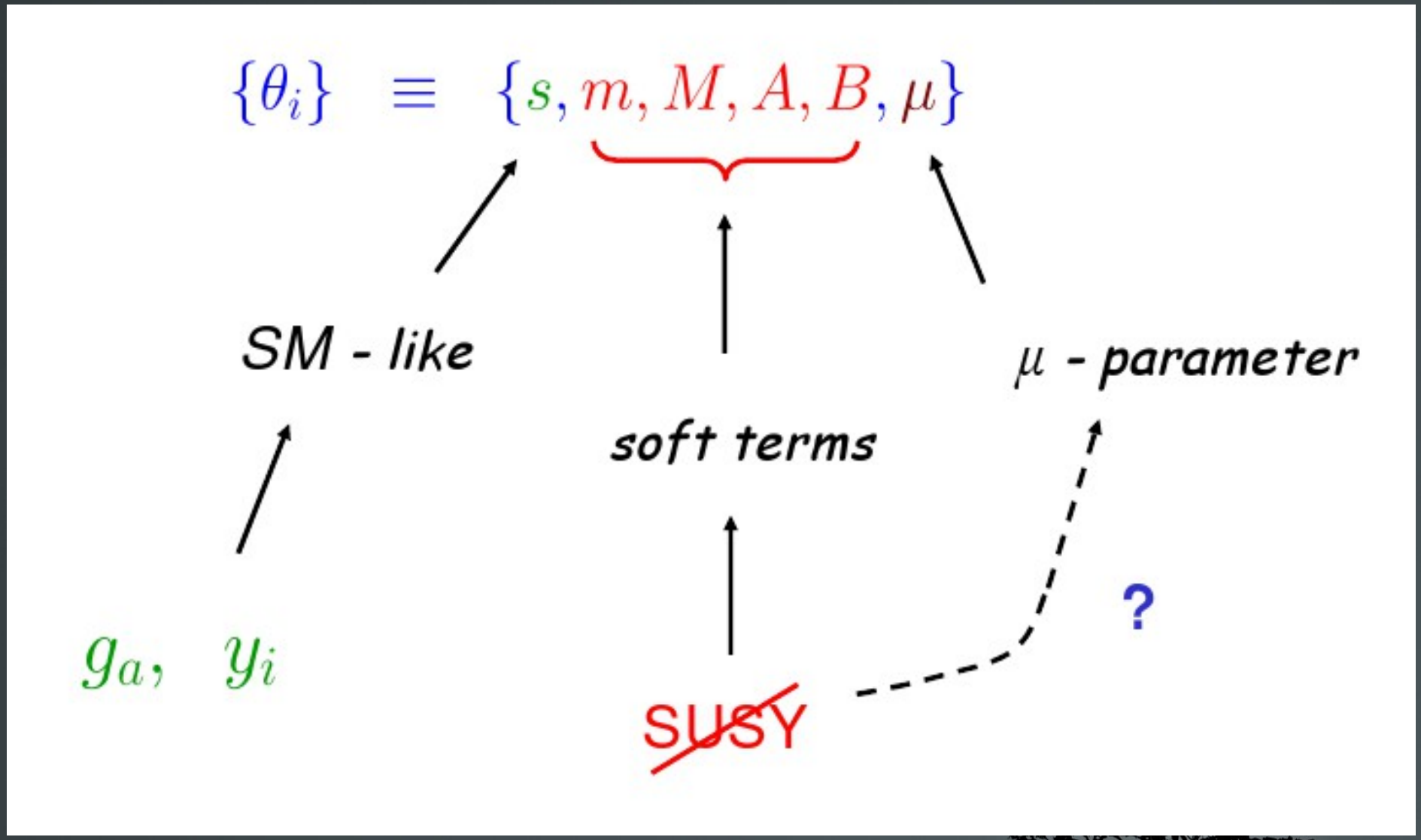
Profile likelihood
Genetic algorithm



Akrami et al (2010)

Feroz, KC, RT et al (2011)

Towards a more refined analysis



Bayesian and Naturalness

- Recall an usual assumption

$$\underbrace{m, M, A, B, \mu}_{(\equiv M_{\rm soft})} \quad \text{should be} \quad \lesssim \mathcal{O}(TeV)$$

In oder to get a Natural Electroweak symmetry Breaking (with no fine-tunnings)

$$V(H_1, H_2) = m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 - 2B\mu H_1 H_2 + \frac{1}{8}(g^2 + g'^2)\left(|H_1|^2 - |H_2|^2\right)^2$$

$$M_Z^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2\beta}{\tan^2\beta - 1} - 2\mu^2$$

$$\sin 2\beta = \frac{2\mu}{B}\left(m_{H_1}^2 + m_{H_2}^2 + 2\mu_{\rm low}^2\right)$$

Unnatural fine-tuning unless $M_{\rm soft} \lesssim \mathcal{O}(\text{TeV})$

•••

- Instead solving μ^2 in terms of M_Z and the other soft-terms, treat as another exp. Data
- Approximate the likelihood as

$$\mathcal{L} = N_Z\, e^{-\frac{1}{2}\left(\frac{M_Z - M_Z^{\rm exp}}{\sigma_Z}\right)^2} \mathcal{L}_{\rm rest}$$

$$\simeq \delta(M_Z - M_Z^{\rm exp})\, \mathcal{L}_{\rm rest}$$

- Use M_Z to marginalize μ

$$p(s,m,M,A,B|\,\text{data}) = \int d\mu\, p(s,m,M,A,B,\mu|\text{data})$$

$$\simeq \mathcal{L}_{\rm rest} \left[\frac{d\mu}{dM_Z}\right]_{\mu_Z} p(s,m,M,A,B,\mu_Z)$$

$$p(s,m,M,A,B|\,\text{data}) = 2\,\mathcal{L}_{\rm rest}\, \frac{\mu_Z}{M_Z}\, \frac{1}{c_\mu}\, p(s,m,M,A,B,\mu_Z)$$

$$c_\mu = \frac{\partial\, \ln M_Z^2}{\partial\, \ln \theta_i}$$

$c^{-1} \sim$ Probability of cancellation between the various contributions to get $M_Z \sim \mathcal{O}(90\text{GeV})$

Putting all together

$$\{\mu, y_t, B\} \xrightarrow{J} \{M_Z, m_t, \tan\beta\}$$

$$p(m_t, m, M, A, \tan\beta|\text{data}) = \underbrace{J|_{\mu=\mu_Z}\, p(y_t, m, M, A, B, \mu_Z)}_{p_{\text{eff}}(m_t, m, M, A, \tan\beta)}\, \mathcal{L}_{\text{rest}}$$

$$p_{\text{eff}}(m_t, m, M, A, \tan\beta) \propto \underbrace{\left[\frac{E}{R_\mu^2}\right] \frac{y}{y_{\text{low}}} \frac{t^2-1}{t(1+t^2)} \frac{B_{\text{low}}}{\mu_Z}}_{J}\, p(m, M, A, B, \mu = \mu_Z)$$

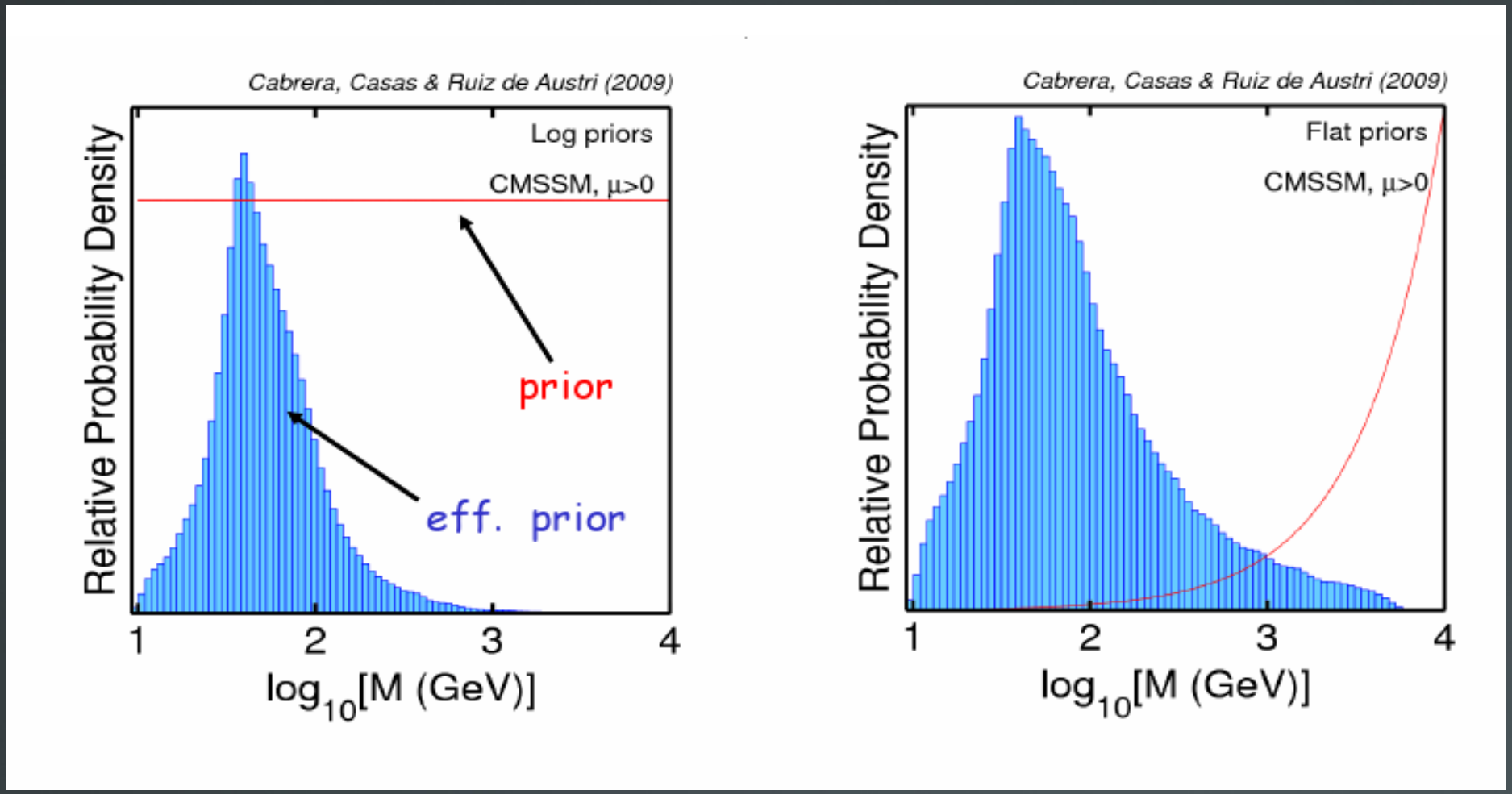
still undefined

model-independent part !

It contains the fine-tuning penalization

It penalizes large tan β

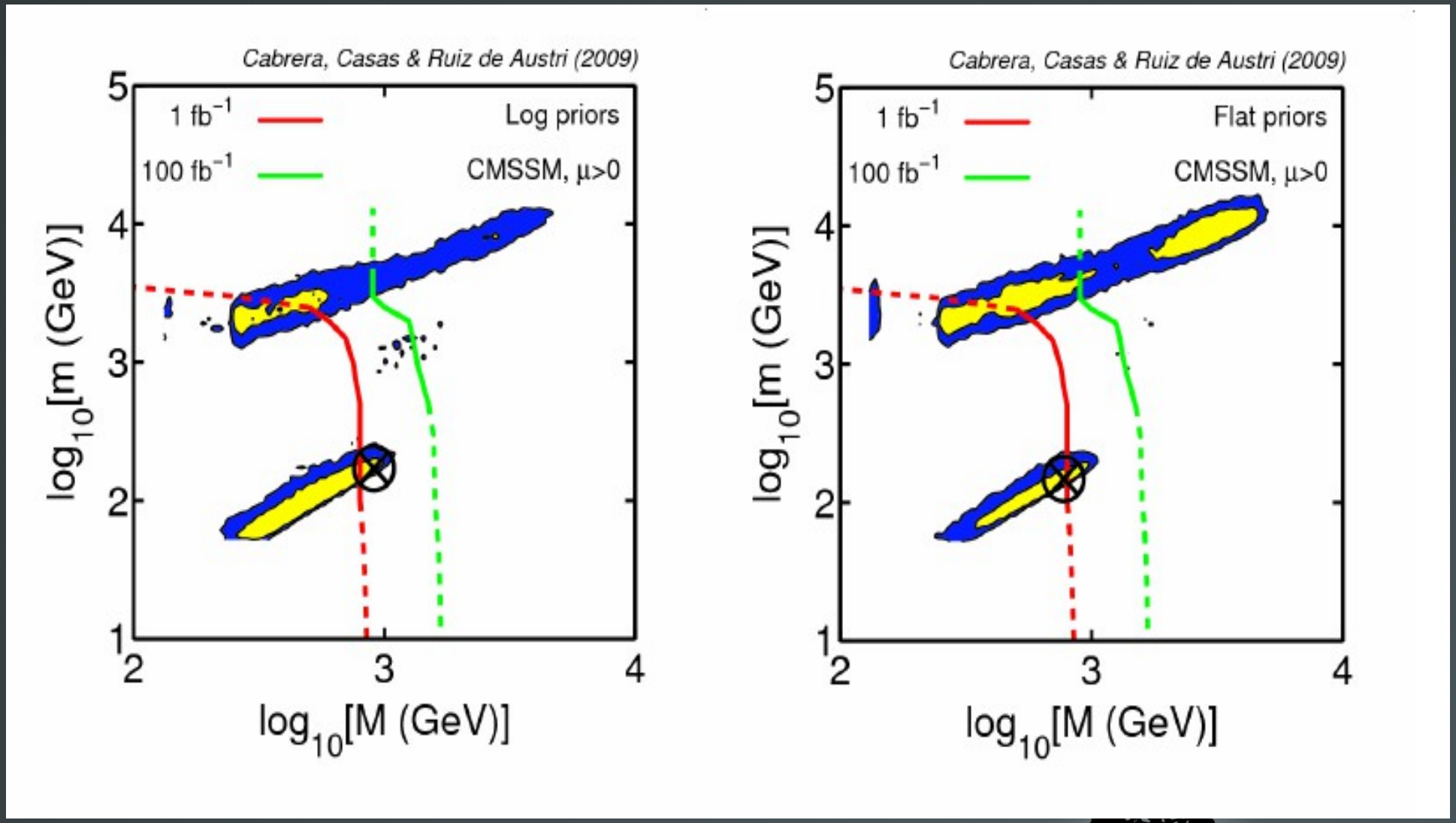
The ElectroWeak Scale



$M_Z^{\rm exp}$ brings SUSY to the LHC region

- We may vary $M_{\rm soft}$ up to M_X the results do not depend on the range chosen
- This suggests that large soft-masses are disfavoured

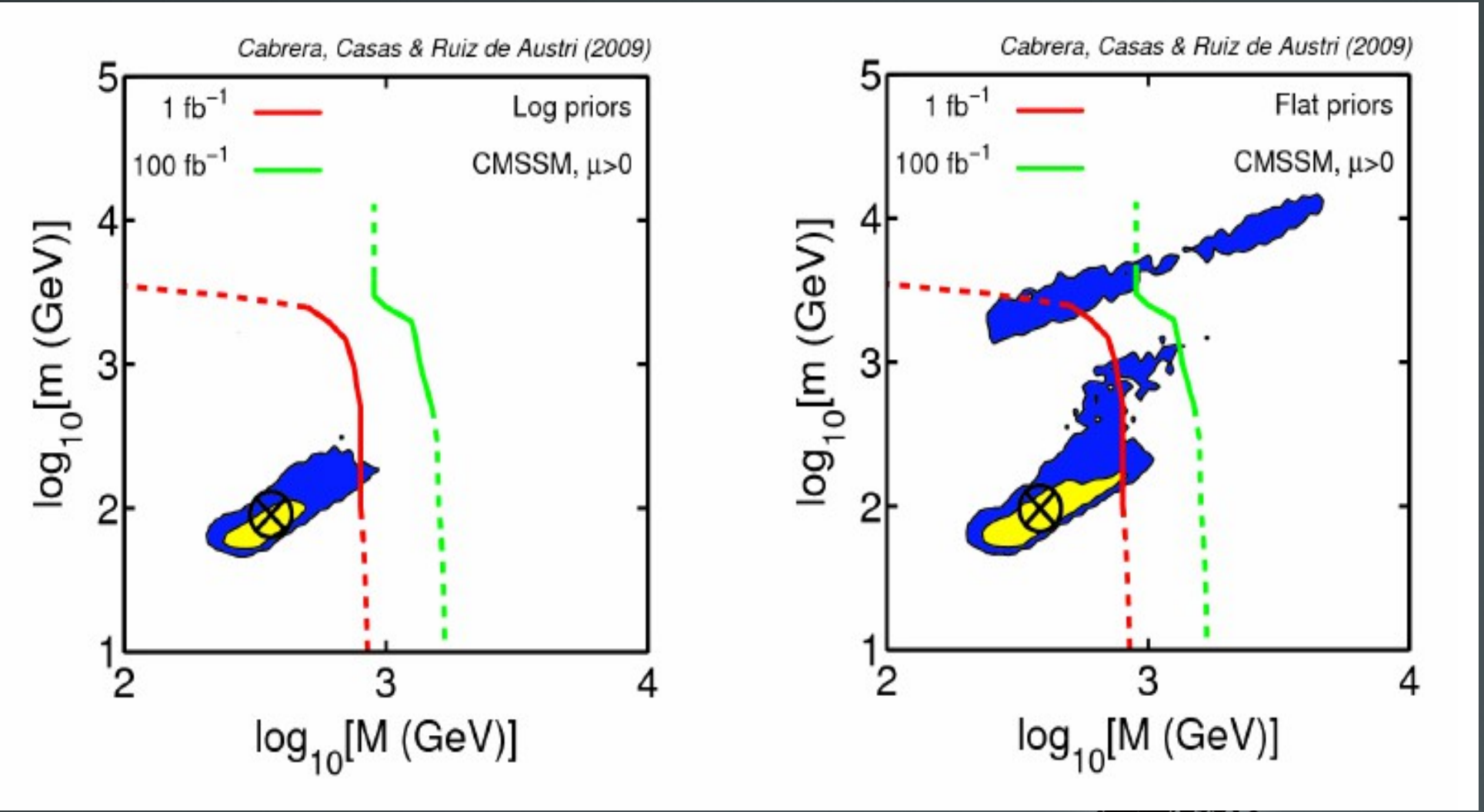
With Ω_{DM} but not g-2



Events with more or equal to 2 jets [Baer et al 0907.1922]

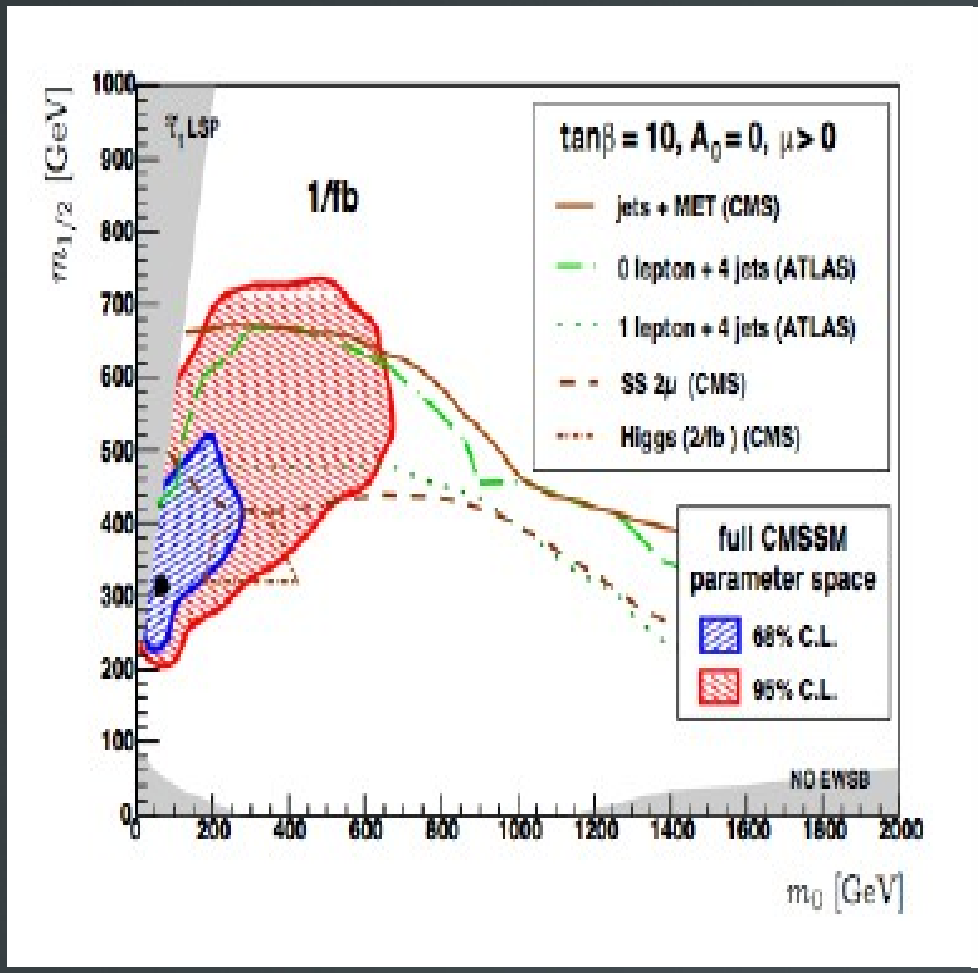
(LHC contours at 14 TeV C.M.)

Adding all data

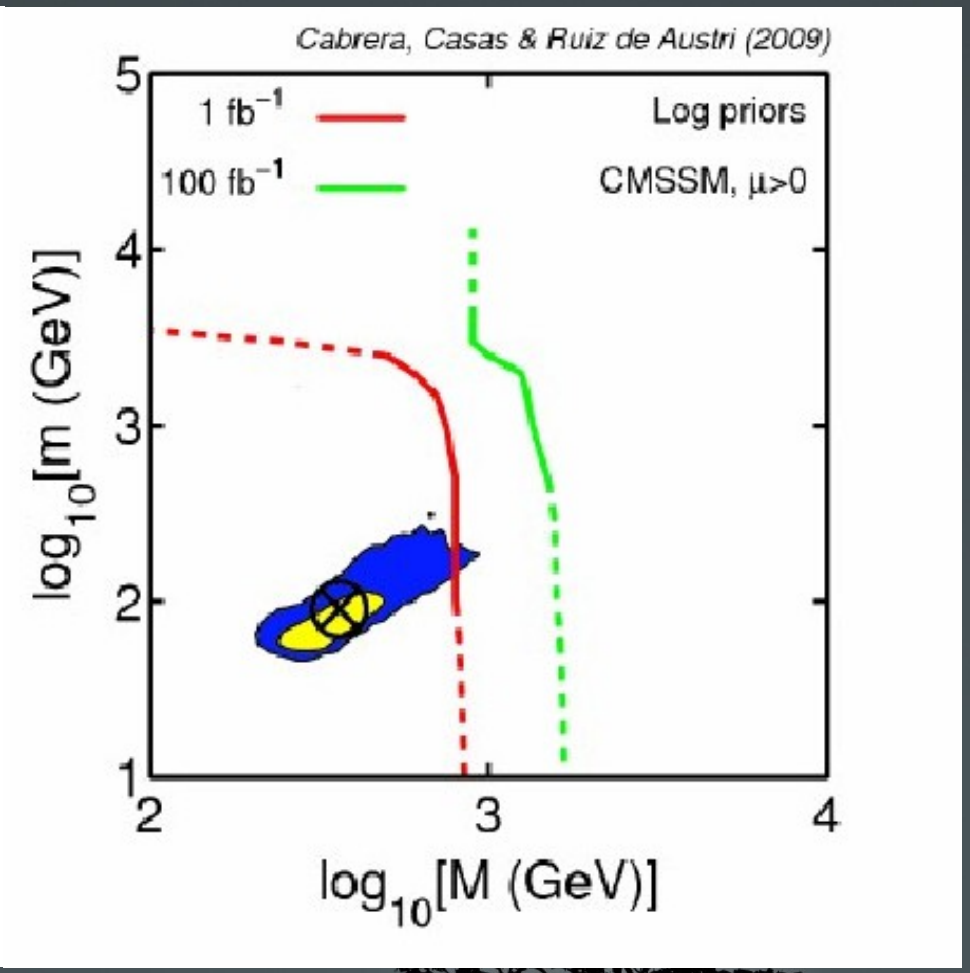


Comparison with Likelihood based inference

Buchmueller et al. (2009)

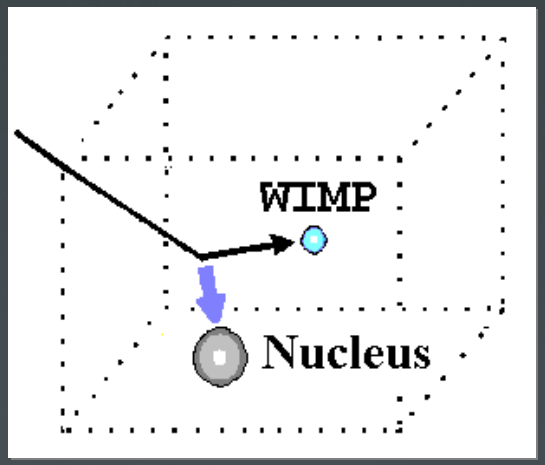


Cabrera et al. (2009)



Dark Matter Direct Detection

$$R = \int_{E_T}^{\infty} dE_R \frac{\rho_0}{m_N m_\chi} \int_{v_{min}}^{\infty} v f(v) \frac{d\sigma_{WN}}{dE_R}(v, E_R)\, dv$$

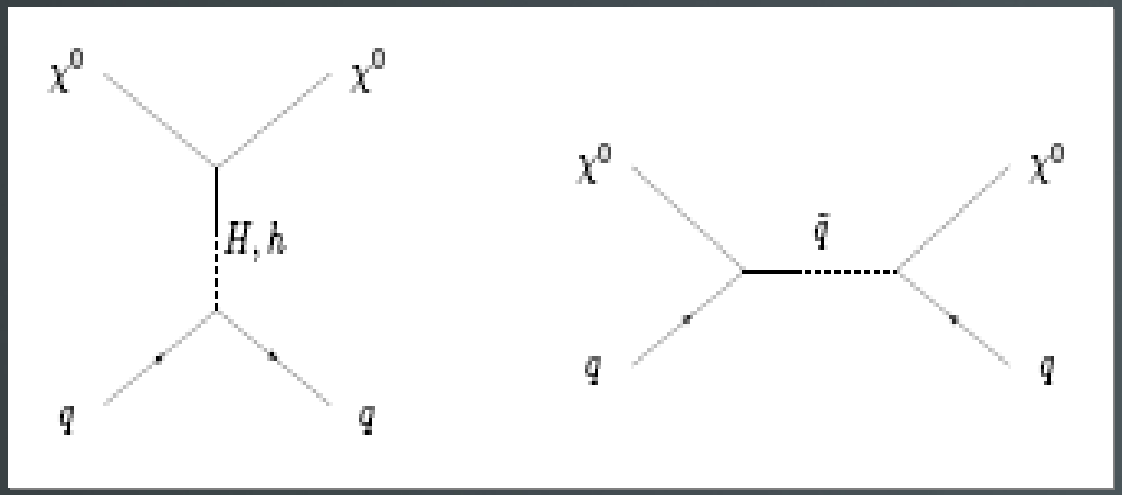


Astrophysics Input: local WIMP density and vel. distrib.

Particle Physics Input: cross sections and form factors

Detector dependence: Energy threshold

Neutralino-nuclei elastic scattering can occur through Higgs and squark exchange diagrams:

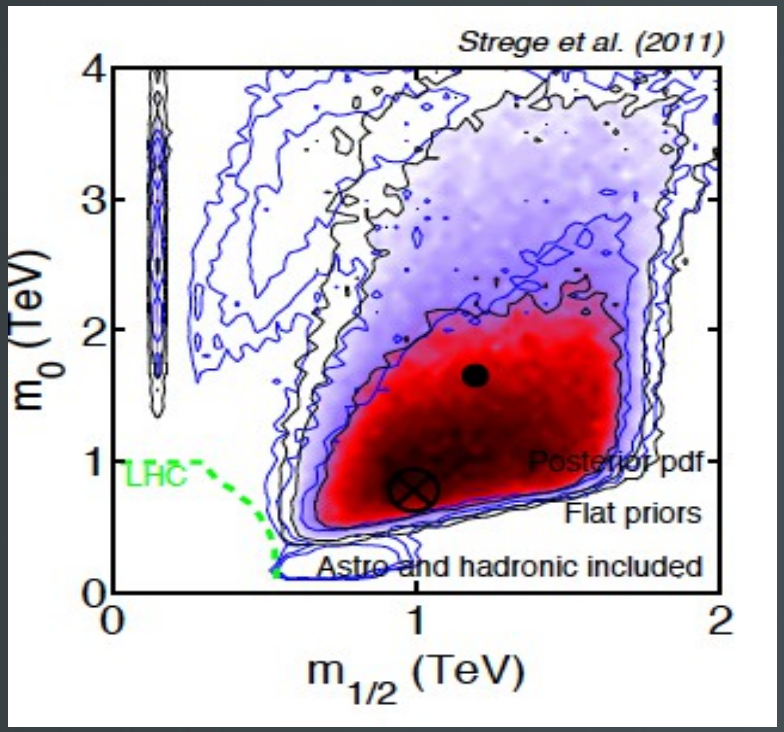


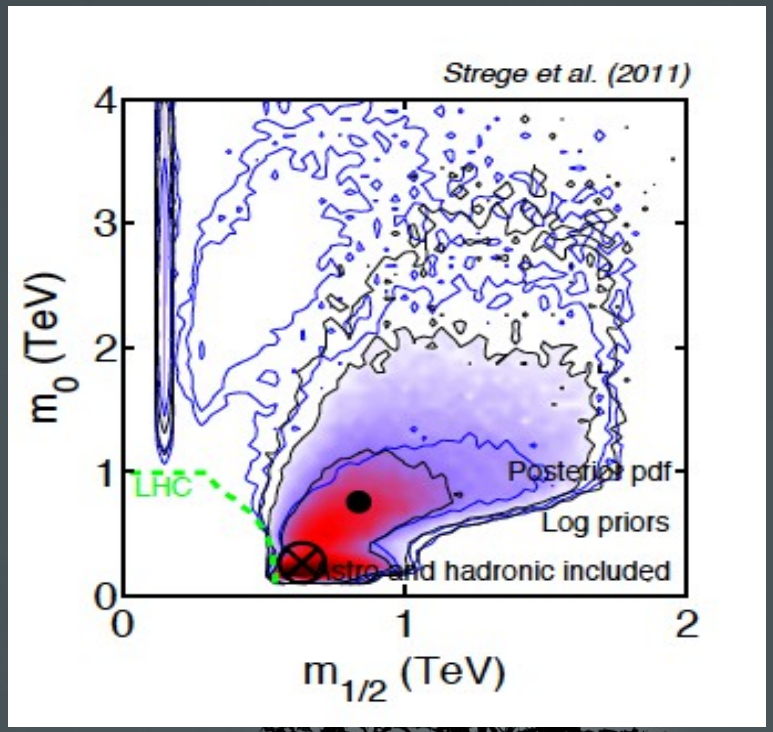
$$\sigma_{\chi N} \sim \frac{g_1^2 g_2^2 |N_{11}|^2 |N_{13}|^2 m_N^4}{4\pi m_W^2 \cos^2\beta\, m_H^4}$$

$$\sigma_{\chi N} \sim \frac{g_1^2 g_2^2 |N_{11}|^2 |N_{13}|^2 m_N^4}{4\pi m_W^2 \cos^2\beta\, m_{\tilde{q}}^4}$$

Xenon100 + LHC Data

- LHC with 1 fb^-1 and 7 TeV has a very limited impact on the posterior
- Direct Detection data disfavor the Focus Point => The prior dependency is reduced significantly

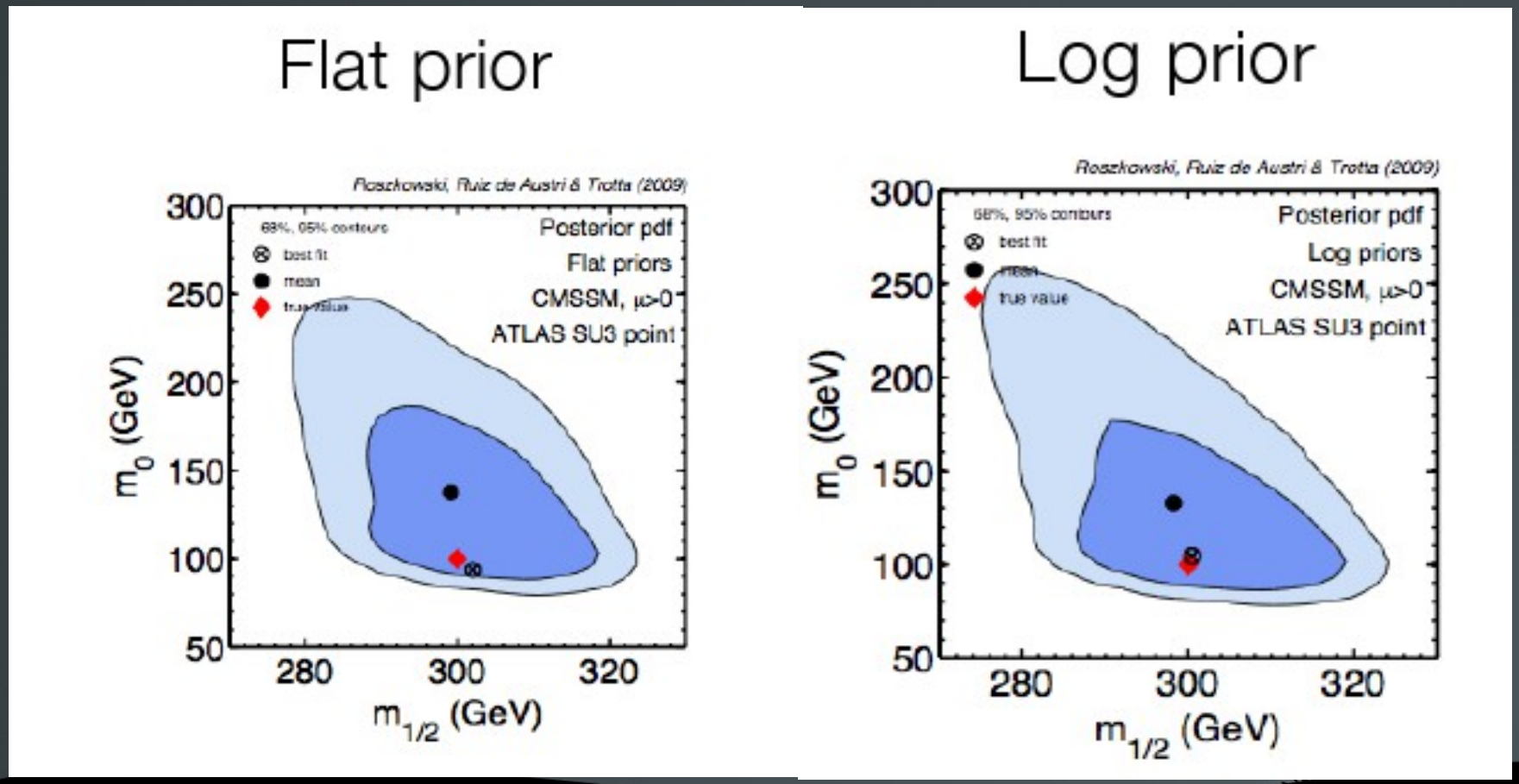




LHC will solve the prior dependency

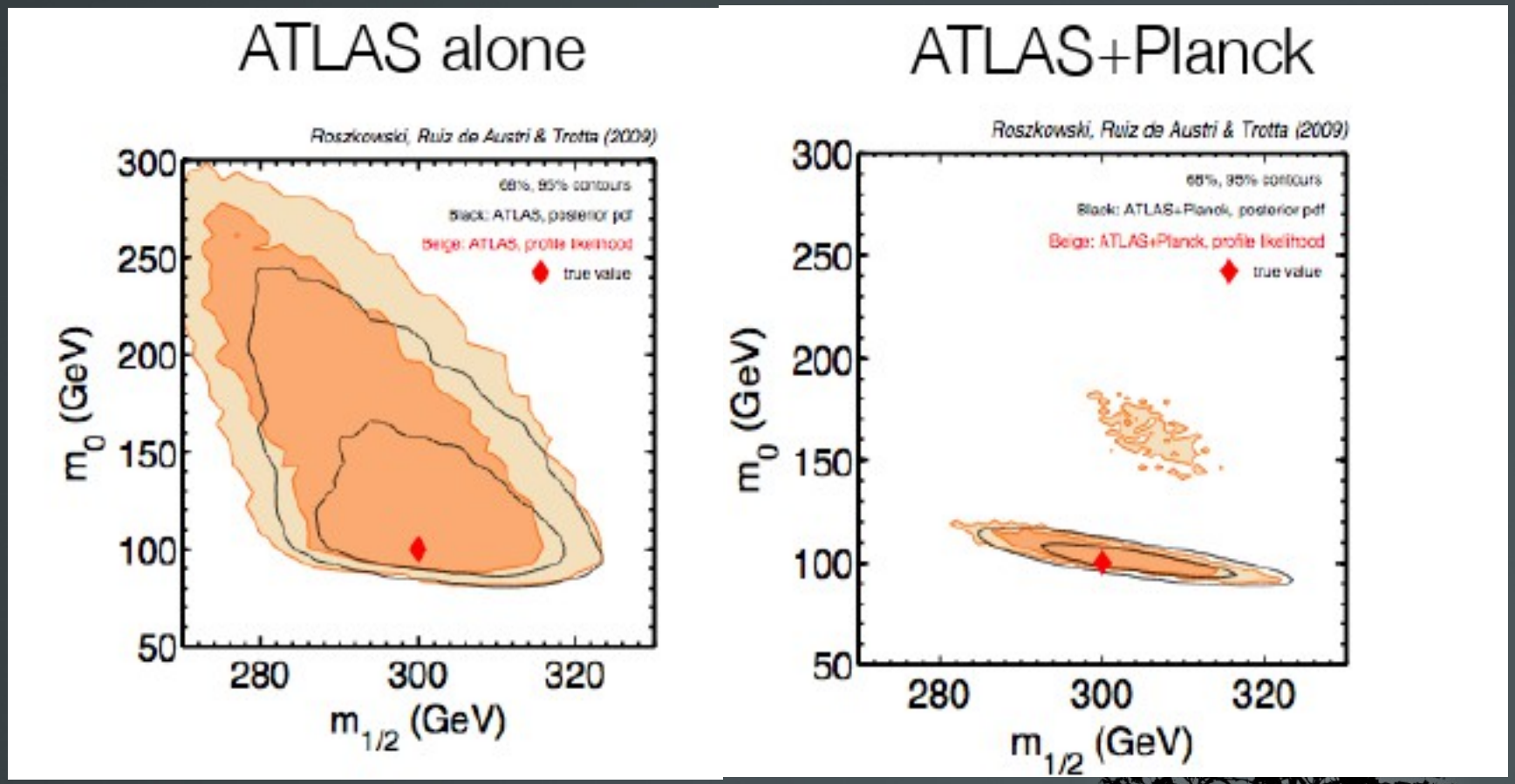
- Projected constraints from ATLAS (dilepton and lepton+jets edges 1 fb^-1 luminosity)

$$\theta = \{m_{\chi_1^0}, m_{\chi_2^0} - m_{\chi_1^0}, \tilde{m}_{\tilde{l}} - m_{\chi_1^0}, \tilde{m}_{\tilde{q}} - m_{\chi_1^0}\},$$



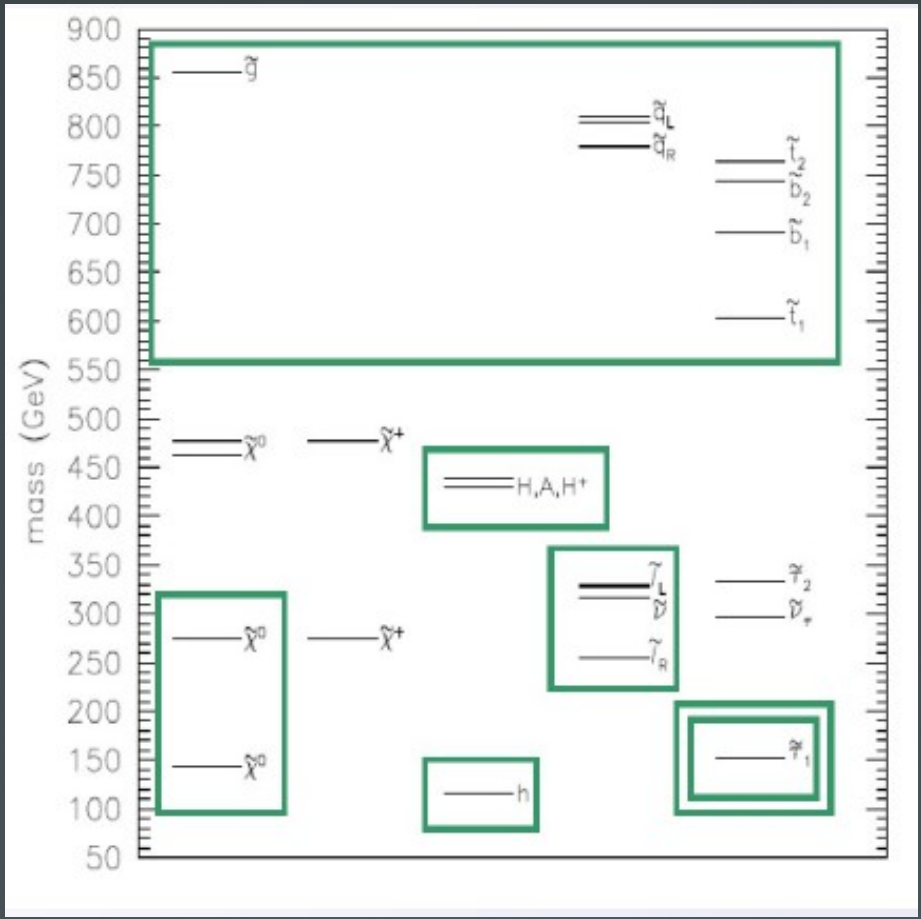
Residual dependency on the statistics

- Marginal posterior and profile likelihood will remain somewhat discrepant using ATLAS alone. Much better agreement from ATLAS+Planck CDM determination.



DM identification with DD + LHC data

- LHC might be able to determine part of the SUSY spectrum... but not all
- Some masses can be accurately determined through the study of kinematical endpoints
- Even the mass difference between the LSP and NLSP could be accurately reconstructed
- However... the neutralino composition might not be determined!
- Using hypothetical LHC data, we have tested the reconstruction of the relic abundance



Procedure:

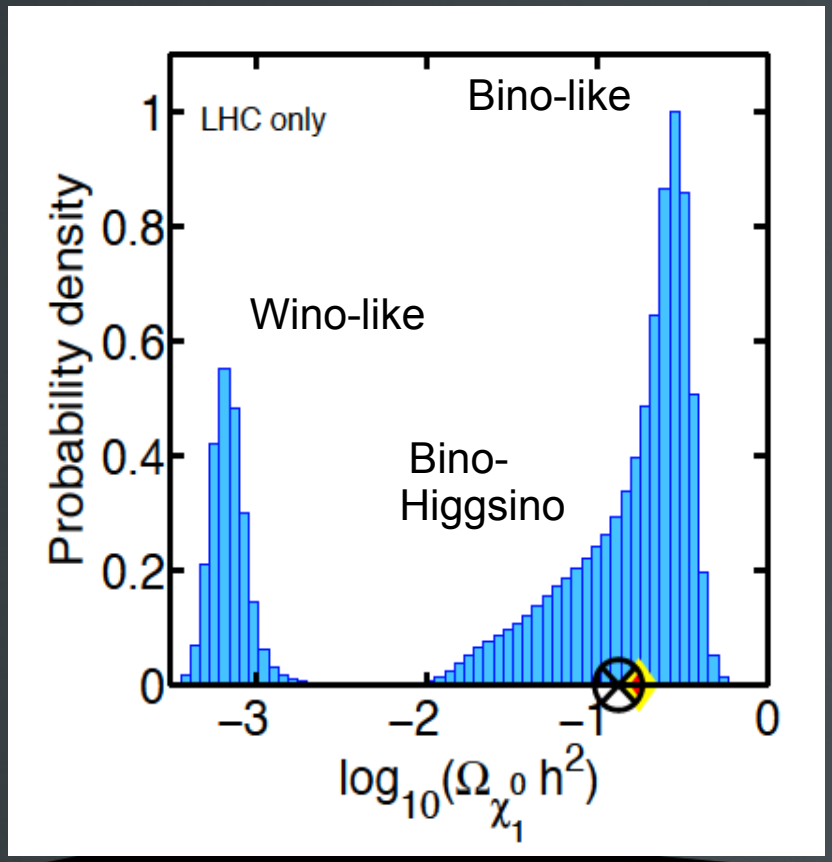
1. Choose the LCC3 in the CMSSM
2. Use determination of masses a experimental constraints
3. Perform a scan in a general MSSM (with 24 parameters) in order to find the regions with the best fit to the “data”
4. Determine the posterior distribution of the predicted relic abundance

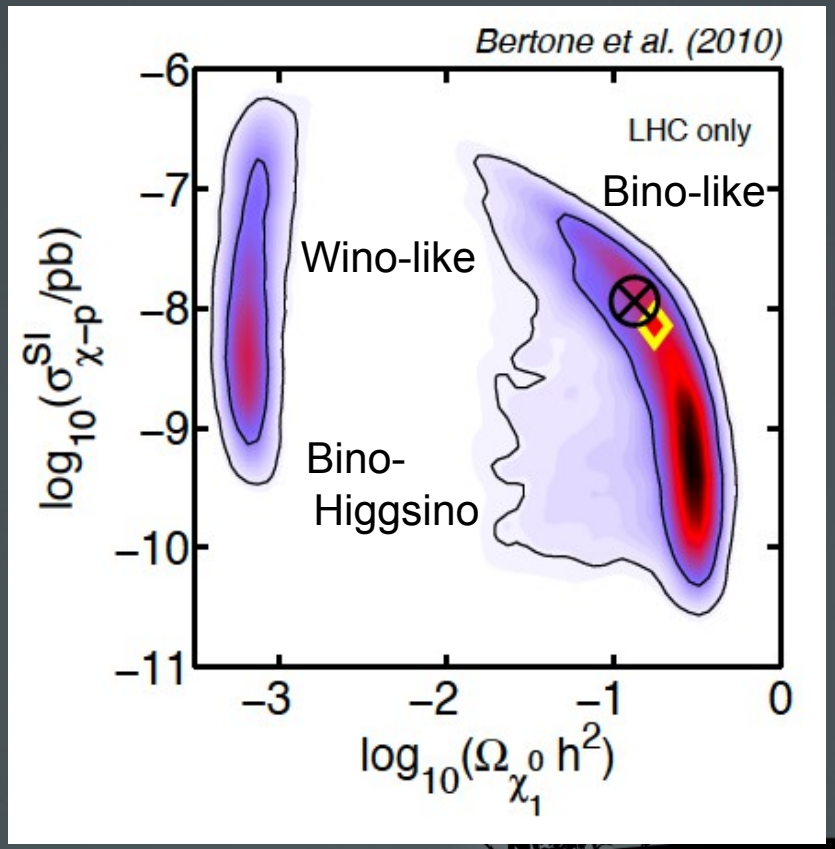
LHC data alone

- The LHC alone misses the determination of some properties of SUSY particles
- This is reflected in the uncertainty in the determination of the relic density

Fitting the LHC data can lead to several maxima

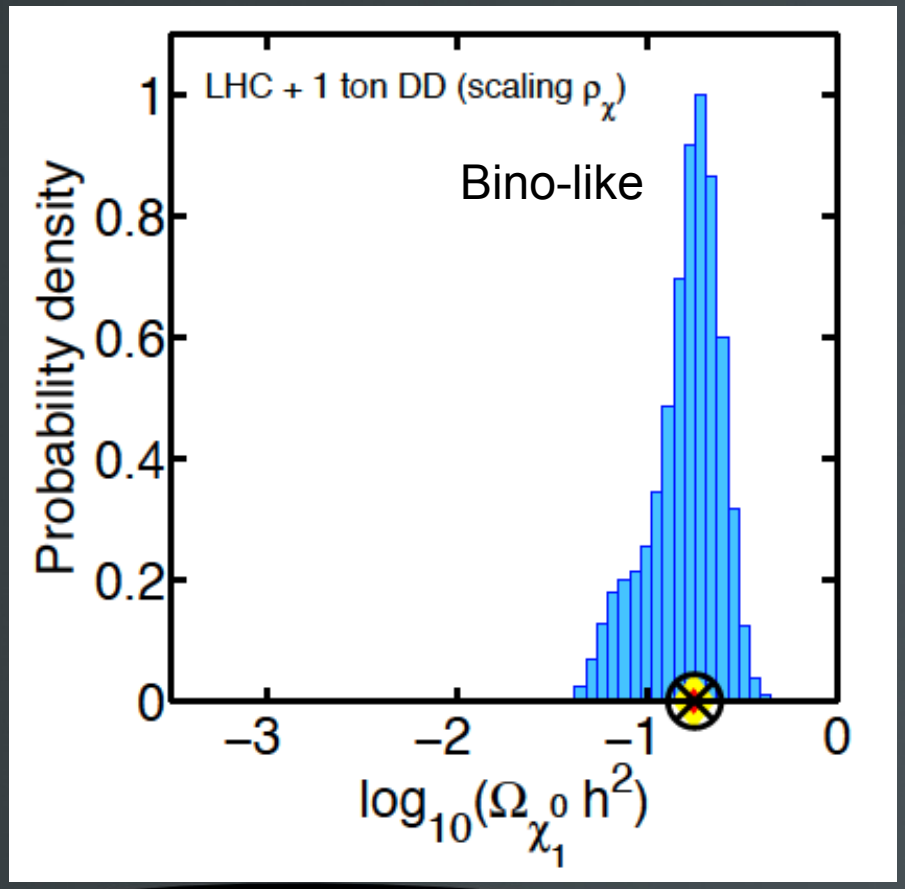
This uncertainty leads to an indetermination of the scattering Xs

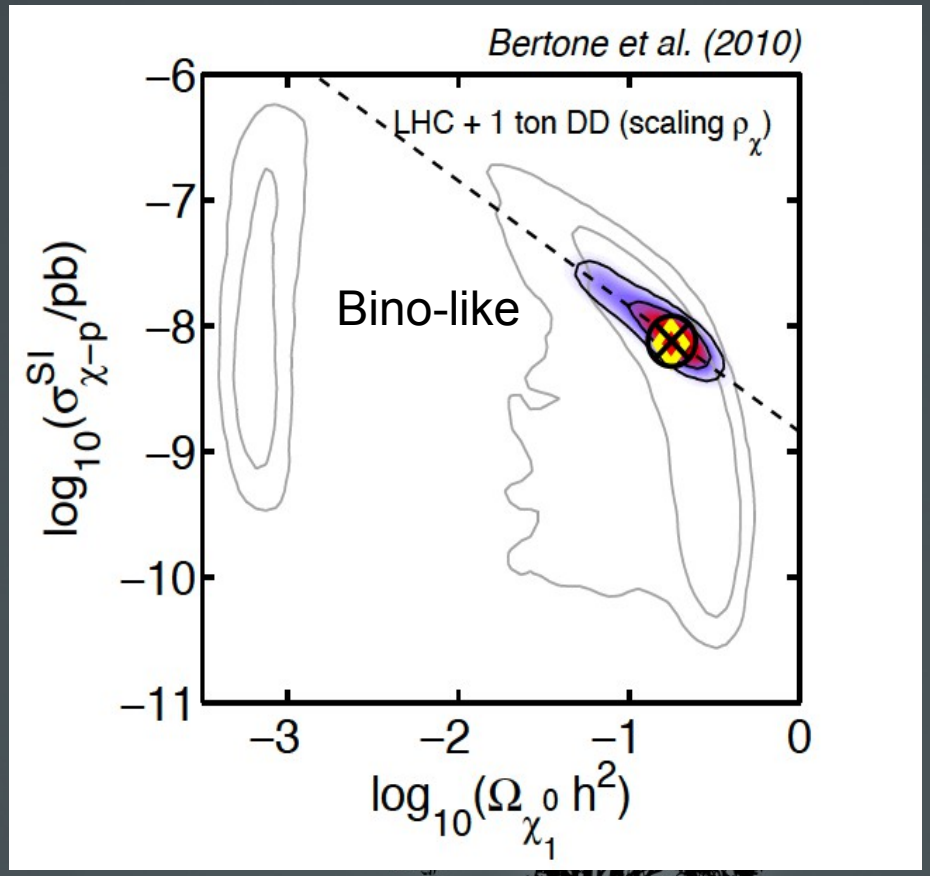




LHC + DM DD data

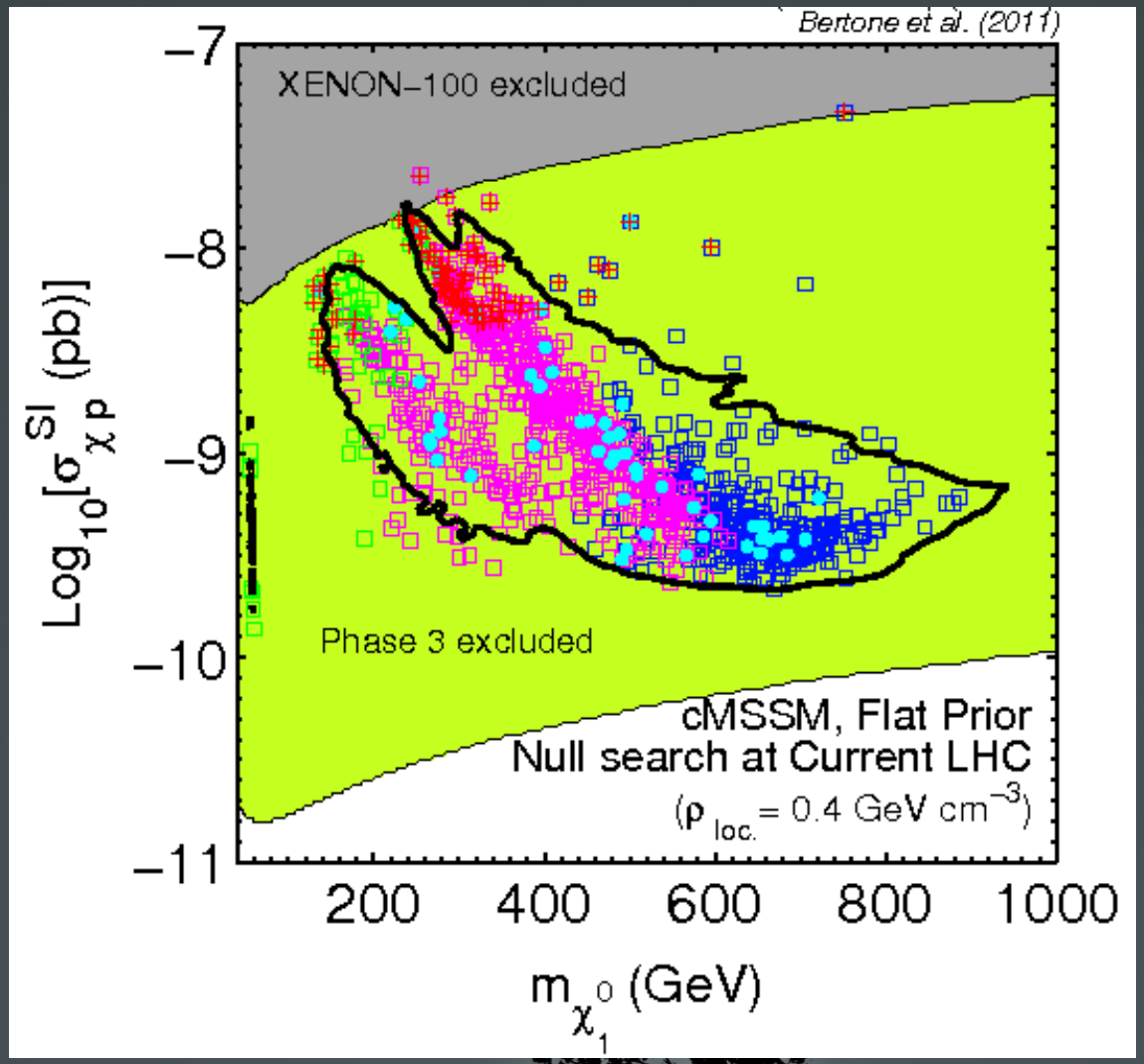
- Assuming the Scaling Ansatz: that the local density scales with the cosmological abundance $\rho_{\tilde{\chi}_1^0}/\rho_{\rm DM} = \Omega_{\tilde{\chi}_1^0}/\Omega_{\rm DM}$
- Scaling Ansatz => $\sigma^{\rm SI}_{\chi-p} \propto \Omega^{-1}_{\tilde{\chi}_1^0}$ breaks degeneracies in parameter space





Nightmare Scenario

- No single probe can cover the whole favoured parameter space, not even the LHC.
- Astroparticle probes (direct and indirect detection) can increase the coverage of the favoured parameter space, and deliver increased statistical robustness.
- High complementarity of LHC reach with direct detection methods.



Conclusions

- SUSY phenomenology provides a timely and challenging problem for parameter inference and model selection. A considerably harder problem than cosmological parameter extraction!
- Bayesian advantages: higher efficiency, inclusion of nuisance parameters, predictions for derived quantities, model comparison
- CMSSM only a case study: Bayesian analysis naturally penalizes fine-tunings. The exp. value of MZ brings the relevant parameter space to the low-energy region (~ accessible to LHC).
- CMSSM starts to be accessible to DM Direct Detection experiments
- LHC will prove new physics at the TeV scale but can not determine on its own the detection of DM; It needs extra info from DM Direct Detection experiments (COMPLEMENTARY)
- Currently, even the CMSSM is somewhat underconstrained: ATLAS+Planck will take us to "statistics nirvana"

Thank you !!!