Dark Attack

Global Fits - Bayesian

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Outline

- Why SUSY inference in Bayesian sense?
- How is this done?
- cMSSM: case study
- Complementarity of LHC and DM Direct Detection experiments
Supersymmetry

- Perhaps the most theoretically appealing (certainly the most well studied) extension of the Standard Model
- Natural solution to hierarchy problem (stabilizes quadratic divergences to Higgs mass)
- Restores unification of couplings
- Natural theory of gravity (Supergravity)
- Vital ingredient of string theory
- Naturally provides a compelling candidate for dark matter
Minimal Supersymmetric Standard Model: MSSM

Particle Spectrum:

The partners of the gauge and Higgs bosons mix to form the Neutralinos and Charginos.

\[
\begin{pmatrix}
M_1 & 0 & -m_Z \cos \beta \sin \theta_W & m_Z \sin \beta \sin \theta_W \\
0 & M_2 & m_Z \cos \beta \cos \theta_W & -m_Z \sin \beta \cos \theta_W \\
-m_Z \cos \beta \sin \theta_W & m_Z \cos \beta \cos \theta_W & 0 & -\mu \\
m_Z \sin \beta \sin \theta_W & -m_Z \sin \beta \cos \theta_W & -\mu & 0
\end{pmatrix}
\]

Also left- and right-chiral Sfermions mix

124 parameters !!!

All couplings for SUSY particles are the same as for SM particles!
Supersymmetric Dark Matter

- R-parity must be introduced in supersymmetry to prevent rapid proton decay
- Another consequence of R-parity is that superpartners can only be created and destroyed in pairs, making the lightest supersymmetric particle (LSP) stable
- Possible WIMP candidates from supersymmetry include:
  - $\gamma$, $Z$, $h$, $H$
  - $\nu$
  - 4 Neutralinos
  - 3 Sneutrinos
  - Excluded by DM DD
Weakly Interacting Massive Particles (WIMPs)

As a result of the thermal freeze-out process, a relic density of WIMPs is left behind:

\[ \Omega h^2 \sim \frac{x_F}{\langle \sigma v \rangle} \]

For a particle with a GeV-TeV mass, to obtain a thermal abundance equal to the observed dark matter density, we need an annihilation cross section of

\[ \langle \sigma v \rangle \sim \text{pb} \]

Generic weak interaction yields:

\[ \langle \sigma v \rangle \sim \alpha^2 (100 \text{ GeV})^{-2} \sim \text{pb} \]

Numerical coincidence? Or an indication that dark matter originates from EW physics?
Constrained Minimal Supersymmetric Standard Model: CMSSM

- 5 parameters (MSSM has 124) => suitable for pheno analyses
  0. Common Gaugino mass: \( m_{1/2} \)
  1. Common Scalar mass: \( m_0 \)
  2. Common trilinear coupling: \( A_0 \)
  3. Ratio of the Higgs vacuum expectation values: \( \tan \beta \)
  4. Sign of the Higgsino mass parameter \( \mu \)
- First 3 parameters are for some high unification scale \( M_x \).

Goal: inference of the model parameters from collider and DM data but it is difficult problem.

Supersymmetry is spontaneously broken by gravitational couplings with a hidden sector on the Planck scale

\[ \sum \text{Fields get vevs} \]
\[ \sum \text{Analog to Higgs mechanism} \]
Is SUSY exact?

- SUSY theories have to be broken since we have not seen SUSY particles
- To solve the hierarchy problem SUSY has to be broken at the TeV scale \((m_F - m_B \sim 1 \text{ TeV})\)
- Different mechanisms to generate the mass spectrum:
  - **Gravity mediated**, gravity interactions with hidden sector fields, e.g. minimal supergravity (mSUGRA) \(\rightarrow\) CMSSM
  - **Gauge mediated**, gauge interactions with messenger sector, GMSB
  - Others: **Anomaly mediated** ...
Fixed Grid Scans

green: consistent with WMAP-7yr (at 2σ)

all the rest excluded by LEP, BR(B → Xs γ), Ωχ h2, EWSB, charged LSP,...
Why is a difficult problem?

- Inherently 8-dimensional: reducing the dimensionality over-simplifies the problem. Nuisance parameters (in particular mt) cannot be fixed!

- Likelihood discontinuous and multi-modal due to physicality conditions
The Bayesian Approach

- Bayesian approach led by two groups (early work by Baltz & Gondolo, 2004)
- Ben Allanach (DAMPT) et al (Allanach & Lester, 2006 onwards, Cranmer, and others)
- RdA, Roszkowski & Roberto Trotta (2006 onwards)
  SuperBayeS public code (available from: superbayes.org) + Feroz & Hobson (MultiNest), + Silk (indirect detection), + de los Heros (IceCube), + Casas et al. (Naturalness) + Bertone et al. (pmssm)†
The Likelihood-based Approach


R. Lafaye, M. Rauch, T. Plehn, D. Zerwas

H. Flächer, M. Goebel, J. Haller, A. Höcker, K. Mönig, J. Stelzer

P. Bechtle, K. Desch, M. Uhlenbrock, P. Wienemann
Favoured regions: Likelihood-based Approach

- Due to the weak nature of constraints, different scanning techniques and statistical methods will generally give different answers

**Likelihood-based methods**: determine the best fit parameters by finding the minimum of $-2 \log(\text{Likelihood}) = \chi^2$

1. Markov Chain Monte Carlo and Minuit as “afterburner”
2. Simulated annealing
3. Genetic algorithms

- Determine approximate confidence intervals: Local $\Delta(\chi^2)$ method

- **Profile likelihood**: way to treat nuisance

$L(x,y) \Rightarrow PL(x) = \max. L(x,y)$ for fixed $x$ in $y$
Bayes Theorem

- **H**: hypothesis
- **D**: data
- **I**: external information
- **Prior**: what we know about H (given information I) before seeing the data
- **Likelihood**: the probability of obtaining data d if hypothesis H is true
- **Posterior**: the probability of obtaining data d if hypothesis H is true
- **Evidence**: normalization constant (independent of H), crucial for model comparison

$$P(H|d,I) = \frac{P(d|H,I)P(H|I)}{P(d|I)}$$
Priors

Ignoring the prior and identifying implicitly amounts to

\[ p(\theta_i | \text{data}) \equiv p(\text{data} | \theta_i) \]

But e.g.

\[ p(\theta_i) = \text{const.} \equiv \text{"flat"} \]

\[ \theta_i \rightarrow \theta_i^2 \]

\[ \text{"flat"} \rightarrow \text{"non-flat"} \]

There is a vast literature on priors: Jeffreys’, conjugate, non-informative, ignorance, etc.

If data are good enough to select a small region of \( \{\theta\} \) then the prior \( p(\theta) \) becomes irrelevant.

\[ p(\theta_i | \text{data}) \equiv p(\text{data} | \theta_i) \]
Key advantages

- **Efficiency**: computational effort scales $\sim N$ rather than $k^N$ as in grid-scanning methods. Orders of magnitude improvement over previously used techniques.

- **Marginalisation**: integration over hidden dimensions comes for free. Suppose we have $\theta_i$ and are interested in $p(\theta_1|\text{data})$

\[
p(\theta_1|\text{data}) = \int d\theta_2 \cdots d\theta_N p(\theta_i|\text{data})
\]

- **Inclusion of nuisance parameters**: simply include them in the scan and marginalise over them. Notice: nuisance parameters in this context must be well constrained using independent data.

- ** Derived quantities**: probabilities distributions can be derived for any function of the input variables (crucial for DD/ID/LHC predictions).
Posterior Samples

- **MCMC**: A Markov Chain is a list of samples $\theta_1, \theta_2, \theta_3, \ldots$ whose density reflects the (unnormalized) value of the posterior.

- **Crucial property**: A Markov Chain converges to a stationary distribution, i.e. one that does not change with time. In our case, the posterior.

- **Different algorithms**: MH, Gibbs... all need a proposal distribution $\Rightarrow$ difficult to find a good one in complex problems.

- **Nested**: New technique for efficient evidence evaluation (and posterior samples) (Skilling 2004).

- **MultiNest**: Also an extremely efficient sampler for multi-modal likelihoods!
  
Favoured regions: Bayesian-based Approach

- **Bayesian methods:** the best-fit has no special status. Focus on region of large posterior probability mass instead.

- Determine posterior credible regions: e.g. symmetric interval around the mean containing 68% of samples
The Gaussian case

- Life is easy (and boring) in Gaussianland:

Profile likelihood

Marginal posterior
Marginalization vs profiling (maximising)

Physical analogy: (thanks to Tom Loredo)

Likelihood = hottest hypothesis
Posterior = hypothesis with most heat

Heat: \[ Q = \int c_V(x) T(x) dV \]

Posterior: \[ P \propto \int p(\theta) L(\theta) d\theta \]

(2D plot depicts likelihood contours - prior assumed flat over wide range)
**MCMC**

- **MCMC**: Markov Chain Monte Carlo with Metropolis-Hastings

1. Select a random point in the parameter space, $\theta_0$
   
   Compute $P(\theta_1) = \text{Like} \times \text{Prior}$

2. Propose a new point, $\theta_1$ with transition probability $T$, satisfying
   
   $$T(\theta_0, \theta_1) = T(\theta_1, \theta_0)$$

3. Evaluate $P(\theta_1) = \text{Like} \times \text{Prior}$

4. If $P(\theta_1) > P(\theta_0)$ move to $\theta_1$
   
   Else
   
   Move to $\theta_1$ with probability $P(\theta_1)/P(\theta_0)$
   
   Otherwise stays in $\theta_0$

   => Obtain a Markov Chain: $\theta_i$, $i = 1, 2, ..., t$

   $t \rightarrow \infty$ chain $\Rightarrow$ **target distribution**, $P(\theta)$
Proposal width low or directions bad:
slow random walk through space

Proposal width too large:
low probability of proposing similar likelihood, low acceptance rate

Proposal distribution must be able to reach all points in parameter space

Can never cross gap!
The Nested Sampling algorithm

- Skilling (2006) introduced Nested Sampling as an algorithm originally aimed at the efficient computation of the model likelihood (Skilling, 2006).
- The idea is to map a multi-dimensional integral onto a 1D integral which is easy to compute numerically.
- The method requires to sample uniformly from the fraction of the prior volume $X(\mu)$ above the iso-likelihood level $\mu$

\[
X(\mu) = \int_{L(\theta) > \mu} P(\theta) d\theta
\]

\[
P(d) = \int d\theta L(\theta) P(\theta) = \int_0^1 X(\mu) d\mu
\]

Feroz et al (2008), arxiv: 0807.4512
Trotta et al (2008), arxiv: 0809.3792
Nested Sampling in action

Nested sampling pseudo-code

Initialization
- Draw N “live points” from the prior (typically, N ~ 2000)
- Compute the likelihood for each live point

[Loop beings]
- Select the live point with the lowest likelihood value, \( \mu \)
- Replace it with a new live point \( \theta \) drawn from the prior with the constraint \( L(\theta) > \mu \)
- Save the previous live point, together with \( \mu \) and the prior volume fraction \( X(\mu) \)
- If \( \text{Lmax} \times X < \text{tolerance} \), exit

[Loop ends]
Sampling of multi-modal likelihoods

- Thanks to the multi-modal ellipsoidal decomposition, MultiNest is an extremely efficient sampler for highly challenging multi-modal likelihoods!

<table>
<thead>
<tr>
<th>Dimensionality (D)</th>
<th>Likelihood evaluations (N)</th>
<th>Efficiency</th>
<th>N^{1/D}</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7,000</td>
<td>70%</td>
<td>83</td>
</tr>
<tr>
<td>10</td>
<td>53,000</td>
<td>34%</td>
<td>3</td>
</tr>
<tr>
<td>20</td>
<td>255,000</td>
<td>15%</td>
<td>1.8</td>
</tr>
<tr>
<td>30</td>
<td>753,000</td>
<td>8%</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Target likelihood (D=2)

Sampled likelihood with MultiNest
The SuperBayeS package
(superbayes.org)

- Supersymmetry Parameters Extraction Routines for Bayesian Statistics
- Implements the CMSSM, but can be easily extended to the general MSSM
- Currently linked to SoftSusy 2.0.18, DarkSusy 4.1, MICROMEGAS 2.2, FeynHiggs 2.5.1, Hdecay 3.102. **New release (v 1.5.1)**
- Includes up-to-date constraints from all observables
- Bayesian MCMC, MULTI-MODAL NESTED SAMPLING or grid scan mode.
- MULTI-MODAL NESTED SAMPLING (Feroz & Hobson 2008), efficiency increased by a factor 200. **A full 8D scan now takes 3 days on a single CPU** (previously: 6 weeks on 10 CPUs)
- Fully parallelized, MPI-ready, user-friendly interface a la cosmomc (thanks Sarah Bridle & Antony Lewis)
- **SuperEGO**: SuperBayeS Enhanced Graphical Output as a MATLAB graphical user interface for statistical analysis and plotting
Global CMSSM fits
**Analysis Pipeline**

**SCANNING ALGORITHM**

4 CMSSM parameters
\[ \theta = \{ m_0, m_{1/2}, A_0, \tan\beta \} \]
(fixing sign(\(\mu\)) > 0)

4 SM “nuisance parameters”
\[ \Psi = \{ m_t, m_b, \alpha_s, \alpha_{EM} \} \]

Data:
Gaussian likelihoods for each of the \(\Psi_j\)
\((j=1 \ldots 4)\)

RGE

Non-linear numerical function
via SoftSusy 2.0.18
DarkSusy 5.0
MICROMEGAS 2.2
FeynHiggs 2.5.1
Hdecay 3.102

Observable quantities
\[ f_1(\theta, \Psi) \]
CDM relic abundance
BR’s
EW observables
\(g-2\)
Higgs mass
sparticle spectrum
(gamma-ray, neutrino, antimatter flux, direct detection x-section)

Joint likelihood function

Data:
Gaussian likelihood
(CDM, EWO, \(g-2\), \(b \to s\gamma\), \(\Delta M_{\mu\beta}\))
other observables have only lower/upper limits

Likelihood = 0

\[ \uparrow \]

NO

Physically acceptable?
EWSB, no tachyons,
neutralino CDM

\[ \downarrow \]

YES
Samples from priors only

- No data in the likelihood, non-physical points discarded priors

\[ p(\mathcal{F}) = p(m) \left| \frac{dm}{d\mathcal{F}} \right| \Rightarrow \text{flat priors on log means} \]

\[ p(m) \propto m^{-1} \]
### Analysis

**Prior ranges**

<table>
<thead>
<tr>
<th>flat priors: CMSSM parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$50 \text{ GeV} &lt; m_0 &lt; 4 \text{ TeV}$</td>
</tr>
<tr>
<td>$50 \text{ GeV} &lt; m_{1/2} &lt; 4 \text{ TeV}$</td>
</tr>
<tr>
<td>$</td>
</tr>
<tr>
<td>$2 &lt; \tan \beta &lt; 62$</td>
</tr>
</tbody>
</table>

**Likelihood function**

$$L = p(\sigma, c | \xi(m)) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{\chi^2}{2}\right]$$

$$\chi^2 = \frac{[\xi(m) - c]^2}{\sigma^2} \quad \sigma \rightarrow s = \sqrt{\sigma^2 + t^2}$$

**Data: Indirect observables**

- **SM parameters**

<table>
<thead>
<tr>
<th>Observable</th>
<th>Mean value</th>
<th>Uncertainties (exper.)</th>
<th>Uncertainties (theor.)</th>
<th>ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin^2 \theta_{\text{eff}}$</td>
<td>0.23153</td>
<td>$16 \times 10^{-5}$</td>
<td>$15 \times 10^{-5}$</td>
<td>[30]</td>
</tr>
<tr>
<td>$m_{\mu}$</td>
<td>106 GeV</td>
<td>1.0</td>
<td>1.0</td>
<td>[31]</td>
</tr>
<tr>
<td>$BR(B \rightarrow X_s \gamma)$</td>
<td>3.55</td>
<td>0.26</td>
<td>0.21</td>
<td>[32]</td>
</tr>
<tr>
<td>$\Delta M_B_d$</td>
<td>17.77 ps⁻¹</td>
<td>0.12 ps⁻¹</td>
<td>2.4 ps⁻¹</td>
<td>[33]</td>
</tr>
<tr>
<td>$BR(B_d \rightarrow \tau \nu)$</td>
<td>1.32</td>
<td>0.49</td>
<td>0.38</td>
<td>[32]</td>
</tr>
<tr>
<td>$\Omega h^2$</td>
<td>0.1099</td>
<td>0.0062</td>
<td>0.1Ωχh²</td>
<td>[34]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Limit (95% CL)</th>
<th>Uncertainty (theor.)</th>
<th>ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BR(B_s \rightarrow \mu^+ \mu^-)$</td>
<td>$&lt; 5.8 \times 10^{-5}$</td>
<td>14%</td>
</tr>
<tr>
<td>$m_h$</td>
<td>&gt; 114.4 GeV (SM-like Higgs)</td>
<td>3 GeV</td>
</tr>
<tr>
<td>$\zeta_h^2$</td>
<td>$f(m_h)$ (see text)</td>
<td>negligible</td>
</tr>
<tr>
<td>$m_{\tilde{q}}$</td>
<td>&gt; 375 GeV</td>
<td>5%</td>
</tr>
<tr>
<td>$m_{\tilde{g}}$</td>
<td>&gt; 289 GeV</td>
<td>5%</td>
</tr>
<tr>
<td>other sparticle masses</td>
<td>As in table 4 of ref. [6].</td>
<td></td>
</tr>
</tbody>
</table>
CMSSM in 2011: Frequentist fits

SuperBayeS: profile likelihood
(~41M samples)

**Best fit**
incl. ATLAS 35 fb⁻¹ and WMAP-7:
\(m_0 \sim 80..100 \text{ GeV}\)
\(m_{10} \sim 360..370 \text{ GeV}\)
\(A_0 \sim -280..-400 \text{ GeV}\)
\(\tan \beta \sim 11..14\)

MasterCode: profile likelihood
(~30M samples)

**Best fit**
incl. ATLAS/CMS 35 fb⁻¹ and WMAP:
\(m_0 \sim 80..100 \text{ GeV}\)
\(m_{10} \sim 370..400 \text{ GeV}\)
\(A_0 \sim -300..-400 \text{ GeV}\)
\(\tan \beta \sim 11..14\)

Buchmueller et al, arxiv: 1102.4585

Low-energy SUSY preferred though Focus Point region cannot be ruled out at 99%

Bertone et al (2011)

arxiv: 1107.1715
Low-energy SUSY remains favoured in both approaches. Tentative convergence between Frequentist and Bayesian scans, although prior effects remain present (in particular, the relative viability of the Focus Point region cannot be robustly established with LHC constraints).
The cMSSM at the end of 2011

Global fits of the Constrained MSSM (4+4 parameters) including relic abundance data, rare processes, g-2, EW precision observables and constraints from 1/fb ATLAS data and Higgs limits from 5 1/fb

Higher luminosity LHC data are putting pressure on the best fit point
Xenon100 Data

- 48 Kg in 100.9 days of exposure
- 3 events observed and bkg = 0.8 ± 0.6
- Likelihood:

\[ \mathcal{L}_{\text{Xe100}}(\Theta) \propto p(\hat{N}|\lambda) = \frac{\lambda^N}{N!} e^{-\lambda} \]

Posterior
The LHC exclusion limit cuts deep into cMSSM parameter space. The Higgs mass constraint pushes contours towards significantly larger values of $m_{1/2}$. The best-fit point remains at low masses. Posterior pdfs become less dependent on the choice of priors and are in good agreement with PL results.
Akrami et al (0910.3950) adopted a genetic algorithm (GA) to map out the profile likelihood.

This allows to find isolated spikes in the likelihood in the focus point region: is this something other frequentist fits might have missed?
Challenges of profile likelihood evaluation

- MCMC/MultiNest are not designed to find the best-fit point. Bayesian algorithms are designed to map out regions of significant posterior probability mass.

- Even for a simple Gaussian toy model, this becomes difficult to do as the number of dimensions of the parameter space increases.

- Profiling with vanilla MCMC or MultiNest scans has to be done with caution!

Toy multinormal likelihood

![Graph showing the difference between true max and recovered max for Vanilla MCMC and Slice sampling. The x-axis represents the number of parameters, and the y-axis represents the difference between the true max and recovered max.](image)
Posterior pdf from MultiNest scans

- MultiNest is primarily aimed at evaluation of the posterior pdf. It does an excellent job even for multi-modal problems. 8D toy case (Feroz, KC, RT et al, in prep)

- The tolerance parameter (tol) determines the stopping criterium (based on the incremental change of the value of the local evidence). Lower tol gives a finer exploration around the peak, important for profile likelihood evaluation

**red**: analytical  
**blue**: MN, tol=0.5  
**black**: MN, tol=0.0001
Profile likelihood from MultiNest scans

- A fairly accurate profile likelihood can be obtained with MultiNest by tuning the tolerance (lower, $tol=0.0001$) and the number of live points (higher, $n_{\text{live}}=20,000$) (Feroz, KC, RT et al, in prep), even for highly multi-modal distributions. 8D toy:

```
red: analytical  blue: MN, tol=0.5  black: MN, tol=0.0001
```

![Graphs showing profile likelihood and log profile likelihood with different tolerances and live points.](image)

Tuesday, 17 July 12
Profile likelihood from MultiNest

- MultiNest scan with 20,000 live points (usually: 4,000) and tolerance 0.0001 (usually: 0.5) results in 5.5 million likelihood evaluations (Akrami et al, GA: 3 million), and best-fit chi-square = 9.26 (Akrami et al, GA: 9.35).

**MultiNest finds a better best-fit + smoother contours than GA.**

Profile likelihood MultiNest, tol=10^{-4}
Merged log and flat priors scans

Profile likelihood Genetic algorithm

Feroz, KC, RT et al (2011)

Akrami et al (2010)
Towards a more refined analysis

\[ \{\theta_i\} \equiv \{s, m, M, A, B, \mu\} \]

SM-like

soft terms

\[ g_a, \ y_i \]

\[ \text{SUSY} \]

\[ \mu \text{-parameter} \]

?
Bayesian and Naturalness

Recall an usual assumption

\[ m, M, A, B, \mu \]

should be \( \lesssim \mathcal{O}(\text{TeV}) \)

In order to get a Natural Electroweak symmetry Breaking
(with no fine-tunings)

\[
V(H_1, H_2) = m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 - 2B\mu H_1 H_2 + \frac{1}{8}(g^2 + g'^2) (|H_1|^2 - |H_2|^2)^2
\]

\[
M_Z^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - 2\mu^2
\]

\[
\sin 2\beta = \frac{2\mu}{B} (m_{H_1}^2 + m_{H_2}^2 + 2\mu_{\text{low}}^2)
\]

Unnatural fine-tuning unless \( M_{\text{soft}} \lesssim \mathcal{O}(\text{TeV}) \)
Instead solving $\mu^2$ in terms of $M_Z$ and the other soft-terms, treat as another exp. Data

Approximate the likelihood as

$$
\mathcal{L} = N_Z \ e^{-\frac{1}{2} \left( \frac{M_Z - M_Z^{\text{exp}}}{\sigma_Z} \right)^2} \mathcal{L}_{\text{rest}} \\
\approx \delta(M_Z - M_Z^{\text{exp}}) \mathcal{L}_{\text{rest}}
$$

Use $M_Z$ to marginalize $\mu$

$$
p(s, m, M, A, B \mid \text{data}) = \int d\mu \ p(s, m, M, A, B, \mu \mid \text{data}) \\
\approx \mathcal{L}_{\text{rest}} \left[ \frac{d\mu}{dM_Z} \right]_{\mu_Z} p(s, m, M, A, B, \mu_Z)
$$

$$
p(s, m, M, A, B \mid \text{data}) = 2 \mathcal{L}_{\text{rest}} \frac{\mu_Z}{M_Z} \frac{1}{c_\mu} p(s, m, M, A, B, \mu_Z)
$$

$c_\mu = \frac{\partial}{\partial \ln \theta_i} \ln M_Z^2$

$c^{-1} \sim$ Probability of cancellation between the various contributions to get

$M_Z \sim \mathcal{O}(90 \text{GeV})$

$\mathcal{O}$
Putting all together

\( \{\mu, y_t, B\} \xrightarrow{J} \{M_Z, m_t, \tan \beta\} \)

\[
p(m_t, m, M, A, \tan \beta | \text{data}) = J_{|\mu=\mu_Z} p(y_t, m, M, A, B, \mu_Z) \mathcal{L}_{\text{rest}}
\]

\[
p_{\text{eff}}(m_t, m, M, A, \tan \beta) \propto \left[ \frac{E}{R^2_\mu} \right] \frac{y}{y_{\text{low}}} \frac{t^2 - 1}{t(1 + t^2)} \frac{B_{\text{low}}}{\mu Z} p(m, M, A, B, \mu = \mu_Z)
\]

model-independent part!
It contains the fine-tuning penalization
It penalizes large \( \tan \beta \)

still undefined
The ElectroWeak Scale

\[ \exp_M^Z \] brings SUSY to the LHC region

- We may vary \[ M_{\text{soft}} \] up to \[ M_X \] the results do not depend on the range chosen
- This suggests that large soft-masses are disfavoured
Events with more or equal to 2 jets  [Baer et al 0907.1922]

(LHC contours at 14 TeV C.M.)
Adding all data
Comparison with Likelihood based inference

Dark Matter Direct Detection

\[ R = \int_{E_d}^{\infty} dE_R \frac{d\omega}{m_N m_\chi} \int_{v_{\text{min}}}^{\infty} v f(v) \frac{d\sigma_{W_N}}{dE_R}(v, E_R) dv \]

Astrophysics Input: local WIMP density and vel. distrib.

Particle Physics Input: cross sections and form factors

Detector dependence: Energy threshold

Neutralino-nuclei elastic scattering can occur through Higgs and squark exchange diagrams:

\[ \sigma_{XN} \sim \frac{g_1^2 g_2^2 |N_{11}|^2 |N_{13}|^2 m_N^4}{4\pi m_W^2 \cos^2 \beta m_H^4} \]

\[ \sigma_{XN} \sim \frac{g_1^2 g_2^2 |N_{11}|^2 |N_{13}|^2 m_N^4}{4\pi m_W^2 \cos^2 \beta m_{\tilde{q}}^4} \]
**Xenon100 + LHC Data**

- LHC with 1 fb^-1 and 7 TeV has a very limited impact on the posterior
- Direct Detection data disfavor the Focus Point $\Rightarrow$ The prior dependency is reduced significantly
LHC will solve the prior dependency

- Projected constraints from ATLAS (dilepton and lepton+jets edges 1 fb^-1 luminosity)

\[ \theta = \{ m_{\chi_1^0}, m_{\chi_2^0} - m_{\chi_1^0}, \tilde{m}_\tau - m_{\chi_1^0}, \tilde{m}_q - m_{\chi_1^0} \} \]

Flat prior

Log prior
Residual dependency on the statistics

- Marginal posterior and profile likelihood will remain somewhat discrepant using ATLAS alone. Much better agreement from ATLAS+Planck CDM determination.
DM identification with DD + LHC data

- LHC might be able to determine part of the SUSY spectrum... but not all
- Some masses can be accurately determined through the study of kinematical endpoints
- Even the mass difference between the LSP and NLSP could be accurately reconstructed
- However... the neutralino composition might not be determined!
- Using hypothetical LHC data, we have tested the reconstruction of the relic abundance

Procedure:

1. Choose the LCC3 in the CMSSM
2. Use determination of masses as experimental constraints
3. Perform a scan in a general MSSM (with 24 parameters) in order to find the regions with the best fit to the “data”
4. Determine the posterior distribution of the predicted relic abundance
The **LHC alone misses** the determination of some properties of SUSY particles.

This is reflected in the uncertainty in the determination of the relic density.

Fitting the LHC data can lead to several maxima.

This uncertainty leads to an indetermination of the scattering Xs.
LHC + DM DD data

- Assuming the **Scaling Ansatz**: that the local density scales with the cosmological abundance
  \[
  \frac{\rho_{\chi_1}^0}{\rho_{DM}} = \frac{\Omega_{\chi_1}^0}{\Omega_{DM}}
  \]

- **Scaling Ansatz** \(\Rightarrow\) breaks degeneracies in parameter space

![Probability density histogram](image1)

![Contour plot](image2)
Nightmare Scenario

- No single probe can cover the whole favoured parameter space, not even the LHC.
- Astroparticle probes (direct and indirect detection) can increase the coverage of the favoured parameter space, and deliver increased statistical robustness.
- High complementarity of LHC reach with direct detection methods.
Conclusions

- SUSY phenomenology provides a timely and challenging problem for parameter inference and model selection. A considerably harder problem than cosmological parameter extraction!

- Bayesian advantages: higher efficiency, inclusion of nuisance parameters, predictions for derived quantities, model comparison

- CMSSM only a case study: Bayesian analysis naturally penalizes fine-tunings. The exp. value of MZ brings the relevant parameter space to the low-energy region (~ accessible to LHC).

- CMSSM starts to be accessible to DM Direct Detection experiments

- LHC will prove new physics at the TeV scale but can not determine on its own the detection of DM; It needs extra info from DM Direct Detection experiments (COMPLEMENTARY)

- Currently, even the CMSSM is somewhat underconstrained: ATLAS+Planck will take us to “statistics nirvana”
Thank you !!!