Understanding and improving the Effective Mass for LHC searches

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[arXiv:1207.0435]
Outline

1. Introduction
2. $M_{\text{eff}}$
3. Kinematic Analysis
4. The correlation $M_{\text{susy}} - M_{\text{eff}}$
5. Improving $M_{\text{eff}}$
6. Conclusions
Introduction

- The LHC is working with impressive performance.
- Is there any signal of New Physics? In a positive case, Which New Physics is it?
- Supersymmetry $\rightarrow$ multijets and large missing energy.
- Kinematic variables.
- In ATLAS analyses the Effective Mass variable ($M_{\text{eff}}$) plays a very important role. [arXiv:0901.0512, arXiv:1102.5290, arXiv:1109.6572]
The Effective Mass

Defined as,

\[ M^{(4)}_{\text{eff}} = \sum_{j=1}^{4} |p_T^j| + |p_T^{\text{miss}}|, \]

where \( j \) denote the four hardest jets.

There is a correlation between \( M^{(4)}_{\text{eff}} |_{\text{max}} \) and the SUSY mass \( M_{\text{susy}} \) defined as,

\[ M_{\text{susy}} = \min\{m_{\tilde{q}}, m_{\tilde{g}}\}. \]
Systematically $M_{\text{eff}}^{(4)}|_{\text{max}} \sim 1.9 M_{\text{susy}}^{(\text{HPSSY})}$ for a large collection of CMSSM models.

- The definition of $M_{\text{susy}}$ is rather arbitrary.

- So far, there is no theoretical explanation for the strong correlation found between the effective mass and the supersymmetric mass.

- For more generic MSSM models (beyond the CMSSM) such correlation fails in many instances. [Tovey, hep-ph/0006276]

Let’s try to understand the behavior of the Effective Mass.
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Kinematic Analysis: $W$ boson production

The simplest case involving visible and invisible particles

Transverse Energy

$$(E_T^a)^2 = m_a^2 + (p_T^a)^2 = (E^a)^2 - (p_z^a)^2$$

Rapidity

$$y^a = \frac{1}{2} \ln \left( \frac{E_a^a + p_z^a}{E_a^a - p_z^a} \right) .$$

Invariant Mass:

$$M_{inv}^2 = m_e^2 + m_{\nu}^2 + 2 \left[ E_T^e E_T^\nu \cosh(y^e - y^\nu) - \vec{p}_T^e \cdot \vec{p}_T^\nu \right].$$
Kinematic Analysis: $W$ boson production

The differential cross: \[ d\sigma \propto d^2\Omega \]

\[ d^2\Omega = d\cos\theta d\phi = \frac{d\cos\theta}{dp_e^T} dp_T^e d\phi, \]

where $\theta, \phi$ are the polar and azimuthal angles.

The Jacobian factor of the production of an $e-\nu$ pair at CM frame:

\[
\frac{d\cos\theta}{dp_T^e} = \frac{-p_T^e |p_e^e|}{\sqrt{|p_e^e|^2 - (p_T^e)^2}} \rightarrow \frac{-M_w p_T^e}{\sqrt{M_w^2 - 4(p_T^e)^2}}
\]

A pole in $p_T^e = \frac{1}{2} M_w$
Kinematic Analysis: Production of SUSY particles

Considering two pseudo-particles $J$ and $X$

Visible $\rightarrow$ $(E^J_T)^2 = (p^J_T)^2 + m^2_J = (E^J)^2 - (p^J_z)^2$

Invisible $\rightarrow$ $(E^X_T)^2 = (p^X_T)^2 + m^2_X = (E^X)^2 - (p^X_z)^2$
Kinematic Analysis: Pair production of SUSY

In analogy with the \textbf{W boson} case

\[
\frac{d\sigma}{dp_T^J} \propto \frac{1}{\sqrt{E^2 - (E_T^J + E_T^X)^2}}
\]

with \(E\) denoting the total energy in the CM. We define a global “transverse energy” as

\[
\mathcal{E}_T = E_T^J + E_T^X.
\]

The differential cross section \((d\sigma/d\mathcal{E})\) shows a pole at

\[
\mathcal{E}_T|_{\text{pole}} = E = M_{\text{inv}} \simeq m_1 + m_2.
\]
Kinematic Analysis: Pair production of SUSY

**At CM frame**

A histogram in the $\mathcal{E}_T$ must show a peak at $\mathcal{E}_T = M_{\text{inv}}$

The two histograms are pretty similar, exhibiting a peak at the expected value.
Introduction

\(M_{\text{eff}}\)

Kinematic Analysis

The correlation \(M_{\text{susy}} - M_{\text{eff}}\)

Improving \(M_{\text{eff}}\)

Conclusions
Understanding the correlation $M_{\text{susy}} - M_{\text{eff}}$

A more convenient definition of $M_{\text{susy}}$

$$M_{\text{susy}} = \frac{\sum_{a,b} \sigma_{ab} (m_a + m_b)}{\sum_{a,b} \sigma_{ab}},$$

- Weighting the masses with their cross section, [Tovey, hep-ph/0006276].
- Works fine when the dominant productions have similar masses.
Understanding the correlation $M_{\text{susy}} - M_{\text{eff}}$

Remember that definitions

$$M_{\text{eff}} = M_{\text{eff}}^J + |p_T^{\text{miss}}|, \quad \mathcal{E}_T = E_T^J + E_T^X$$

where $M_{\text{eff}}^J \equiv \sum_j |p_T^j|$.

**A connection between $\mathcal{E}_T$ and $M_{\text{eff}}$**

- Massless jets
- All jets with the same rapidity
- Massless LSP
- Both LSP's in same direction

$$E_T^J = M_{\text{eff}}^J$$

$$E_T^X = p_T^{\text{miss}}$$
Understanding the correlation $M_{\text{susy}} - M_{\text{eff}}$

In the peak of the histogram satisfy $E_T^J = E^J = \sum_j E^j \simeq \sum_j |\vec{p}^j|$ then,

$$\frac{M_{\text{eff}}^J}{E_T^J} \simeq \frac{\sum_j |p_T^j|}{\sum_j |\vec{p}^j|}.$$

Then the differential probability that a jet occurs at a particular $\theta$ is $\sin \theta d\theta$. In average

$$\langle p_T^j \rangle = |\vec{p}^j| \int d\theta \sin^2 \theta = (\pi/4)|\vec{p}^j|$$

at the peak of the histogram

$$\frac{\langle M_{\text{eff}}^J \rangle}{E_T^J} \simeq \frac{\pi}{4} = 78.5\%.$$
Proposal of new kinematic variable

But $E_T^X$ is not measurable.

It turns out that if $m_X$ are fairly smaller than their momenta then

$$\langle E_T^X \rangle \simeq 2 |p_T^{\text{miss}}|.$$ 

We propose a kinematic variable,

$$E_T^{\text{eff}} = E_T^J + 2 |p_T^{\text{miss}}|,$$

The extension of this variables to other scenarios of new physics is straightforward.
The simulation

We will simulate LHC signals at 14 TeV center-of-mass energy using SOFTSUSY and PYTHIA version 6.419

1. At least three jets with $p_T > 50$ GeV.

2. The hardest jet with $p_T > 100$ GeV and $|\eta| < 1.7$.

3. $p_T^{\text{miss}} > 100$ GeV.

4. $\Delta \phi (\text{jet}_1 - p_T^{\text{miss}}) > 0.2$, $\Delta \phi (\text{jet}_2 - p_T^{\text{miss}}) > 0.2$, $\Delta \phi (\text{jet}_3 - p_T^{\text{miss}}) > 0.2$.

For the construction of the jets we will use FASTJET.
Supersymmetry

CMSSM

\[ \{\theta_i\} = \{m, M, A, B, \mu\} . \]

- \( m, M, A \) and \( B \): soft parameters
- \( \mu \): Higgs mass term in the superpotential,

\[ \Rightarrow \text{Two Higgs doublets } H_u, H_d \]

\[ v^2 = 2(v_u^2 + v_d^2) \]

where \( \tan \beta \equiv v_u / v_d \)

\[ m_u \sim y_u v_u = y_u v \sin \beta \]
\[ m_d \sim y_d v_d = y_d v \cos \beta \]
Supersymmetry: The CMSSM

Benchmark point SU9

\[ m = 300 \text{ GeV}, \quad M_{1/2} = 425 \text{ GeV}, \quad A = 20 \text{ GeV}, \quad \tan \beta = 20, \quad \mu > 0 \]

\[ M_{\text{susy}} = 1898 \text{ GeV} \]
Supersymmetry: The CMSSM

How do $\mathcal{E}_T^{\text{eff}}$ behaves?
Testing $\mathcal{E}_{T}^{\text{eff}}$: The CMSSM

A remarkable correlation between $\mathcal{E}_{T}^{\text{eff}}$ at the maximum of the histogram and $M_{\text{susy}}$. !
Supersymmetry

MSSM

Allowing non-universal soft parameters at the $M_X$ scale

$M_1, M_2, M_3$: gaugino masses

$A_t, A_b, A_\tau$: trilinear couplings

$m_{\tilde{q}_L, \tilde{q}_R}, m_{\tilde{t}_L, \tilde{b}_L}, m_{\tilde{t}_R}, m_{\tilde{b}_R}$: squark masses,

$m_{\tilde{l}_L, \tilde{l}_R}, m_{\tilde{\tau}_L, \tilde{\nu}_L}, m_{\tilde{\tau}_R}$: slepton masses,

$m_{H_u, H_d}$: Higgs masses

$\tan \beta$. 
Neglecting the neutralino mass one is missing a potential important piece.
Testing $E_T^{\text{eff}}$ 

One need a better estimation of $E_T^X$,

$$\langle E_T^X \rangle \simeq \sqrt{4(p_T^X)^2 + 4m_X^2} \simeq 2p_T^X + 2m_X^2/E_T^X$$

A conservative correction

$$2m_X^2/E_T^X \sim 4m_X^2/M_{\text{susy}}$$

then, the maximum of the histogram must be around

$$M_{\text{susy}} \rightarrow M_{\text{susy}} - 4 \frac{m_X^2}{M_{\text{susy}}}$$
Testing $\mathcal{E}_T^{\text{eff}}$: The MSSM

The correlation is there for a more general MSSM models !!
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Conclusions

- We have found an explanation for the correlation

\[ M_{\text{eff}} \mid_{\text{max}} \simeq 80\% \, M_{\text{susy}}, \]

- This understanding has allowed us to propose a new kinematic variable

\[ E_{T}^{\text{eff}} = E_{T}^{J} + 2|p_{T}^{\text{miss}}|. \]

- We have examine the correlation of \( E_{T}^{\text{eff}} \) in the CMSSM and a more general MSSM showing that it is a simple kinematic variable to determine the characteristic SUSY masses.

- \( E_{T}^{\text{eff}} \) and \( M_{\text{eff}} \) are complementary variables, rather than competitors.