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PHY585 Exercises
Sheet 1

Exercise 1 *Yukawa and Coulomb potential*

a) The Yukawa potential

$$V(r) = V_0 \frac{e^{-\mu r}}{\mu r}$$

describes the nucleon-nucleon interactions in the atomic nucleus. Its typical reach is $1/\mu$. Derive the scattering amplitude in the Born approximation and show that the scattering cross section is

$$\frac{d\sigma}{d\omega} = \left(\frac{2mV_0}{\mu 4\pi^2 h}\right)^2 \frac{1}{[2k^2(1-\cos\Theta)+\mu^2]^2}$$

b) Derive the scattering cross section for the Coulomb potential by considering the limit $\mu \to 0$ with

$$\frac{V_0}{\mu} = \frac{Ze^2}{4\pi\epsilon_0}$$

fixed.

Exercise 2 *Born Approximation*

a) Consider a spherically symmetric potential $V(\vec{r}) = g\delta(\vec{r})$, with g constant, and show that the Born approximation for the scattering amplitude is given by

$$f(\theta) = -\frac{m}{2\pi\hbar}g$$

b) Derive an expression for g if this potential is used to describe the scattering of thermal neutrons with scattering length b.

Exercise 3 *Time evolution operator*

The time evolution operator $U(t, t_0)$ fulfils the differential equation:

$$i\partial_t\, U(t,t_0) = H_I(t)\, U(t,t_0)\ .$$

Show that

a) $U(t,t_0)$ is unitary.

b) $U(t,t_0)$ can be expressed as Neumann series (use equivalent integral equation):

$$\begin{aligned} U(t,t_0) &= \mathbf{1} + (-i)\int_{t_0}^{t} dt_1 H_I(t_1) \\ &+(-i)^2 \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\, H_I(t_1)H_I(t_2) \\ &+\dots \\ &+(-i)^n \int_{t_0}^{t} dt_1 \dots \int_{t_0}^{t_{n-1}} dt_n\, H_I(t_1)H_I(t_2)\dots H_I(t_n) \\ &+\dots \end{aligned}$$

c) $U(t,t_0)$ is given by the time-ordered product (insert into differential equation):

$$U(t,t_0) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{t_0}^{t} dt_1 \dots \int_{t_0}^{t} dt_n\, T\left(H_I(t_1)\dots H_I(t_n)\right) \ .$$

Exercise 4 *Width of the diffraction maximum*

We assume that in a linear crystal on every lattice point $\vec{\rho} = m\vec{a}$, $m \in \mathbb{Z}$, there is an identical point-like scattering centre. The total amplitude of the scattered radiation is proportional to $F = \sum \exp\left(-im\vec{a}\cdot\Delta\vec{k}\right)$. The sum over M lattice points has the value

$$F = \frac{1-\exp\left(-iM\vec{a}\cdot\Delta\vec{k}\right)}{1-\exp\left(-i\vec{a}\cdot\Delta\vec{k}\right)} \tag{1}$$

when we use the series expansion

$$\sum_{m=0}^{M-1} x^m = \frac{1-x^M}{1-x}. \tag{2}$$

a) The scattered intensity is proportional to $|F|^2$. Show that

$$|F|^2 \equiv F^*F = \frac{\sin^2\left(\frac{1}{2}M\vec{a}\cdot\Delta\vec{k}\right)}{\sin^2\left(\frac{1}{2}\vec{a}\cdot\Delta\vec{k}\right)}. \tag{3}$$

b) For $\vec{a}\cdot\Delta\vec{k} = 2\pi h$, $h\in\mathbb{Z}$, a diffraction maximum appears. We change $\Delta\vec{k}$ slightly and define ε in $\vec{a}\cdot\Delta\vec{k} = 2\pi h + \varepsilon$ such that ε gives the first zero-crossing of the function $\sin\left(\frac{1}{2}M\vec{a}\cdot\Delta\vec{k}\right)$. Show that $\varepsilon = 2\pi/M$. What does this mean for the width of the diffraction maximum?