KTII - Exercise 2

Universität Zürich

Due: 6 March 2020

For particle information, including hadronic quark content, particle masses, particle lifetimes, or other physical constants not given in the problem, please consult the Particle Data Group's Review of Particle Physics. It is available for free on their website: http://pdg.lbl.gov/.

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1. **(5 points)**

   Consider the Lagrangian:

   $$\mathcal{L} \;=\; -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \;+\; \overline{\Psi}(i\not{D} - m)\Psi.$$

   Where $F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$ and the covariant derivative is:

   $$D_\mu = \partial_\mu - ieA_\mu$$

   .

   This Lagrangian is invariant under a gauge transformation defined by:

   $$\Psi(x) \to \Psi'(x) = e^{i\alpha(x)}\Psi(x)$$

   ,

   $$A_\mu \to A'_\mu = A_\mu + \frac{1}{e}\partial_\mu \alpha(x)$$

   .

   (a) Show that D_μ transforms as:

   $$D_\mu \Psi \to e^{i\alpha(x)} D_\mu \Psi$$

   (b) Show that the term $\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ is invariant under the gauge transformation.

   (c) Show that a mass term of the form $\frac{1}{2}m_\gamma^2 A^\mu A_\mu$ is not invariant under the gauge transformation.

2. **(3 points)**

   (a) Write the ratio of the $U(1)_Y$ and $SU(2)_L$ coupling constants (*i.e.* $\frac{g'}{g_W}$) in terms of the masses of the W and Z bosons.

(b) Look up the masses of the bosons and use them to give a numerical value of the ratio.

(c) Imagine a fermion, f, with a mass equal to the W boson mass, $m_f = m_W$. What would the strength of it's Yukawa coupling (g_f) be? What is the ratio of it's Yukawa coupling to the hWW trilinear coupling, g_{HWW}? *i.e.* find $\frac{g_f}{g_{HWW}}$.

3. **(2 points)**

(a) Draw the primary Feynman diagram contributing to the decay $h \to \gamma\gamma$.

(b) Explain briefly (1-2 sentences) why the $h \to \gamma\gamma$ decay was important for the higgs discovery despite the fact that its branching ratio is much smaller than other decay modes, such as $h \to b\bar{b}$.