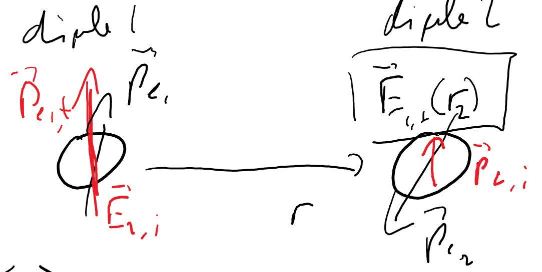
Van der Waals interaction



snapshot @ time t

$\vec{p}_e = q\vec{d}$

↳ el. dipole moment

$\langle q \rangle = 0$

$\langle \vec{p}_e \rangle = 0$

$\langle \vec{p}_e^{\,2} \rangle \neq 0$ fluctuating dipole moment

$\vec{p}_{e_1,t} \rightarrow \vec{E}_{1,t} \sim \frac{\vec{p}_{1,t}}{r^3}$

field @ position of dipole 2 $\vec{E}_1(r_2) = A\frac{\vec{p}_1}{r_1^3}$

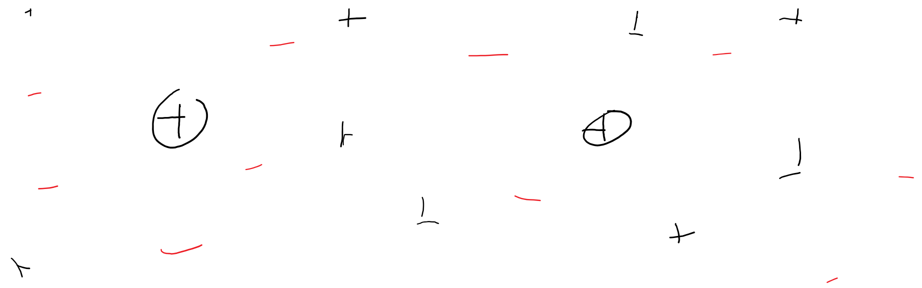
↳ induced dipole $\vec{p}_{1,i} = \alpha \vec{E}_1(r_2) \rightarrow \vec{E}_{2,ind} \sim \frac{\vec{p}_{1,i}}{r^3}$

(α: polarizability)

↳ $\vec{E}_{1,ind}(r_1) = A\frac{\vec{p}_{1,i}}{r_2^3} = A\frac{\alpha \vec{E}_1(r_1)}{r_2^3} = A^2\alpha\frac{\vec{p}_1}{r_2^6}$

↳ $E_{pot} = -\vec{p}_1 \cdot \vec{E}_{1,ind} = -A^2\alpha\frac{\langle \vec{p}_1^{\,2} \rangle}{r_1^6}$

Screened Coulomb interaction



$$n_\pm = n_0 e^{\pm \frac{qV}{kT}} \qquad \rho = e n_+ - e n_-$$

$$\vec{\nabla}^2 V = -\rho/\varepsilon_0 \qquad \text{Poisson-Eq.} \qquad \vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$$

$$\vec{E} \to -\vec{\nabla} V$$

$$\vec{\nabla}^2 V = -\frac{e}{\varepsilon_0}(n_+ - n_-) = -\frac{n_0 e}{\varepsilon_0}\left(e^{+\frac{eV}{kT}} - e^{-\frac{eV}{kT}}\right)$$

$eV \ll kT \to$ expand exponential

$$1 + \frac{eV}{kT} - \left(1 - \frac{eV}{kT}\right) = \frac{2eV}{kT}$$

$$\boxed{\vec{\nabla}^2 V = -\frac{2e^2 n_0}{\varepsilon_0 kT} V}$$

$$\frac{2e^2 n_0}{\varepsilon_0 kT} \to 1/\lambda^2 \to \boxed{\lambda = \sqrt{\frac{\varepsilon_0 kT}{2e^2 n_0}}}$$