Exercise 1. ***Hydrogen Atom in an Electric field***

Consider a hydrogen atom in its ground state. A uniform electric field aligned along the positive z direction is turned on at $t = 0$.

Compute the probability to find the atom in each of the following states at $t = t_f > 0$.

(a) $n = 1$, $l = 0$

(b) $n = 2$, $l = 1$, $m_l = -1$

(c) $n = 2$, $l = 1$, $m_l = 0$

(d) $n = 2$, $l = 1$, $m_l = +1$.

You can use the following expressions for the wave functions Ψ_{nlm}:

$$\Psi_{100} = 2\left(\frac{1}{a}\right)^{3/2} e^{-r/a} Y_0^0(\theta,\phi) = \frac{1}{\sqrt{\pi}}\left(\frac{1}{a}\right)^{3/2} \mathrm{e}^{-r/a}$$

$$\Psi_{21-1} = \frac{1}{2\sqrt{6}}\left(\frac{1}{a}\right)^{3/2}\left(\frac{r}{a}\right) e^{-r/(2a)} Y_1^{-1}(\theta,\phi) = \frac{1}{8\sqrt{\pi}}\left(\frac{1}{a}\right)^{3/2}\left(\frac{r}{a}\right)\mathrm{e}^{-r/2a}\sin\theta\mathrm{e}^{-i\phi}$$

$$\Psi_{210} = \frac{1}{2\sqrt{6}}\left(\frac{1}{a}\right)^{3/2}\left(\frac{r}{a}\right) e^{-r/(2a)} Y_1^{0}(\theta,\phi) = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a}\right)^{3/2}\left(\frac{r}{a}\right)\mathrm{e}^{-r/2a}\cos\theta$$

Exercise 2. ***The Driven Harmonic Oscillator***

At $t = 0$ a 1-dimensional harmonic oscillator with natural frequency ω is driven by the perturbation.

$$H_1(t) = -Fxe^{-i\Omega t} \tag{1}$$

The oscillator is initially in its ground state at $t = 0$.

(a) Using the lowest order perturbation theory to get a nonzero result, find the probability that the oscillator will be in the 2nd excited state $n = 2$ at time $t > 0$. Assume $\omega \neq \Omega$.

(b) Now begin again and do the simpler case, $\omega = \Omega$. Again, find the probability that the oscillator will be in the 2nd excited state $n = 2$ at time $t > 0$.

(c) Expand the result of part (a) for small times t, compare with the results of part (b), and interpret what you find.
In discussing the results see if you detect any parallels with the driven classical oscillator.