University of Zurich[UZH]	**Solid State Physics**	FS16
	Exercise Sheet 7	Prof. Dr. Johan Chang

Discussion on 13^{th} April | Due on 20^{th} April

Exercise 1 *Debye temperature*

The Debye approximation is assuming a phonon dispersion with $\omega = vk$ where v is the sound velocity.

a) Calculate in two dimensions the density of state $D(\omega)$, the frequency cut-off ω_D and the Debye temperature T_D.

b) The sound velocity is given by $v = \sqrt{B/\rho}$ where B is the bulk modulus and ρ is the mass density. In a previous lecture we saw how argon is bonded together through van-der-Waals interaction and take an fcc structure with nearest neighbour distance 3.76 Å. We derived $B \approx 75\epsilon/\sigma^3$ where (for Argon) $\epsilon \approx 10$ meV and $\sigma = 4/1.09 \approx 4$ Åare the parameters of the Lennard-Jones potential. Show, using rough back-on-the-envelope approximations, that $v \approx 1150\,\text{m/s}$.

c) With this sound velocity, evaluate the Debye temperature for this material. (Remember that opposed to (a) we are now in 3D.)

Exercise 2 *Heat capacity - Debye approximation*

Let's again use the Debye approximation.

a) Calculate in one and two dimensions the temperature dependence of the heat capacity for the limit where T is much smaller than the Debye temperature.

b) The sound velocity of diamond is $12\,000\,\text{m/s}$ whereas lead has $2000\,\text{m/s}$. For a finite fixed temperature in the limit $T \ll T_D$, which of the two materials will have the highest heat capacity? From our knowledge of the phonon dispersion of a mono-atomic 1D chain, give arguments as to why diamond has a higher sound velocity.

Exercise 3 *Heat capacity - Einstein model*

Einstein derived

$$C = 3Nk_B \left(\frac{\hbar\omega_0}{k_BT}\right)^2 \frac{\exp\left(\frac{\hbar\omega_0}{k_BT}\right)}{\left[\exp\left(\frac{\hbar\omega_0}{k_BT}\right) - 1\right]^2}. \tag{1}$$

a) In figure 1, the heat capacity of diamond is plotted as $\frac{C_p}{3Nk_B}$ versus temperature. Einstein's model has just one free parameter: ω_0. Based on these data (given below), determine the energy scale $\hbar\omega_0$ that would give the best description of the experiment.

b) How does this energy scale compare with the energy scale of optical phonons in diamond (see figure 2)?

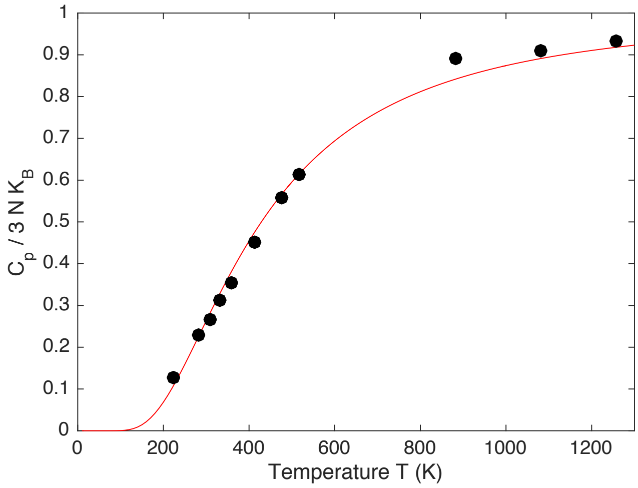

Figure 1: Heat capacity of diamond versus temperature T as of 1906. This was thus the information that Einstein had available when thinking about the problem of heat capacity.

The data points in figure 1 are:
T [K] = 222, 284, 308.5, 333, 358, 411, 477, 518, 880, 1080, 1259
$\frac{C_p}{3Nk_B}$ = 0.1282, 0.2308, 0.2650, 0.3120 , 0.3547, 0.4530, 0.5556, 0.6154, 0.8932, 0.9103, 0.9316

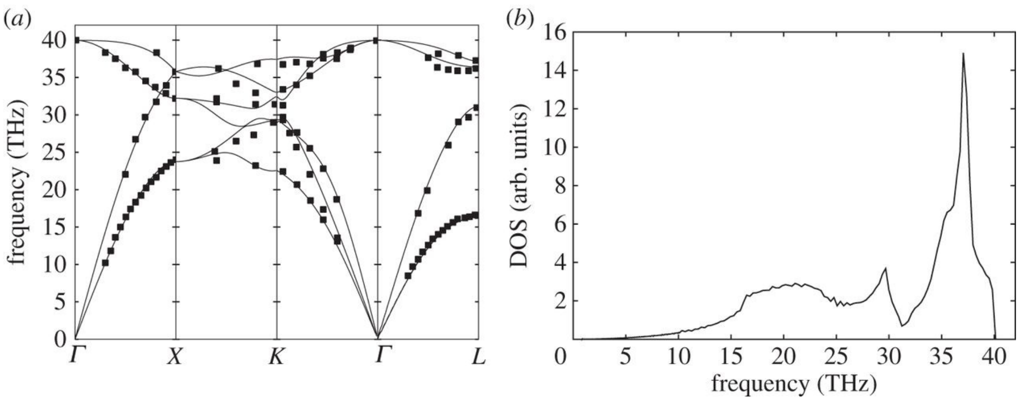

Figure 2: (a) Phonon spectrum of diamond. (b) Phonon density of states of diamond. From: http://rspa.royalsocietypublishing.org/content/470/2169/20140371