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Spurious gauge invariance and γ_5 in Dimensional Regularization

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[PO, Luca Vecchi, arXiv:2406.17013]

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Outline

1. Motivation
2. Breaking of gauge invariance
3. ``Standard” calculation of counterterms
4. Unbreaking gauge invariance
5. Mapping NDR - BMHV
6. Conclusions

Motivation

NDR scheme

- In Dimensional Regularization (DR), we regularize the action extending it to $d = 4 - 2\epsilon$ dimensions.

- Dirac algebra can be extended to d dimensions keeping the usual anticommutation relations for γ_5 (NDR):

$$\{\gamma_\mu, \gamma_\nu\} = 2\,\eta_{\mu\nu} \qquad\qquad \{\gamma_\mu, \gamma_5\} = 0$$

Motivation

NDR scheme

- It is not possible to consistently define γ_5 maintaining these 4-dimensional properties:

  (i) $\{\gamma_\mu, \gamma_5\} = 0$

  (ii) $\mathrm{Tr}[\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma_5] = 4\mathrm{i}\epsilon_{\mu\nu\rho\sigma}$

  (iii) Cyclicity of the trace.

Motivation

NDR scheme

- Traces with γ_5 and six or more gammas are, in fact, ambiguous.

$$\mathrm{Tr}[\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\ldots] = \mathrm{Tr}[\gamma_\mu\gamma_\nu\gamma_\rho\ldots\gamma_5] + \mathcal{O}(\epsilon)$$

- Matching weakly-coupled extensions into SMEFT dimension six **is not** ambiguous.
  - Ambiguity is proportional to $\epsilon_{\mu\nu\rho\sigma} \to \phi^\dagger\phi X_{\mu\nu}\tilde{X}^{\mu\nu}$ or $XX\tilde{X}$.
  - Imposing a real Wilson Coefficient fixes the ambiguity.

Motivation

NDR scheme

- There are some reading-point prescriptions to keep using NDR.

  [J. G. Körner, D. Kreimer, and K. Schilcher, Z. Phys. C54 (1992) 503–512]

  [D. Kreimer, arXiv:hep-ph/9401354]

- Recent modifications/extensions for shortcomings in multi-loop calculations.

  [L. Chen, arXiv:2304.13814]

  [L. Chen, arXiv:2409.08099]

- There is no proof that this works in general.

Motivation

BMHV scheme

- The Breitenlohner-Maison-t'Hooft-Veltman (BMHV) prescription is consistent at all others.
- γ_μ matrices are split into a 4- and $(d-4)$-dimensional part:

$$\gamma_\mu = \gamma_{\overline{\mu}} + \gamma_{\hat{\mu}}$$

- Consistency is obtained by requiring:

$$\{\gamma_{\overline{\mu}}, \gamma_5\} = 0 \qquad [\gamma_{\hat{\mu}}, \gamma_5] = 0$$

Motivation

Classical action in DR

- Consider the regularized fermionic action:

$$S_F^{(d)} = \int d^d x \, \bar{f} \, \mathrm{i}\gamma^\mu \partial_\mu f = \int d^d x \, (\overline{f_X} \, \mathrm{i}\gamma^{\bar{\mu}} \partial_{\bar{\mu}} f_X + \overline{f_X} \, \mathrm{i}\gamma^{\hat{\mu}} \partial_{\hat{\mu}} f_{\overline{X}})$$

with $f_X = P_X f, \quad f_{\overline{X}} = (1 - P_X) f \qquad (X = L, R)$

- In a chiral gauge theory, this constitutes a explicit breaking of gauge invariance.

$$\bar{f}_L \hat{\partial\!\!\!/} f_R \to \bar{f}_L U_L^\dagger U_R \hat{\partial\!\!\!/} f_R$$

Breaking of gauge invariance

Variation of the effective action

- Defining the gauge variation of a functional F as:

$$\delta_\alpha F = \int dx\, \alpha^a(x)\, L_a(x)\, F$$

- Gauge invariance is manifest in the "Ward identity" for F:

$$L_a(x)\, F = 0$$

- For instance, for the classical action:

$$\delta_\alpha S_F^{(d)} = \alpha_a\, \bar{f}\, T_A^a \gamma_5\, \hat{\partial\!\!\!/} f \qquad (T_A^a \equiv T_R^a - T_L^a)$$

Breaking of gauge invariance

Variation of the effective action

- Quantization of a gauge theory leaves the action invariant only under BRST transformations.

- The theory now obeys the Slavnov-Taylor identities, also broken by regularization even at the classical level.

- Various works in the literature restoring BRST symmetry.

  [H. Bélusca-Maïto, D. Stöckinger et al., arXiv:2303.09120] -> Review

  [H. Bélusca-Maïto, D. Stöckinger et al., arXiv:2004.14398] -> One-loop YM

  [D. Stöckinger and M. Weißwange, arXiv:2312.11291] -> Three loop abelian

  [D. Stöckinger, M. Weißwange et al., arXiv:2411.02543] -> Evanescent schemes

Breaking of gauge invariance

Variation of the effective action

- We can, however, use the background-field method:

$$\phi \to \phi + \tilde{\phi}$$

  to keep the action background-gauge invariant.

- The only source of gauge-symmetry breaking is then the gauge variation of the regularized action $S^{(d)}$.

Breaking of gauge invariance

Variation of the effective action

- The gauge variation of the 1PI effective action is:

  [C. Cornella, F. Feruglio, L. Vecchi, 2205.10381]

$$L_a \Gamma^{(d)}[\phi] = \frac{\int D\tilde{\phi}\, L_a S_F^{(d)}[\phi + \tilde{\phi}]\, e^{iS^{(d)}[\phi+\tilde{\phi}]}}{\int D\tilde{\phi}\, e^{iS^{(d)}[\phi+\tilde{\phi}]}}$$

- We can restore gauge invariance perturbatively by the addition of some local counterterms:

$$L_a \Gamma^{(n)} = - L_a S^{(n)}_{c.t.}$$

- Therefore, the goal is to compute $L_a \Gamma^{(1)}$ to identify $S^{(1)}_{c.t.}$.

Calculation of the counterterms

Structure of the gauge variation

- $L_a\,\Gamma$ is conformed by an unintegrated basis of local, independent operators (or, equivalently, an integrated basis of all the fields *and* $\alpha_a(x)$).

$$L_a\,\Gamma = \Sigma\,\xi_a^i I_i \qquad\qquad \delta_\alpha\,\Gamma = \int dx\,\alpha^a\,\Sigma\,\xi_a^i I_i$$

- $S_{c.t.}$ will contain a subset of the former operators because some of them can be reduced with integration by parts:

$$S_{c.t} = \Sigma\,\hat{\xi}^i \mathcal{O}_i$$

Calculation of the counterterms

Structure of the gauge variation

- We impose the condition:

$$L_a\, S_{c.t.} = -\, L_a\, \Gamma \qquad \rightarrow \qquad \Sigma\, (c_i(\hat{\xi}) + \xi_i)\, I_i = 0$$

- The necessary steps are:
  - Computing $L_a \Gamma$.
  - ``Formal integration''.

Calculation of the counterterms

Example

- Consider a generic gauge theory with a set of fermions f:

$$S[\phi + \tilde{\phi}] = S_{YM} + S_{g.f.} + S_{gh} + S_F$$

- A basis of monomials (containing fermions) for the gauge variation and the action:

$$I_1 = \alpha_a \, \bar{f}_{X,i} \overleftarrow{\partial\!\!\!/} f_{X,j}$$
$$I_2 = \alpha_a \, \bar{f}_{X,i} \overrightarrow{\partial\!\!\!/} f_{X,j}$$
$$I_3 = \alpha_a \, \bar{f}_{X,i} \gamma^\mu f_{X,j} \, A^c_\mu$$

$$\mathcal{O}_1 = \bar{f}_{X,i} \overrightarrow{\partial\!\!\!/} f_{X,j}$$
$$\mathcal{O}_2 = \bar{f}_{X,i} \gamma^\mu f_{X,j} \, A^a_\mu$$

Calculation of the counterterms

Example

- We need to compute all 1PI amplitudes with an insertion of the gauge variation of the fermionic action:

$$\delta_\alpha S_F^{(d)} = \alpha_a \bar{f}\, T_A^a \gamma_5 \hat{\partial\!\!\!/} f \qquad\qquad (T_A^a \equiv T_R^a - T_L^a)$$

- Non-trivial results will arise with divergent integrals, for which we can extract their hard region.

- We can compute it automatically using *Matchmakereft*.

Matchmakereft

Automated matching

- Matchmakereft is a fully automated tool to perform tree-level and one-loop matching between arbitrary models and arbitrary EFTs.

  [Carmona, Lazopoulos, PO, Santiago '21]

- It can also compute one-loop RGEs of arbitrary EFTs and check the off-shell independence of a set of operators.

- Since it is prepared to compute the hard region of one-loop actions, it can be used to compute these counterterms.

Matchmakereft

Automated matching

- Define UV and EFT theories:

```
ngi = 2*yalf[ff1,ff2] ( fbar[sp1,ff1].del[right[f[sp2,ff2]],mu] -
      fbar[sp1,ff1].del[left[f[sp2,ff2]],mu]) alf Ga[mu,sp1,sp2];
```

```
alphac12L[ff1,ff2] alf fbar[sp1,ff1].del[left[f[sp2,ff2]],mu] Ga[mu,sp1,sp2]
+ alphac12R[ff1,ff2] alf fbar[sp1,ff1].del[right[f[sp2,ff2]],mu] Ga[mu,sp1,sp2]
```

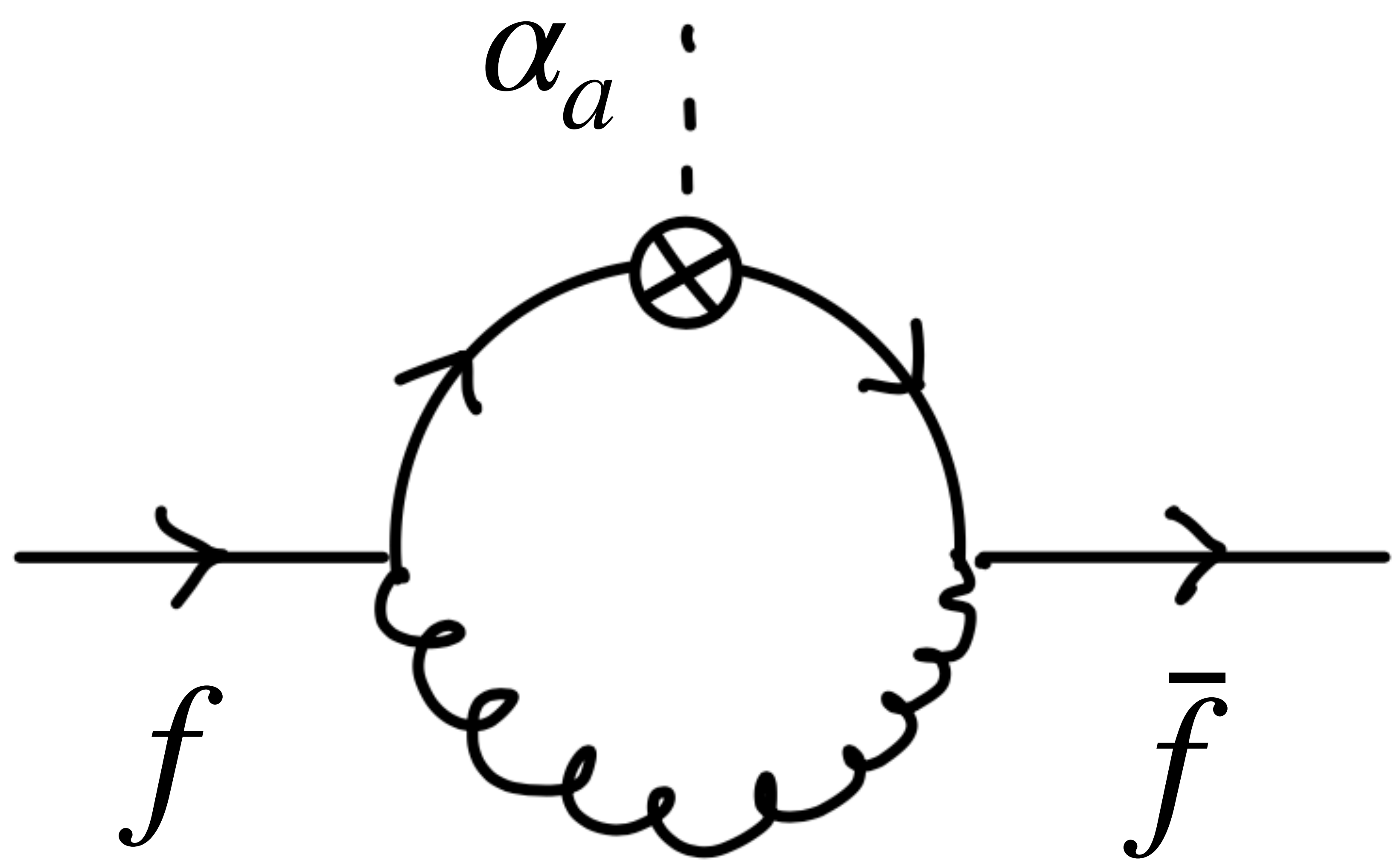
Calculation of the counterterms

Example

- We need to compute the following diagrams:

$$I_1 = \alpha_a \, \bar{f}_{X,i} \overleftarrow{\not\partial} f_{X,j}$$

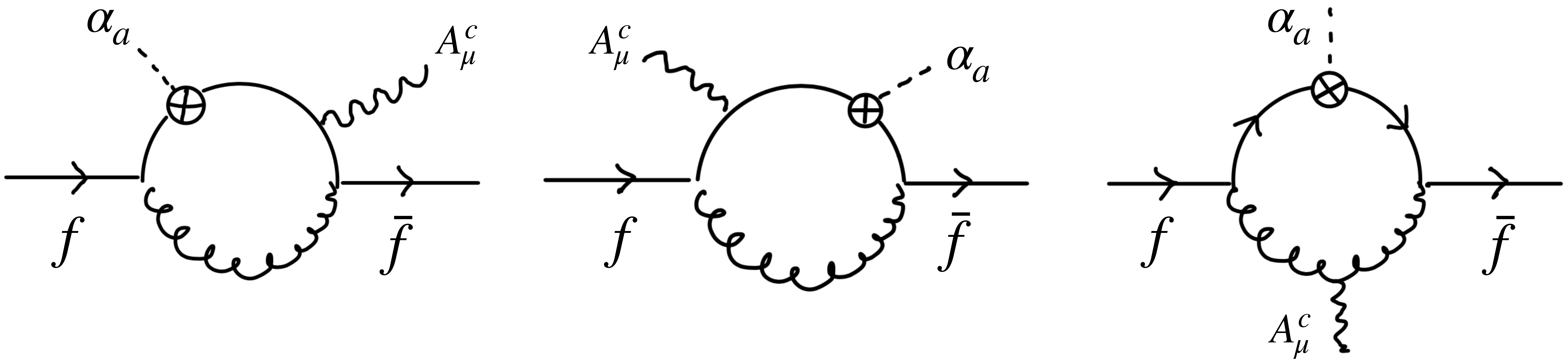
$$I_2 = \alpha_a \, \bar{f}_{X,i} \overrightarrow{\not\partial} f_{X,j}$$



Calculation of the counterterms

Example

- We need to compute the following diagrams:



$$I_3 = \alpha_a \, \bar{f}_{X,i} \gamma^\mu f_{X,j} \, A^c_\mu$$

Calculation of the counterterms

Example

- For instance:

$$I_2^{(L)} = \alpha_a \bar{f}_{L,i} \overrightarrow{\partial\!\!\!/} f_{L,j}$$

$$I_2^{(R)} = \alpha_a \bar{f}_{R,i} \overrightarrow{\partial\!\!\!/} f_{R,j}$$

```
In[30]:= alphaxi2L[1, 2] /. MatchingResult[[1, 2, 1]]
         alphaxi2R[1, 2] /. MatchingResult[[1, 2, 1]]
```

Out[30]= $-\frac{TA_{fl1,fl3}\, TL_{1,fl1}{}^{fl2}\, TL_{fl3,2}{}^{fl2}}{8\,\pi^2}$

Out[31]= $\frac{TA_{fl1,fl3}\, TR_{1,fl1}{}^{fl2}\, TR_{fl3,2}{}^{fl2}}{8\,\pi^2}$

Calculation of the counterterms

Example

- All counterterms with fermions and gauge bosons have been crosschecked with Matchmakereft.

[C. Cornella, F. Feruglio, L. Vecchi, 2205.10381]

I^0_a	$\Box\partial^\mu A_{a\mu}$
I^1_{ab}	$\epsilon^{\mu\nu\alpha\beta}(\partial_\alpha A_{a\mu})(\partial_\beta A_{b\nu})$
I^2_{ab}	$A_{a\mu}(\partial^\mu\partial^\nu - \Box g^{\mu\nu})A_{b\nu}$
I^3_{ab}	$A_{a\mu}\Box A^\mu_b$
I^4_{ab}	$(\partial_\nu A_{a\mu})(\partial^\nu A^\mu_b)$
I^5_{ab}	$(\partial_\nu A_{a\mu})(\partial^\mu A^\nu_b)$
I^6_{ab}	$(\partial^\mu A_{a\mu})(\partial^\nu A_{b\nu})$
I^7_{abd}	$(\partial_\mu A^\mu_a)A_{b\nu}A^\nu_d$

I^8_{abd}	$(\partial_\mu A^\nu_a)A_{b\mu}A^\nu_d$
I^9_{abd}	$\epsilon^{\mu\nu\alpha\beta}(\partial_\beta A_{a\mu})A_{b\nu}A_{d\alpha}$
I^{10}_{abde}	$A_{a\mu}A^\mu_b A_{d\nu}A^\nu_e$
I^{11}_{abde}	$\epsilon^{\mu\nu\rho\sigma}A_{a\mu}A_{b\nu}A_{d\rho}A_{e\sigma}$
I^{12}_{Xij}	$\bar{f}_{Xi}\overrightarrow{\not\partial} f_{Xj}$
I^{13}_{Xij}	$\bar{f}_{Xi}\overleftarrow{\not\partial} f_{Xj}$
I^{14}_{Xaij}	$\bar{f}_{Xi}\not{A}_a f_{Xj}$

Unbreaking gauge invariance

Spurion technique

- We can formally maintain gauge invariance introducing a spurious field:
  [PO, L. Vecchi, 2406.17013]

$$S_F^{(d)} = \int d^d x\, \bar{f}\, \mathrm{i}\gamma^\mu \partial_\mu f = \int d^d x\, \left[\, \overline{f_X}\, \mathrm{i}\gamma^{\bar{\mu}} \partial_{\bar{\mu}} f_X + \overline{f_L}\, \mathrm{i}\Omega\, \hat{\partial\!\!\!/} f_R + \overline{f_R}\, \mathrm{i}\Omega^\dagger\, \hat{\partial\!\!\!/} f_L \right]$$

as long as Ω transforms as $\Omega \to e^{i\alpha_a T_L^a}\, \Omega\, e^{-i\alpha_a T_R^a}$.

- We can take Ω to be a special unitary matrix $\Omega\Omega^\dagger = 1$.

Unbreaking gauge invariance

Spurion technique

- There is some freedom in the interactions with gauge bosons:

$$S_F^{(d)} = \int d^d x \left[\bar{f}\, \mathrm{i}\gamma^{\bar{\mu}} D_{\bar{\mu}} f + \overline{f_L}\, \mathrm{i}\Omega\, \hat{\partial\!\!\!/} f_R + \overline{f_R}\, \mathrm{i}\Omega^\dagger\, \hat{\partial\!\!\!/} f_L + c(\overline{f_L}\, \Omega\, \hat{A\!\!\!/}^A T_R^A f_R + \overline{f_R}\, \Omega^\dagger \hat{A\!\!\!/}^A T_L^A f_L) \right]$$

- For $c = 1$ the theory is invariant under the full D-dimensional spurious gauge symmetry.
- A recent work explores the different alternatives.

[D. Stöckinger, M. Weißwange et al., arXiv:2411.02543] -> Evanescent schemes

Unbreaking gauge invariance

Spurion technique

- The theory is also endowed with spurious versions of P and C:

$$A_\mu^A(x) \xrightarrow{\text{P}} \mathcal{P}_\mu^\nu A_\nu^A(\mathcal{P}x) \qquad A_\mu^A(x) \xrightarrow{\text{C}} -A_\mu^A(x)$$

$$\Psi_i(x) \xrightarrow{\text{P}} \gamma^0 \Psi_i(\mathcal{P}x) \qquad \Psi_i(x) \xrightarrow{\text{C}} C\overline{\Psi}_i^t(x)$$

$$\phi_a(x) \xrightarrow{\text{P}} \phi_a(\mathcal{P}x) \qquad \phi_a(x) \xrightarrow{\text{C}} \phi_a(x)$$

$$\Omega_{ij}(x) \xrightarrow{\text{P}} \Omega_{ji}^*(\mathcal{P}x) \qquad \Omega_{ij}(x) \xrightarrow{\text{C}} \Omega_{ij}^*(x)$$

$$T_{ij}^A \xrightarrow{\text{P}} \overline{T}_{ij}^A \qquad T_{ij}^A \xrightarrow{\text{C}} T_{ij}^{A*}$$

$$\overline{T}_{ij}^A \xrightarrow{P} T_{ij}^A \qquad \overline{T}_{ij}^A \xrightarrow{\text{C}} \overline{T}_{ij}^{A*}$$

$$[T_\phi^A]_{ab} \xrightarrow{\text{P}} [T_\phi^A]_{ab} \qquad [T_\phi^A]_{ab} \xrightarrow{\text{C}} [T_\phi^A]_{ab}^*$$

$$Y_{ij}^a \xrightarrow{\text{P}} Y_{ji}^{a*} \qquad Y_{ij}^a \xrightarrow{\text{C}} Y_{ij}^{a*}$$

Unbreaking gauge invariance

Spurion technique

- We define the effective action including a non-dynamical Ω:

$$e^{i\Gamma_{\text{inv}}[\phi,\Omega]} = \int_{1PI} D\tilde{\phi}\, e^{iS^{(d)}_{\text{reg}}[\phi+\tilde{\phi},\Omega]+iS^{\text{div}}_{\text{ct}}[\phi+\tilde{\phi},\Omega]+iS^{\text{fin}}_{\text{ct}}[\phi+\tilde{\phi},\Omega]}$$

- S_{ct} is defined so that chiral invariance is ensured in the limit $\Omega \to 1$:

$$\Gamma_{\text{inv}}[\phi] = \Gamma[\phi,1] + S_{\text{ct}}[\phi,1]$$

Unbreaking gauge invariance

Spurion technique

- In general, we can split Γ into:

$$\Gamma_{\not\Omega}[\phi] \qquad\qquad \Gamma_{\Omega}[\phi, \Omega]$$

- With:

$$\delta\Gamma_{\not\Omega}[\phi] = 0 \qquad\qquad \delta\Gamma_{\Omega}[\phi, \Omega] = 0$$

- But then:

$$\delta\Gamma[\phi, 1] = -\,\delta S_{\mathrm{ct}} \qquad \Rightarrow \qquad \boxed{\Gamma_{\Omega}[\phi, 1] = -\,S_{\mathrm{ct}}}$$

Unbreaking gauge invariance

Spurion technique

- It is convenient to work with $\Omega \equiv \Omega_0$. The fermionic propagator reads:

$$\frac{i}{p^2}\left[\gamma^{\bar{\mu}} p_{\bar{\mu}} + (\Omega_0 P_L + \Omega_0^\dagger P_R)\,\gamma^{\hat{\mu}} p_{\hat{\mu}}\right]$$

- Then we cannot recover terms with $\partial_\mu \Omega$.

- We write a basis of $S_{\text{ct}}[\phi, \Omega]$ and evaluate it at $\Omega = \Omega_0$ to fix the coefficients.

Unbreaking gauge invariance

Spurion technique

- Summing up, the steps are the following:
  - Compute the Ω_0-dependent part of $\Gamma_\Omega[\phi, \Omega_0]$.
  - Write a basis of counterterms $S_{\rm ct}[\phi, \Omega]$ with arbitrary coefficients and extract the Ω_0-dependent part.
  - Equate to fix the coefficients.

Unbreaking gauge invariance

Spurion technique

- Let us consider a general theory with a set of scalars φ_a:

$$\mathscr{L} \supset \frac{1}{2}(D_\mu \varphi^a)^2 + \left[Y^a_{ij}\, \varphi_a\, \bar{f}_{L,i}\, f_{R,j} + \mathrm{h.c.} \right]$$

- And define the following spurion:

$$\Phi_{ij} \equiv Y^a_{ij}\, \varphi_a \qquad\qquad \Phi \to e^{i\alpha_a T^a_L}\, \Phi\, e^{-i\alpha_a T^a_R}$$

Unbreaking gauge invariance

Results

$\boldsymbol{D^4}$	
$\langle L_{\bar\mu\bar\nu}\Omega R^{\bar\mu\bar\nu}\Omega^\dagger\rangle$	0
$i\langle L_{\bar\mu\bar\nu}D^{\bar\mu}\Omega D^{\bar\nu}\Omega^\dagger + R_{\bar\mu\bar\nu}D^{\bar\mu}\Omega^\dagger D^{\bar\nu}\Omega\rangle$	$-\frac{1}{2}$
$\langle D_{\bar\mu}\Omega D^{\bar\mu}\Omega^\dagger D_{\bar\nu}\Omega D^{\bar\nu}\Omega^\dagger + D_{\bar\mu}\Omega^\dagger D^{\bar\mu}\Omega D_{\bar\nu}\Omega^\dagger D^{\bar\nu}\Omega\rangle$	$-\frac{1}{6}$
$\langle D_{\bar\mu}\Omega D_{\bar\nu}\Omega^\dagger D^{\bar\mu}\Omega D^{\bar\nu}\Omega^\dagger\rangle$	$+\frac{1}{12}$
$\langle D_{\bar\mu}D^{\bar\mu}\Omega D_{\bar\nu}D^{\bar\nu}\Omega^\dagger\rangle$	0
$\langle D_{\bar\mu}D_{\bar\nu}\Omega D^{\bar\mu}D^{\bar\nu}\Omega^\dagger\rangle$	$+\frac{1}{6}$

$\boldsymbol{\phi^4}$		$\boldsymbol{\phi^2 D}$	
$\langle(\Phi\Omega^\dagger)^4\rangle + \text{h.c.}$	$-\frac{1}{12}$	$\langle\Phi D_{\bar\mu}\Omega^\dagger\Phi D^{\bar\mu}\Omega^\dagger\rangle + \text{h.c.}$	$+\frac{1}{3}$
$\langle(\Phi\Omega^\dagger)^2\Phi\Phi^\dagger\rangle + \text{h.c.}$	$-\frac{2}{3}$	$\langle(\Phi\Omega^\dagger)^2 D_{\bar\mu}\Omega D^{\bar\mu}\Omega^\dagger\rangle + \text{h.c.}$	$-\frac{1}{3}$
		$\langle\Phi\Phi^\dagger D_{\bar\mu}\Omega D^{\bar\mu}\Omega^\dagger + \Phi^\dagger\Phi D_{\bar\mu}\Omega^\dagger D^{\bar\mu}\Omega\rangle$	$-\frac{1}{3}$
		$\langle\Phi\Omega^\dagger D_{\bar\mu}\Omega\Phi^\dagger\Omega D^{\bar\mu}\Omega^\dagger\rangle$	$+\frac{1}{3}$
		$\langle D_{\bar\mu}\Phi D^{\bar\mu}\Omega^\dagger\Phi\Omega^\dagger + D_{\bar\mu}\Phi\Omega^\dagger\Phi D^{\bar\mu}\Omega^\dagger\rangle + \text{h.c.}$	$+\frac{2}{3}$
		$\langle D_{\bar\mu}\Phi\Omega^\dagger D^{\bar\mu}\Phi\Omega^\dagger\rangle + \text{h.c.}$	$+\frac{1}{6}$
		$\langle\Phi \overleftrightarrow{D}_{\bar\mu}\Phi^\dagger\Omega D^{\bar\mu}\Omega^\dagger + \Phi^\dagger \overleftrightarrow{D}_{\bar\mu}\Phi\Omega^\dagger D^{\bar\mu}\Omega\rangle$	$+\frac{1}{3}$

$\boldsymbol{\psi^2 D}$		$\boldsymbol{\psi^2\phi}$	
$\overline{\Psi}\gamma^{\bar\mu}T_L^A\Omega iD_{\bar\mu}\Omega^\dagger T_L^A P_L\Psi + \text{P.c.}$	1	$[\overline{\Psi}T_R^A\Omega^\dagger\Phi\Omega^\dagger T_L^A P_L\Psi + \text{P.c.}] + \text{h.c.}$	-2
$\overline{\Psi}\gamma^{\bar\mu}Y^a\Omega^\dagger iD_{\bar\mu}\Omega[Y^a]^\dagger P_L\Psi + \text{P.c.}$	$\frac{1}{2}$		

Unbreaking gauge invariance

Results

- We can also write a WZW term:

$$S_{\rm ct}^{\rm Fin}[\xi,\Omega]\big|_{\rm WZW} = \frac{n}{48\pi^2}\left\{\int {\rm d}^4x\ \epsilon^{\bar\mu\bar\nu\bar\alpha\bar\beta}\, Z_{\bar\mu\bar\nu\bar\alpha\bar\beta} + \dots\right\}$$

- Our result gives $n = 1$. This result is exact at all orders.

Unbreaking gauge invariance

SM embedding

- We can describe the particle content in a fermion and scalar multiplets:

$$\Psi = \begin{pmatrix} u \\ d \\ \nu \\ e \end{pmatrix}, \qquad H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_3 + i\phi_4 \\ \phi_1 + i\phi_2 \end{pmatrix}$$

Unbreaking gauge invariance

SM embedding

$$T_L^{\{1,2,3\}}\Big|_{\mathrm{SU}(2)} = \left\{ \frac{1}{2}\begin{pmatrix} & \mathbb{1} & & \\ \mathbb{1} & & & \\ & & & 1 \\ & & 1 & \end{pmatrix}, \frac{1}{2}\begin{pmatrix} & -i\mathbb{1} & & \\ +i\mathbb{1} & & & \\ & & & -i \\ & & i & \end{pmatrix}, \frac{1}{2}\begin{pmatrix} \mathbb{1} & & & \\ & -\mathbb{1} & & \\ & & 1 & \\ & & & -1 \end{pmatrix} \right\},$$

$$T_R^{\{1,2,3\}}\Big|_{\mathrm{SU}(2)} = \mathbb{0}$$

$$T_L|_{\mathrm{U}(1)} = \begin{pmatrix} \frac{1}{6}\mathbb{1} & & & \\ & \frac{1}{6}\mathbb{1} & & \\ & & -\frac{1}{2} & \\ & & & -\frac{1}{2} \end{pmatrix}, \qquad T_R|_{\mathrm{U}(1)} = \begin{pmatrix} \frac{2}{3}\mathbb{1} & & & \\ & -\frac{1}{3}\mathbb{1} & & \\ & & 0 & \\ & & & -1 \end{pmatrix}$$

Unbreaking gauge invariance

SM embedding

$$T_\phi^{\{1,2,3\}}\Big|_{\mathrm{SU}(2)} = \left\{ \frac{1}{2}\begin{pmatrix} & & & i \\ & & -i & \\ & i & & \\ -i & & & \end{pmatrix}, \frac{1}{2}\begin{pmatrix} & & i & \\ & & & i \\ -i & & & \\ & -i & & \end{pmatrix}, \frac{1}{2}\begin{pmatrix} & -i & & \\ i & & & \\ & & & i \\ & & -i & \end{pmatrix} \right\} \qquad T_\phi\big|_{\mathrm{U}(1)} = \frac{1}{2}\begin{pmatrix} & i & & \\ -i & & & \\ & & & i \\ & & -i & \end{pmatrix}$$

$$Y^1 = \begin{pmatrix} +y_u\mathbb{1} & & & \\ & +y_d\mathbb{1} & & \\ & & 0 & \\ & & & +y_e \end{pmatrix}, \qquad Y^2 = \begin{pmatrix} -iy_u\mathbb{1} & & & \\ & +iy_d\mathbb{1} & & \\ & & 0 & \\ & & & +iy_e \end{pmatrix}$$

$$Y^3 = \begin{pmatrix} & +y_d\mathbb{1} & & \\ -y_u\mathbb{1} & & & \\ & & & +y_e \\ & & 0 & \end{pmatrix}, \qquad Y^4 = \begin{pmatrix} & +iy_d\mathbb{1} & & \\ +iy_u\mathbb{1} & & & \\ & & & +iy_e \\ & & 0 & \end{pmatrix}$$

Unbreaking gauge invariance

What is left?

- Compute generic results for general EFT.
- How does matching work? Method of regions VS counterterms?
- Automatize the calculation.

Mapping BMHV & NDR

BMHV vs NDR

- In practice, the difference between the two schemes lies in the following anticommutator:

$$\{\gamma_\mu, \gamma_5\} = \begin{cases} 0 & \text{for NDR,} \\ 2\gamma_{\hat{\mu}}\gamma_5 & \text{for BMHV.} \end{cases}$$

- The two schemes, if computed consistently, can be *translated*.

[S. Di Noi et al., Phys.Rev.D 109 (2024) 9, 095024]
[M. Ciuchini et al., Nucl. Phys. B 415, 403 (1994)]

Mapping BMHV & NDR

SMEFT

- We can define a shift in the SMEFT Wilson Coefficients so that:

[S. Di Noi, R. Gröber and PO, in preparation]

$$\Delta S^{(1)}_{\mathrm{SMEFT}} = \int dx \, \hat{c}^{(1)}_i \mathcal{O}_i$$

$$\Gamma^{(1)}_{\mathrm{BMHV}} \equiv \Gamma^{(1)}_{\mathrm{NDR}} + \Delta S^{(1)}_{\mathrm{SMEFT}}$$

- We define the SMEFT in BMHV keeping all gamma matrices four-dimensional.

Mapping BMHV & NDR

SMEFT

- In the limit $g_1, g_2 \to 0$, the SMEFT is vector-like and we do not have to compute symmetry-restoring counterterms in BMHV.
- The SMEFT still has, however, an $\mathrm{SU}(2) \otimes \mathrm{U}(1)$ global symmetry that is going to be broken by regularization.
- Recovering this global symmetry is particularly simple introducing a spurious Ω.

Conclusions

- The BMHV prescription is the only one proven to be consistent at all orders.
- We have developed a novel method to restore gauge-invariance in the BMHV prescription and computed counterterms for SM.
- It would be interesting to explore the relation/translation between the two schemes
- In the future, the results for more counterterms and the automatization will make BMHV much easier to use.