MMP I
Tutorial 8

HS 2017
Prof. M. Grazzini

S. Devoto, J. Yook, J. Mazzitelli
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Exercise 1: Metric spaces (4 Pts.)
Consider $X = C[a, b]$, the set of continous functions over the real interval $[a, b]$ and the functions:

a) $d_1(x, y) = \max_{t \in C[a,b]} |x(t) - y(t)|$

b) $d_2(x, y) = \int_a^b |x(t) - y(t)]dt$.

Show that both d_1 and d_2 are metrics.

Exercise 2: Metrics on X (7 Pts.)
Consider $X = \mathbb{R}$ and establish which of the following mappings from $\mathbb{R} \times \mathbb{R} \to \mathbb{R}$ are metrics.

a) $d_1(x, y) = (x - y)^2$

b) $d_2(x, y) = \sqrt{|x - y|}$

c) $d_3(x, y) = |x^2 - y^2|$

d) $d_4(x, y) = |x - 2y|$

e) $d_5(x, y) = |x - y|/(1 + |x - y|)$

Exercise 3: Completion of a metric space $\mathcal{M}$ (5 Pts.)
Consider $\mathcal{M} = (\mathbb{R}, d)$ where $\mathbb{R}$ is the set of real numbers and d is the metric defined as:

$$d(x, y) = |\tanh(x) - \tanh(y)|. \tag{3.1}$$

a) Prove that $d(x, y)$ is indeed a metric.

b) Show that $\mathcal{M}$ is not complete.

c) Find the completion of $\mathcal{M}$.