MMP I
Exercise Sheet 1

HS 21
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Exercise 1 [Fouries series (5 points)]

a) The function $f \in C^2(\mathbb{R}/2\pi\mathbb{Z})$ defined as

$$f(x) = (x^2 - \pi^2)^2, \quad |x| \leq \pi$$

is periodic with a period 2π. Develop f in a Fourier series and calculate

$$\sum_{n=1}^{\infty} \frac{1}{n^4}.$$

b) The function h in $\mathbb{R}$ is defined as

$$h(x) = \begin{cases} 1, & \text{for} \quad 0 < x < \pi \\ 0, & \text{for} \quad \pi < x < 2\pi \\ \frac{1}{2}, & \text{for} \quad x = 0 \quad \text{or} \quad x = \pi \end{cases}$$

and is periodic with period 2π. Calculate the Fourier series of h. Which is the value of the Fourier series in $x = 0$?

Exercise 2 [Series (2 points)]

a) Show that for $-\pi < x < \pi$ the following equation is valid:

$$x = 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx \ . \tag{1}$$

b) Using equation (1) derive the following:

$$1 - \frac{1}{3} + \frac{1}{5} - \cdots = \frac{\pi}{4}.$$

– please turn over –

Exercise 3 [Trigonometric series (4 points)]
For the partial sums

$$s_m(x) = \sum_{n=-m}^{m} c_n e^{inx}$$

of the trigonometric series

$$\sum_{n=-\infty}^{\infty} c_n e^{inx},$$

the following properties are fulfilled:

1. For each $\delta > 0$, the s_m converge uniformly in the interval $[-\pi + \delta, \pi - \delta]$ to some $f(x)$.
2. There's a number k such that $|s_m(x)| < k$ for all x and m.

Show now that the series $\sum_{n=-\infty}^{\infty} c_n e^{inx}$ is the Fourier series of $f(x)$.

Exercise 4 [Convergence (2 points)]
Consider the series

$$f(z) = 1 + z + z^2 + \ldots .$$

a) In which points in $\mathbb{C}$ is this series pointwise convergent?

   Hint: Consider the partial sums and study its convergence.

b) Give an example for a subset of $\mathbb{C}$ where $f(z)$ is uniformly convergent.