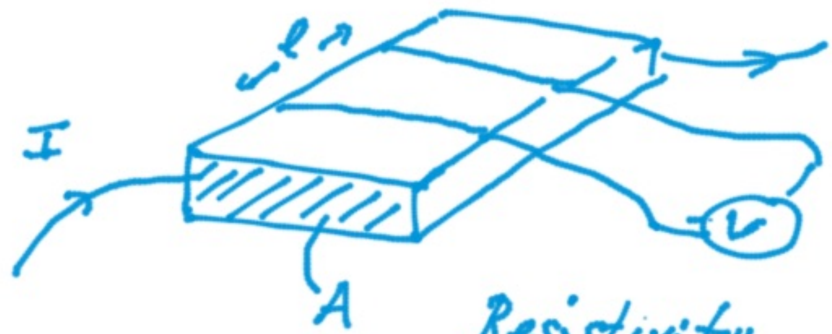
Ohm's LAW

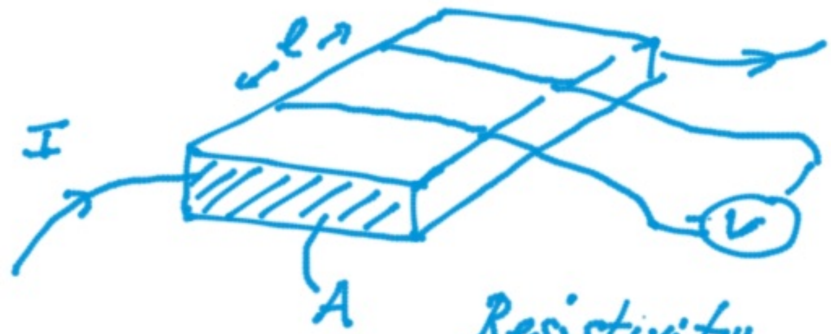
$V = R \cdot I$ or $I = \frac{V}{R}$



Resistivity Experiment

Ohm's LAW

$V = R\cdot I$ or $I = \frac{V}{R}$



Resistivity Experiment

CURRENT DENSITY: $j = \frac{I}{A}$

ELECTRIC FIELD

VOLTAGE BETWEEN TWO POINTS: $V = \int E\,dx = E\cdot \ell$

OHM'S LAW REFORMULATED: $E\ell = R\cdot Aj \Rightarrow E = \frac{A}{\ell}\cdot R\cdot j$

$\Rightarrow E = \rho\cdot j$

or $\boxed{j = \sigma E}$

$\rho = \frac{A}{\ell}\cdot R$ = resistivity

σ = conductivity

DRUDE CONDUCTIVITY: $\sigma = \frac{ne^2\tau}{m}$

Application of Electrical Field E:

$\hookrightarrow$ Electron's sense a force: $F = -eE$

$$m\frac{dv}{dt} = -eE$$

DRUDE CONDUCTIVITY: $\sigma = \frac{ne^2\tau}{m}$

Application of Electrical Field E:

↳ Electron's sense a force: $F = -eE$

$$\Downarrow$$

$$m\frac{dv}{dt} = -eE$$

$$dv = \frac{-eE}{m}dt$$

collision at $t = t_0$

v_0 = velocity just after t_0

τ = average time between collision

$t_0 < t_1 \approx \tau$

$V(t_1) = V_0 - \frac{eE}{m}t_1$

$\langle v \rangle = -\frac{eE}{m}\tau$

DRUDE CONDUCTIVITY: $\sigma = \frac{ne^2\tau}{m}$

Application of Electrical Field E:

Electron's sense a force: $F = -eE$

$$\Downarrow$$

$$m\frac{dv}{dt} = -eE$$

$$dv = \frac{-eE}{m}dt$$

collision at $t = t_0$

v_0 = velocity just after t_0

τ = average time between collision

$V(t) = V_0 - \frac{eE}{m}t_1 \qquad t_0 < t_1 \stackrel{<}{\sim} \tau$

$\langle v \rangle = -\frac{eE}{m}\tau$

$$j = -nve = +\frac{ne^2\tau}{m}E$$

$$\sigma = \frac{ne^2\tau}{m}$$

$$= ne\mu$$

$\mu = \frac{e}{m}\cdot\tau$ = mobility

effective mass

$$v_G = \frac{1}{\hbar}\frac{d\varepsilon}{dk}\;,\; \frac{dv_G}{dt} = \frac{1}{\hbar}\frac{d^2\varepsilon}{dk\,dt} = \frac{1}{\hbar}\frac{d^2\varepsilon}{dk^2}\frac{dk}{dt}$$

$$F = \frac{d}{dt}(\underbrace{\hbar k}_{=p}) = \hbar\frac{dk}{dt} \Rightarrow \frac{dk}{dt} = \frac{F}{\hbar}$$

$$\Rightarrow \frac{dv_G}{dt} = \underbrace{\frac{1}{\hbar^2}\frac{d^2\varepsilon}{dk^2}}_{=:\frac{1}{m^*}} F \Rightarrow F = m^*\frac{dv_G}{dt}$$

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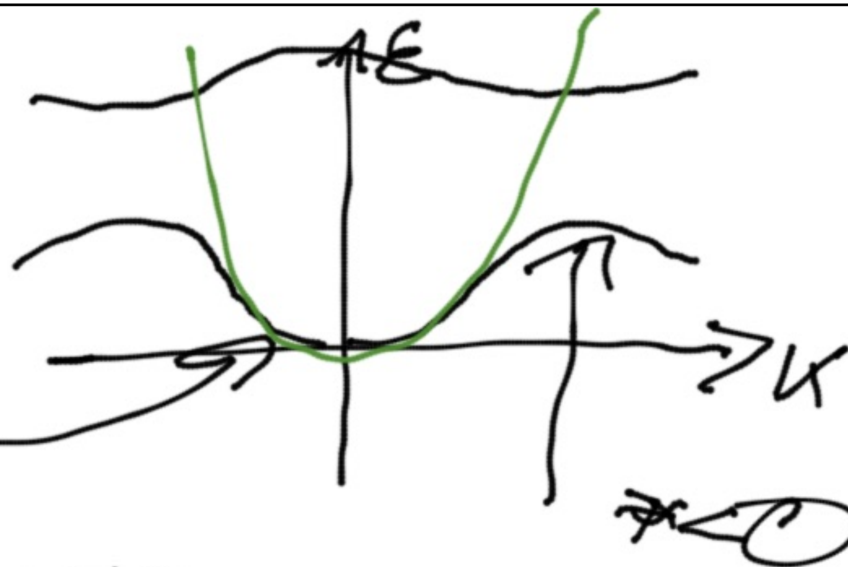
$$\frac{1}{m^*} \sim \frac{d^2\mathcal{E}}{dk^2}$$

$m^* > 0$

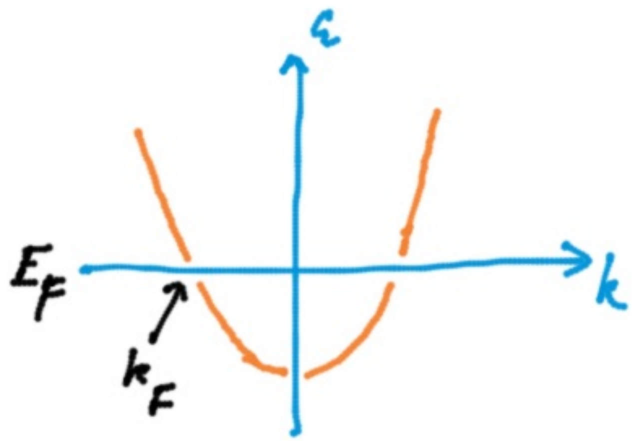


free electron: $\mathcal{E} = \frac{\hbar^2 k^2}{2m}$

$$\frac{1}{\hbar^2}\frac{d^2\mathcal{E}}{dk^2} = \frac{\cancel{\hbar^*}}{m} = \frac{1}{m^*} \Rightarrow \underline{m^* = m}$$

$$m_e = -m_h$$

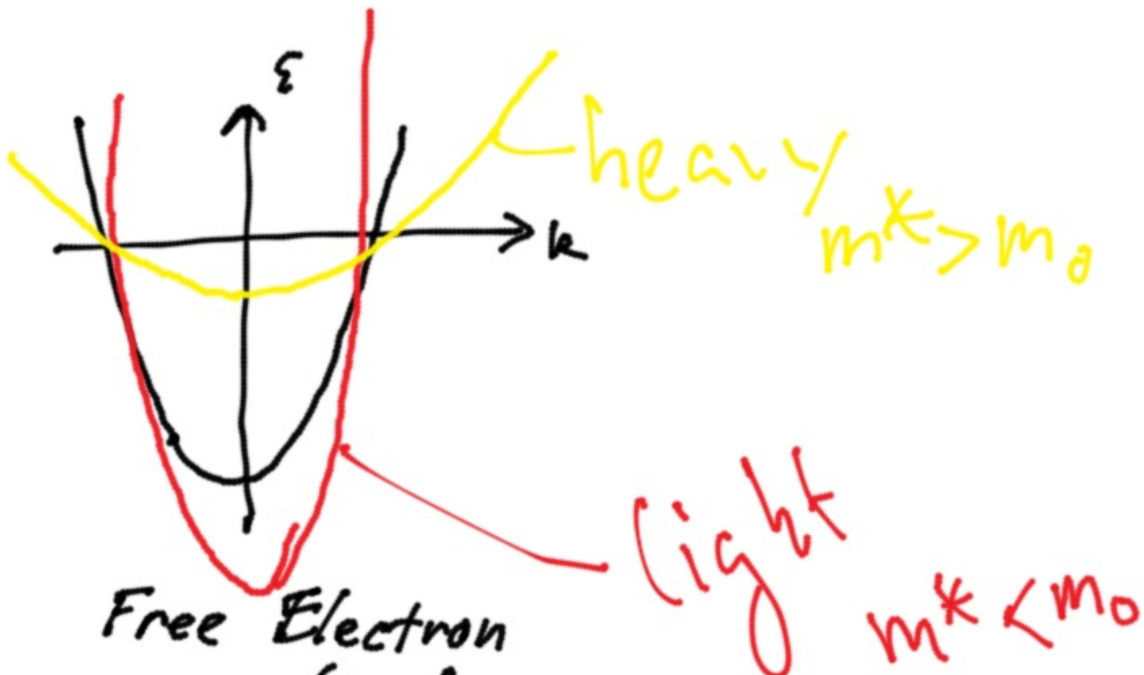
Effective Electron Mass in Crystal



$$\mathcal{E} \propto k^2$$

$$\mathcal{E} = \frac{(\hbar k)^2}{2m^*}$$

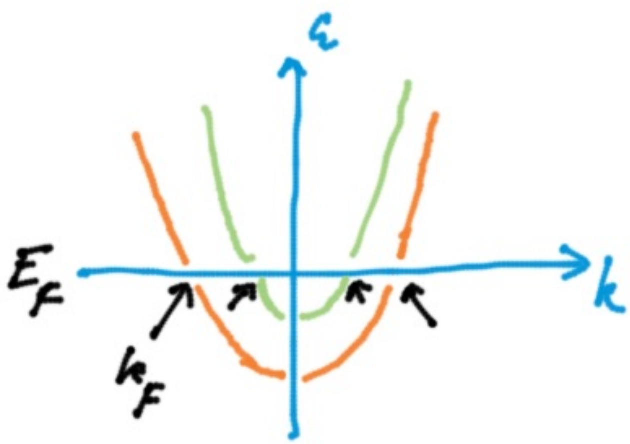
"LIGHT" V "Heavy" Electrons



Free Electron

$$E_k = \frac{(\hbar k)^2}{2m_0}$$

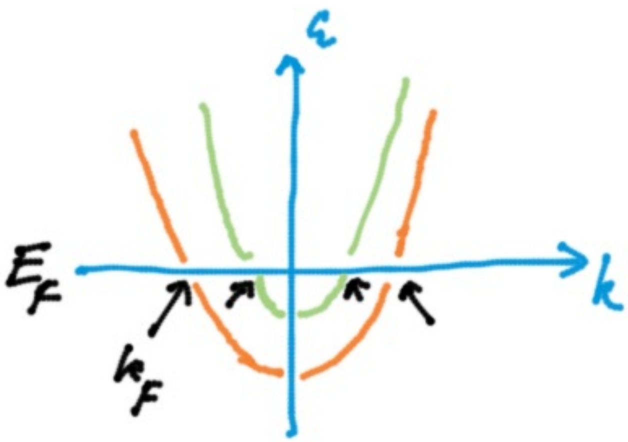
Effective Electron Mass in Crystal

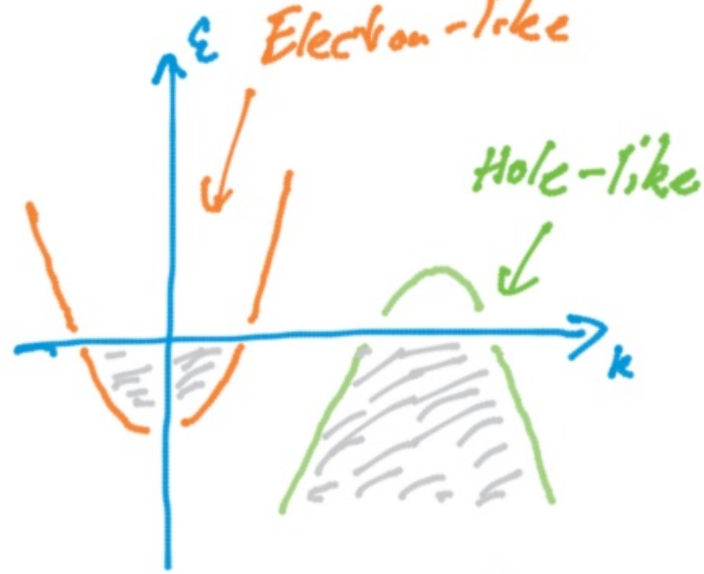


Two bands

Two Fermi surfaces

Effective Electron Mass in Crystal





Two bands

Two Fermi surfaces

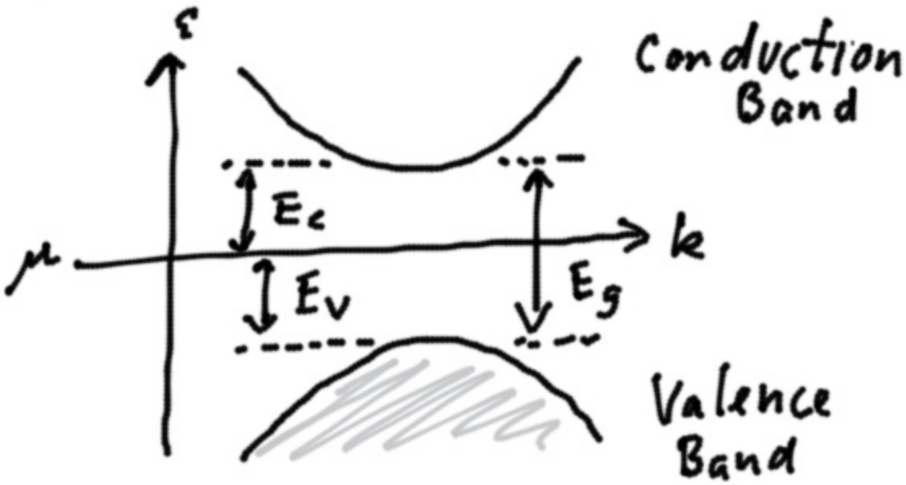
$m^* > 0$ $m^* < 0$

$$m^* \frac{dv}{dt} = -eE$$

$\Downarrow$

$$\frac{dv}{dt} = \frac{-e}{\;} E$$

Semiconductor Charge Carrier Concentration



$$E_g = E_v + E_c$$

GERMANIUM

$$E_g \sim 0.6\,\text{eV} = 600\,\text{meV}$$

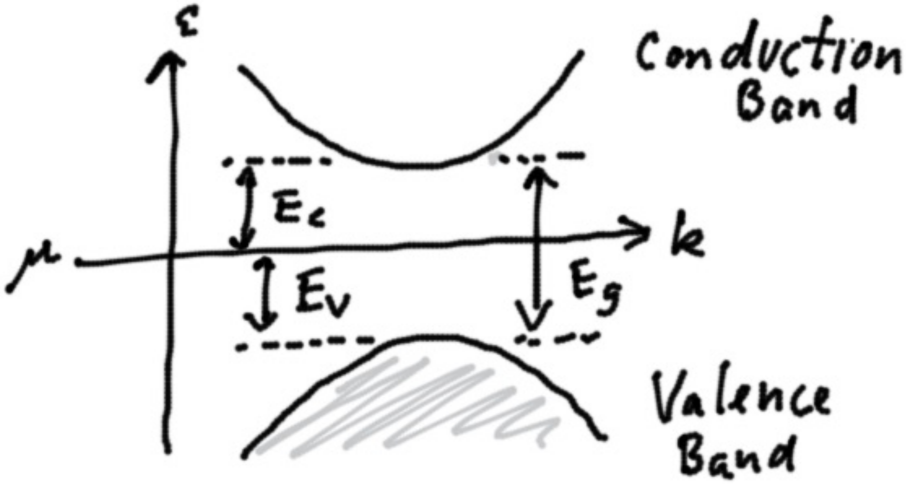
$$T = 300\,\text{K}$$

$$\Downarrow$$

$$k_B T \approx 30\,\text{meV}$$

$$\frac{E_g}{2} + \mu \gg k_B T$$

Semiconductor Charge Carrier Concentration



FERMI-DIRAC DISTRIBUTION

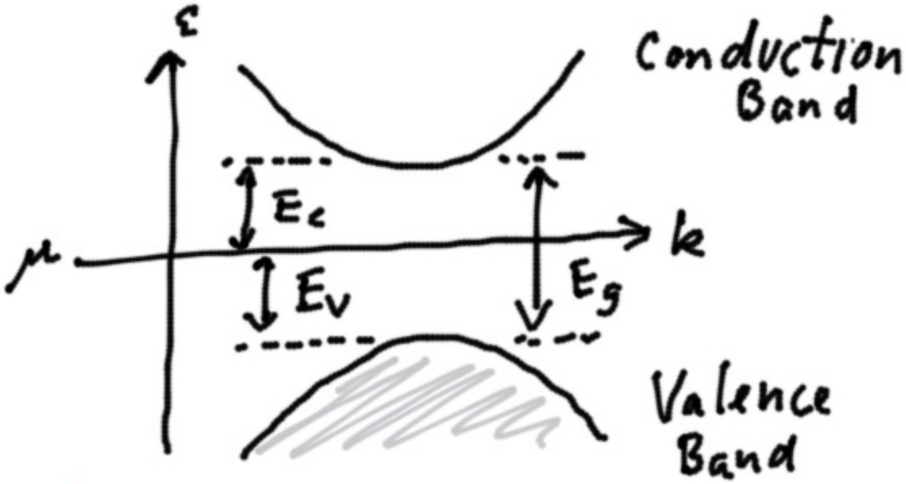
$$f(\varepsilon) = \frac{1}{\exp\left(\frac{\varepsilon-\mu}{k_B T}\right)+1} \approx \exp\left(\frac{\mu-\varepsilon}{k_B T}\right) \equiv f_e(\varepsilon)$$

CONDUCTION BAND DISPERSION

DOS

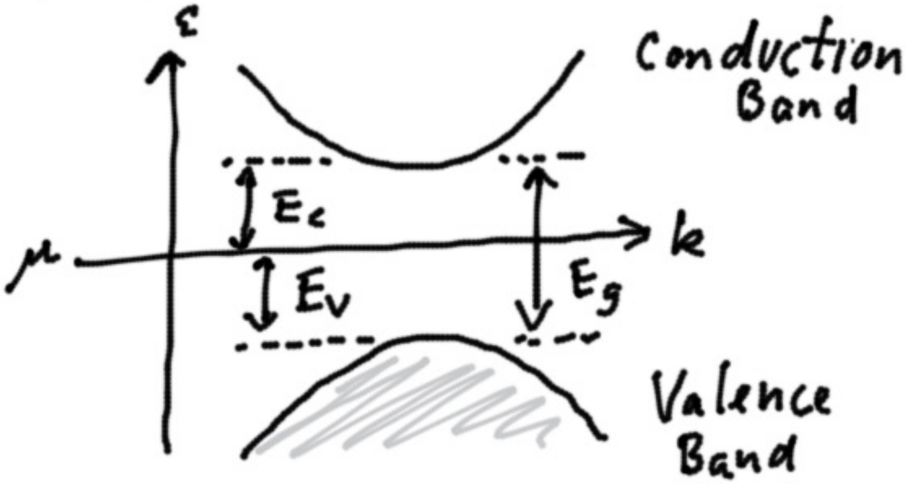
$$\varepsilon_k = E_c + \frac{(\hbar k)^2}{2m_e} \longrightarrow D_e(\varepsilon) = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}(\varepsilon - E_c)^{1/2}$$

Semiconductor Charge Carrier Concentration



$$n = \int_{E_c}^{\infty} D_e(\varepsilon)\, f_e(\varepsilon)\, d\varepsilon = 2\left(\frac{m_c k_B T}{2\pi \hbar^2}\right)^{3/2} \exp\left(\frac{\mu - E_c}{k_B T}\right)$$

Semiconductor Charge Carrier Concentration



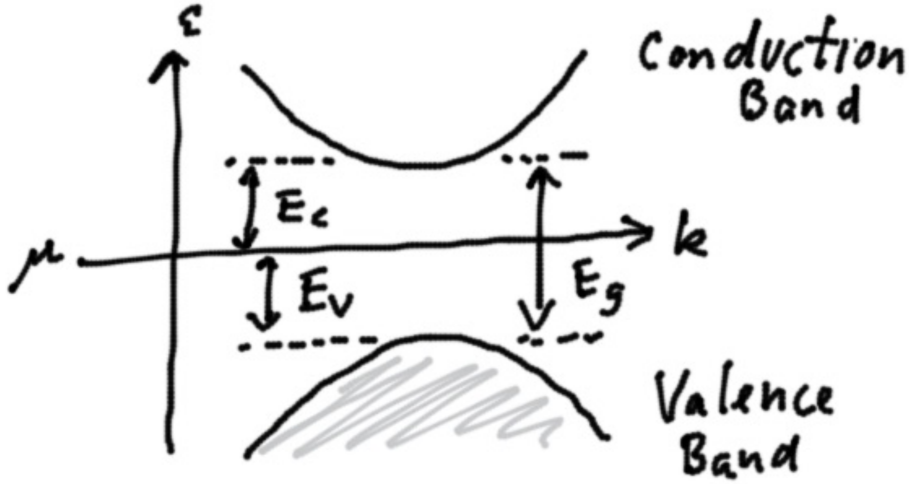
Summary

$$n = \int_{E_c}^{\infty} D_e(\varepsilon) f_e(\varepsilon)\, d\varepsilon = 2\left(\frac{m_c k_B T}{2\pi\hbar^2}\right)^{3/2} \exp\left(\frac{\mu - E_c}{k_B T}\right)$$

$$p(T) = 2\left(\frac{m_h k_B T}{2\pi\hbar^2}\right)^{3/2} \exp\left(\frac{E_v - \mu}{k_B T}\right)$$

$$n(T)\, p(T) = \frac{1}{2}\left(\frac{k_B T}{\pi\hbar^2}\right)(m_e m_h)^{3/2} \exp\left(\frac{-E_g}{k_B T}\right)$$

Semiconductor Charge Carrier Concentration



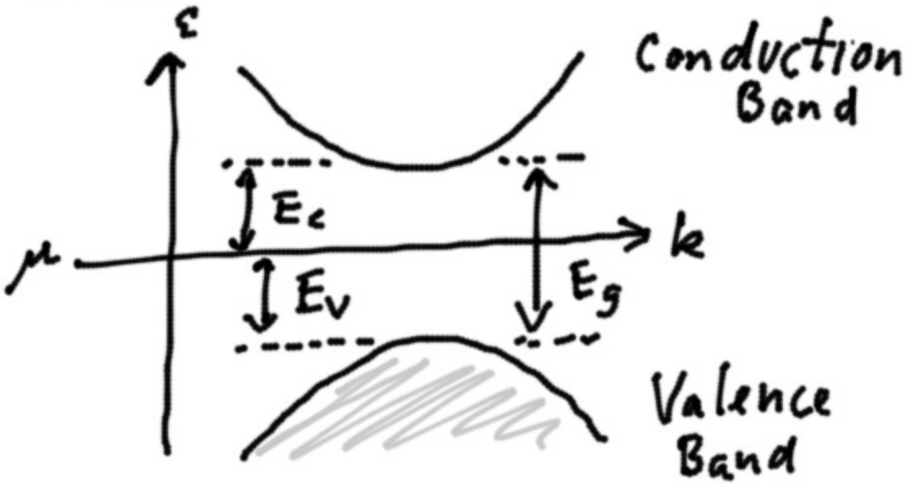
$n = p$

Chemical potential μ

$$\frac{n(T)}{p(T)} = 1 = \left(\frac{m_e}{m_h}\right)^{3/2} \exp\left(\frac{2\mu - (E_c + E_v)}{k_B T}\right)$$

$$\mu = \frac{E_c + E_v}{2} + \frac{3}{4} k_B T \cdot \log\left(\frac{m_e}{m_h}\right)$$

Semiconductor Charge Carrier Concentration



Numerical evaluation: $n(300K)$ Germanium

$E_g \approx 600\,\mathrm{meV}$

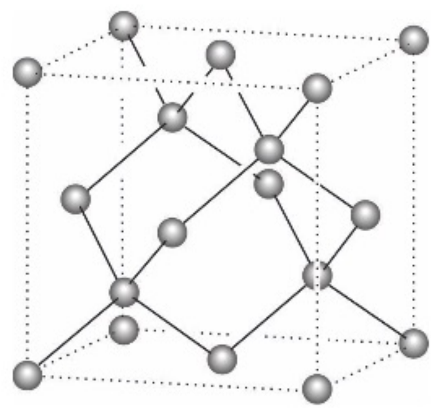
$|\mu - E_c| \sim 300\,\mathrm{meV}$

$k_B T \sim 30\,\mathrm{meV}$ @ 300 K

$m_e = 0.04\, m_0$

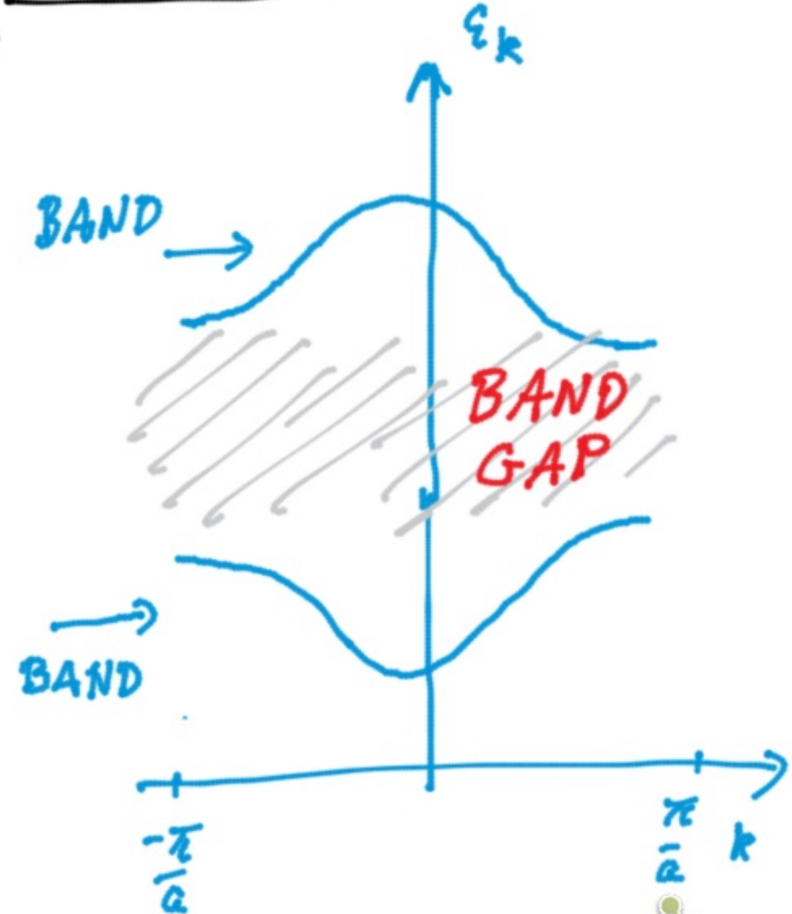
$n(300) = 2.5\cdot 10^{12}\ \mathrm{cm}^{-3}$

CRYSTAL STRUCTURE



PERIODIC POTENTIAL

BAND STRUCTURE



BAND STRUCTURE