Representations of SUSY algebra

Recall the SUSY algebra

$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu$ $\qquad$ $[P_\mu, Q_\alpha] = [P_\mu, \bar{Q}_{\dot\alpha}] = 0$

$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0$ $\qquad$ $[M_{\mu\nu}, Q_\alpha] = i\sigma_{\mu\nu\alpha}{}^\beta Q_\beta$, $[M_{\mu\nu}, \bar{Q}^{\dot\alpha}] = i\bar\sigma_{\mu\nu}{}^{\dot\alpha}{}_{\dot\beta}\bar{Q}^{\dot\beta}$

+ Poincaré ...

- We already know that a SUSY multiplet contains an equal number of bosons and fermions

Massless representations

Choose a frame where $P_\mu = (E, 0, 0, E)$

(massless rep.s of Poincaré determined by helicity, $J_3 = M_{12}$)

$$\sigma^\mu P_\mu = E(\sigma^0 - \sigma^3) = \begin{pmatrix} 0 & 0 \\ 0 & 2E \end{pmatrix}$$

$\{Q_1, \bar{Q}_{\dot 1}\} = \{Q_1, \bar{Q}_{\dot 2}\} = \{Q_2, \bar{Q}_{\dot 1}\} = 0 \implies \langle\phi|\{Q_1, \bar{Q}_{\dot 1}\}|\phi\rangle = \| Q_1|\phi\rangle\|^2 + \|\bar{Q}_{\dot 1}|\phi\rangle\|^2 = 0$

$$Q_1 = \bar{Q}_{\dot 1} \equiv 0$$

$\{Q_2, \bar{Q}_{\dot 2}\} = 4E$ Clifford algebra

$a \equiv \frac{1}{\sqrt{4E}} Q_2$, $\quad a^\dagger \equiv \frac{1}{\sqrt{4E}} \bar{Q}_{\dot 2}$ $\qquad$ $\{a, a^\dagger\} = 1$, $\quad \{a, a\} = \{a^\dagger, a^\dagger\} = 0$

$$[M_{12}, Q_2] = i(\sigma_{12})_2{}^2 Q_2 = \frac{1}{2}\sigma_3 \cdot Q_2 = -\frac{1}{2} Q_2$$

$$[M_{12}, \bar{Q}_{\dot 2}] = i(\bar\sigma_{12})_{\dot 2}{}^{\dot 2} \bar{Q}_{\dot 2} = -\frac{1}{2}\sigma_3 \cdot \bar{Q}_{\dot 2} = +\frac{1}{2}\bar{Q}_{\dot 2}$$

$$\implies Q_2|\lambda\rangle \sim |\lambda - 1/2\rangle, \quad \bar{Q}_{\dot 2}|\lambda\rangle \sim |\lambda + 1/2\rangle$$

Start from a Clifford vacuum $|\lambda_0\rangle$, $a|\lambda_0\rangle = 0$

$\longrightarrow$ $N=1$ supermultiplet $\{|\lambda_0\rangle, |\lambda_0 + 1/2\rangle\}$ $\qquad$ (recall $Q^2 = 0$)

•) Chiral supermultiplet : $\lambda_0 = 0$

$\{|0\rangle, |+1/2\rangle\}$ is not CPT invariant

$\implies \{|0\rangle, |+1/2\rangle\} \oplus \{|0\rangle, |-1/2\rangle\}$: complex scalar + Weyl fermion

•) Vector supermultiplet : $\lambda_0 = 1/2$

$\{|1/2\rangle, |1\rangle\} \oplus \{|-1/2\rangle, |-1\rangle\}$: Weyl fermion + (gauge) vector

Internal symmetries commute with super-algebra $[G, Q] = 0$

$\Longrightarrow$ the Weyl fermion transforms in the same representation of the internal group as the vector (adjoint of the gauge group)

No other $N=1$ multiplet without particles of spin > 1.

If gravity is taken into account, we can have states of helicity $3/2$ and 2.

- Gravitino supermultiplet : $\lambda_0 = 1 \longrightarrow \{|1\rangle, |3/2\rangle\} \oplus \{|-1\rangle, |-3/2\rangle\}$
- Graviton supermultiplet : $\lambda_0 = 3/2 \longrightarrow \{|3/2\rangle, |2\rangle\} \oplus \{|-3/2\rangle, |-2\rangle\}$

N = 2 example : Q^1_α, Q^2_α

$\lambda_0 = -1/2 \longrightarrow \{|-1/2\rangle, \underbrace{|0\rangle, |0\rangle}_{\binom{N}{K} \text{ states with helicity } K/2 + \lambda_0}, |+1/2\rangle\} \oplus$ CPT conjugate (= two N=1 chiral mult.)

$\lambda_0 = 0 \longrightarrow \{|0\rangle, |1/2\rangle, |1/2\rangle, |1\rangle\} \oplus$ CPT conjugate (= chiral + vector N=1 mult.)

▸ no chiral matter is possible in N=2 SUSY

(ex: find the only N=4 multiplet with helicities ≤ 1)

Massive representations

$P_\mu = (m, 0, 0, 0) \Longrightarrow \{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2m\,\delta_{\alpha\dot\beta}$ 2 Clifford algebras

$$a_{1,2} \equiv \frac{1}{\sqrt{2m}} Q_{1,2}, \quad a^\dagger_{1,2} \equiv \frac{1}{\sqrt{2m}} \bar{Q}_{\dot 1,\dot 2}$$

$$[M_{12}, Q_\alpha] \sim \sigma_3 \cdot Q_\alpha \Longrightarrow Q_1|j\rangle \sim |j+1/2\rangle, \quad Q_2|j\rangle \sim |j-1/2\rangle$$

•) Massive chiral multiplet : $j_0 = 0$

$Q_1|j=0\rangle \sim |j=1/2\rangle$, $Q_2|j=0\rangle \sim |j=-1/2\rangle$, $Q_1Q_2|j=0\rangle \sim |j=0\rangle'$

$\{|-1/2\rangle, |0\rangle, |0\rangle', |+1/2\rangle\}$: Majorana fermion + complex scalar

•) <u>Massive vector multiplet</u> : $j_0 = 1/2$

$$\{|1/2\rangle, |0\rangle, |1\rangle, |1/2\rangle'\} \underset{CPT}{\oplus} \{|-1/2\rangle, |0\rangle, |-1\rangle, |-1/2\rangle'\}$$

$$= \{|-1\rangle, 2\times|-1/2\rangle, 2\times|0\rangle, 2\times|+1/2\rangle, |+1\rangle\} \simeq \text{massless chiral + vector mult.}$$

Massive vector = massless vector + massles chiral multiplet

$\longrightarrow$ <u>super-Higgs mechanism</u>

Field representations

$\bar{Q}|\lambda_0\rangle = 0 \quad \rightsquigarrow \quad [\bar{Q}_{\dot\alpha}, \phi] = 0$ consider a scalar field ϕ

ϕ must be complex, otherwise $[Q_\alpha, \phi] = ([\bar{Q}_{\dot\alpha}, \phi])^\dagger = 0$

$\Rightarrow [P_\mu, \phi] \approx [\{Q_\alpha, \bar{Q}_{\dot\beta}\}, \phi] = 0 \quad \Rightarrow \quad \partial_\mu \phi = 0 \quad \phi$ constant!

$[Q_\alpha, \phi] \equiv \psi_\alpha$ fermion field

$\{Q_\alpha, \psi_\beta\} \equiv F_{\alpha\beta}$, $\{\bar{Q}_{\dot\alpha}, \psi_\beta\} \equiv X_{\dot\alpha\beta}$

$$X_{\dot\alpha\beta} = \{\bar{Q}_{\dot\alpha}, [Q_\beta, \phi]\} = -\{Q_\beta, \cancel{[\bar{Q}_{\dot\alpha}, \phi]}\} - [\phi, \{Q_\beta, \bar{Q}_{\dot\alpha}\}] = -[\phi, 2\sigma^\mu_{\beta\dot\alpha} P_\mu]$$
$$= 2\sigma^\mu_{\beta\dot\alpha}[P_\mu, \phi] \sim \partial_\mu \phi \quad \text{(not an independent field)}$$

$$F_{\alpha\beta} = \{Q_\alpha, [Q_\beta, \phi]\} = -\{Q_\beta, [Q_\alpha, \phi]\} - [\phi, \cancel{\{Q_\alpha, Q_\beta\}}]$$
$$\Rightarrow F_{\alpha\beta} + F_{\beta\alpha} = 0 , \qquad F_{\alpha\beta} = \epsilon_{\alpha\beta} F$$

The algebra closes without introducing other fields :

$$[Q_\alpha, F_{\beta\gamma}] = [Q_\alpha, \{Q_\beta, \psi_\gamma\}] = -[Q_\beta, \{Q_\alpha, \psi_\gamma\}] - [\psi_\gamma, \cancel{\{Q_\beta, Q_\alpha\}}] = -[Q_\beta, F_{\alpha\gamma}]$$
$$\Rightarrow \epsilon_{\beta\gamma}[Q_\alpha, F] + \epsilon_{\alpha\gamma}[Q_\beta, F] = 0 \quad \Rightarrow \quad [Q_\alpha, F] = 0$$

$$[\bar{Q}_{\dot\alpha}, F_{\beta\gamma}] = [\bar{Q}_{\dot\alpha}, \{Q_\beta, \psi_\gamma\}] = -[Q_\beta, \{\bar{Q}_{\dot\alpha}, \psi_\gamma\}] - [\psi_\gamma, \{\bar{Q}_{\dot\alpha}, Q_\beta\}]$$
$$= -[Q_\beta, X_{\gamma\dot\alpha}] - 2\sigma^\mu_{\beta\dot\alpha}[\psi_\gamma, P_\mu] = 2i\sigma^\mu_{\gamma\dot\alpha}[Q_\beta, \partial_\mu\phi] - 2i\sigma^\mu_{\beta\dot\alpha}\partial_\mu\psi_\gamma$$
$$= 2i\sigma^\mu_{\gamma\dot\alpha}\partial_\mu\psi_\beta - 2i\sigma^\mu_{\beta\dot\alpha}\partial_\mu\psi_\gamma$$

<u>Chiral supermultiplet</u> (ϕ, ψ_α, F)

Count the degrees of freedom :

$$\left.\begin{array}{lll} \phi = (\mathrm{Re}\,\phi, \mathrm{Im}\,\phi) & : & 2 \text{ bosonic} \\ \psi_\alpha = (\mathrm{Re}\,\psi_1, \mathrm{Im}\,\psi_1, \mathrm{Re}\,\psi_2, \mathrm{Im}\,\psi_2) & : & 4 \text{ fermionic} \\ F = (\mathrm{Re}\,F, \mathrm{Im}\,F) & : & 2 \text{ bosonic} \end{array}\right\} \quad 4_B + 4_F \text{ d.o.f.}$$

For the on-shell state : chiral multiplet $\sim \{|\phi\rangle, |\psi\rangle\}$: $2_B + 2_F$ d.o.f.
(an on-shell fermion has 2 d.o.f. , the Dirac equation fixes 2 constraints !)
$\Rightarrow$ the equations of motion must fix the 2 components of F on-shell
$F = F(\phi, \psi)$, 0 d.o.f. on-shell $\Rightarrow$ F is an <u>auxiliary field</u>

Superfields & superspace

(ϕ, ψ_α, F) transformation properties under SUSY ?
We want to make SUSY manifest ... (as for Lorentz, e.g. $(\phi, \vec{A}) \to A_\mu$)

Is it possible to associate a group to the SUSY algebra ?
$\theta_\alpha, \bar{\theta}_{\dot{\alpha}}$ <u>constant</u> anticommuting Grassmann variables

$$\{\theta^\alpha, \theta^\beta\} = \{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = \{\theta^\alpha, \bar{\theta}_{\dot{\beta}}\} = 0$$

$$\left.\begin{array}{l} [\theta Q, \bar{\theta}\bar{Q}] = 2\theta\sigma^\mu\bar{\theta}\, P_\mu \, , \\ [\theta Q, \theta Q] = [\bar{\theta}\bar{Q}, \bar{\theta}\bar{Q}] = 0 \, . \end{array}\right\} \quad \text{super-Poincaré Lie algebra}$$

Super-Poincaré group : $G(x, \theta, \bar{\theta}, \omega) = \exp\left(ix^\mu P_\mu + \underbrace{i\theta^\alpha Q_\alpha + i\bar{\theta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}}_{\text{transformations in superspace}} + \frac{i}{2}\omega^{\mu\nu}M_{\mu\nu}\right)$

$x^\mu \longleftrightarrow \exp(x^\mu P_\mu)$ Minkowski space

$(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}) \longleftrightarrow \exp(x^\mu P_\mu)\exp(\theta Q + \bar{\theta}\bar{Q})$ superspace

<u>Superfield</u> : a field in superspace $Y(x, \theta, \bar{\theta})$

$$\begin{aligned} Y(x, \theta, \bar{\theta}) = {} & \phi(x) + \theta\psi(x) + \bar{\theta}\bar{\chi}(x) + \theta^2 m(x) + \bar{\theta}^2 n(x) \\ & + \theta\sigma^\mu\bar{\theta}\, v_\mu(x) + \theta^2\bar{\theta}\bar{\lambda}(x) + \bar{\theta}^2\theta\rho(x) + \theta^2\bar{\theta}^2 f(x) \end{aligned}$$

$(\theta_\alpha\theta_\beta\theta_\gamma = 0)$

How do the supercharges $Q, \bar{Q}$ act on superfields ?

(Example : translations

$$\phi(x+a) = e^{-ia^\mu P_\mu} \phi(x) e^{ia^\mu P_\mu} = \phi(x) - ia^\mu [P_\mu, \phi(x)]$$

$$= \phi(x) + a^\mu \partial_\mu \phi(x) + \cdots$$

$$\Rightarrow [\phi(x), P_\mu] = -i\partial_\mu \phi(x) \equiv \mathcal{P}_\mu \phi(x)$$ representation of P_μ as diff. operator

$$\delta_a \phi = ia^\mu [\phi, P_\mu] = ia^\mu \mathcal{P}_\mu \cdot \phi$$)

Translation in superspace : $(x, \theta, \bar{\theta}) \longrightarrow (x + \delta x, \theta + \delta\theta, \bar{\theta} + \delta\bar{\theta})$

$$Y(x+\delta x, \theta + \delta\theta, \bar{\theta} + \delta\bar{\theta}) = e^{-i(\epsilon Q + \bar{\epsilon}\bar{Q})} Y(x, \theta, \bar{\theta}) e^{i(\epsilon Q + \bar{\epsilon}\bar{Q})}$$

$$= e^{-i(\epsilon Q + \bar{\epsilon}\bar{Q})} e^{-i(xP + \theta Q + \bar{\theta}\bar{Q})} Y(0) e^{i(xP + \theta Q + \bar{\theta}\bar{Q})} e^{i(\epsilon Q + \bar{\epsilon}\bar{Q})}$$

$$\exp(ix^\mu P_\mu + i\theta Q + i\bar{\theta}\bar{Q}) \exp(i\epsilon Q + i\bar{\epsilon}\bar{Q})$$

$$= \exp\left(ix^\mu P_\mu + i(\theta+\epsilon)Q + i(\bar{\theta}+\bar{\epsilon})\bar{Q} - \frac{1}{2}[\bar{\theta}\bar{Q}, \epsilon Q] - \frac{1}{2}[\theta Q, \bar{\epsilon}\bar{Q}] + \cdots\right)$$

$$= \exp\left(ix^\mu P_\mu + i(\theta+\epsilon)Q + i(\bar{\theta}+\bar{\epsilon})\bar{Q} + \epsilon\sigma^\mu\bar{\theta} P_\mu - \theta\sigma^\mu\bar{\epsilon} P_\mu + \cdots\right)$$

$$\Rightarrow \begin{cases} \delta x^\mu = i\theta\sigma^\mu\bar{\epsilon} - i\epsilon\sigma^\mu\bar{\theta} \\ \delta\theta^\alpha = \epsilon^\alpha \\ \delta\bar{\theta}^{\dot{\alpha}} = \bar{\epsilon}^{\dot{\alpha}} \end{cases}$$

$\delta x \neq 0$ even if we act only with $Q, \bar{Q}$!
SUSY algebra <u>is not</u> a direct product
Poincaré × something
$\{Q, \bar{Q}\} \sim P$: supercharges act also on spacetime

$$\delta_{\epsilon,\bar{\epsilon}} Y(x, \theta, \bar{\theta}) = i(\theta\sigma^\mu\bar{\epsilon} - \epsilon\sigma^\mu\bar{\theta})\partial_\mu Y + \epsilon^\alpha \frac{\partial}{\partial\theta^\alpha} Y + \bar{\epsilon}^{\dot{\alpha}} \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} Y + \cdots$$

$$\equiv -i\epsilon^\alpha [Q_\alpha, Y] - i\bar{\epsilon}_{\dot{\alpha}} [\bar{Q}^{\dot{\alpha}}, Y] + \cdots$$ $\longleftarrow$ from $e^{-i(\epsilon Q + \bar{\epsilon}\bar{Q})} Y e^{i(\epsilon Q + \bar{\epsilon}\bar{Q})}$

$$\equiv (i\epsilon \mathcal{Q} + i\bar{\epsilon}\bar{\mathcal{Q}}) \cdot Y$$

$\mathcal{Q}_\alpha Y \equiv [Y, Q_\alpha]$, $\bar{\mathcal{Q}}_{\dot{\alpha}} Y \equiv [Y, \bar{Q}_{\dot{\alpha}}]$ (operators acting on Y)

Differential representation :

$$\begin{cases} \mathcal{Q}_\alpha = -i\partial_\alpha - \sigma^\mu_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_\mu \\ \bar{\mathcal{Q}}_{\dot{\alpha}} = +i\bar{\partial}_{\dot{\alpha}} + \theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu \end{cases}$$ $(\partial_\alpha \equiv \partial/\partial\theta^\alpha, \ \bar{\partial}_{\dot{\alpha}} \equiv \partial/\partial\bar{\theta}^{\dot{\alpha}})$

Can check that $Q, \bar{Q}, P_\mu$ satisfy the SUSY algebra :

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = (-i\partial_\alpha - \sigma^\mu_{\alpha\dot\gamma}\bar\theta^{\dot\gamma}\partial_\mu)(i\bar\partial_{\dot\beta} + \theta^\gamma\sigma^\nu_{\gamma\dot\beta}\partial_\nu) + (i\bar\partial_{\dot\beta} + \theta^\gamma\sigma^\nu_{\gamma\dot\beta}\partial_\nu)(-i\partial_\alpha - \sigma^\mu_{\alpha\dot\gamma}\bar\theta^{\dot\gamma}\partial_\mu)$$

$$= \cdots = -i\sigma^\nu_{\alpha\dot\beta}\partial_\nu - i\sigma^\mu_{\alpha\dot\beta}\partial_\mu = 2\sigma^\mu_{\alpha\dot\beta}P_\mu$$

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0$$

The general superfield Y has too many components to describe an irreducible SUSY representation, e.g. the chiral multiplet (ϕ, ψ_α, F)

$\rightsquigarrow$ we have to impose constraints

$\bar\partial_{\dot\alpha} Y = 0$? Keep only the components without $\bar\theta$... ϕ, ψ, m

this constraint is not SUSY-invariant ! $\quad [\bar\partial_{\dot\alpha}, \epsilon Q] = \epsilon^\beta\sigma^\mu_{\beta\dot\alpha}\partial_\mu$

$\Rightarrow \bar\partial_{\dot\alpha} Y$ is not a superfield !

Covariant derivatives

$$\begin{cases} D_\alpha \equiv \partial_\alpha + i\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu \\ \bar{D}_{\dot\alpha} \equiv \bar\partial_{\dot\alpha} + i\theta^\beta\sigma^\mu_{\beta\dot\alpha}\partial_\mu \end{cases}$$

$$\{D_\alpha, \bar{D}_{\dot\beta}\} = 2i\sigma^\mu_{\alpha\dot\beta}\partial_\mu = -2\sigma^\mu_{\alpha\dot\beta}P_\mu$$

$$\{D_\alpha, D_\beta\} = \{\bar{D}_{\dot\alpha}, \bar{D}_{\dot\beta}\} = 0$$

the covariant derivatives anticommute with the SUSY generators

$$\{D_\alpha, Q_\beta\} = \{D_\alpha, \bar{Q}_{\dot\beta}\} = \{\bar{D}_{\dot\alpha}, Q_\beta\} = \{\bar{D}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0$$

$\Rightarrow \quad \delta_{\epsilon,\bar\epsilon}(D_\alpha Y) = D_\alpha\,\delta_{\epsilon,\bar\epsilon} Y \qquad (\delta_{\epsilon,\bar\epsilon} Y = i(\epsilon Q + \bar\epsilon\bar{Q})\cdot Y)$

SUSY-invariant constraints : $D_\alpha Y = 0, \quad \bar{D}_{\dot\alpha} Y = 0$

Chiral superfield : $\quad \bar{D}_{\dot\alpha}\Phi = 0$

it is convenient to change coordinates : $\quad y^\mu \equiv x^\mu + i\theta^\alpha\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}$

$$\bar{D}_{\dot\alpha} y^\mu = (\bar\partial_{\dot\alpha} + i\theta^\beta\sigma^\nu_{\beta\dot\alpha}\partial_\nu)(x^\mu + i\theta^\alpha\sigma^\mu_{\alpha\dot\gamma}\bar\theta^{\dot\gamma}) = -i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha} + i\theta^\beta\sigma^\nu_{\beta\dot\alpha}\delta^\mu_\nu = 0$$

$$\bar{D}_{\dot\alpha}\theta = (\bar\partial_{\dot\alpha} + i\theta\sigma^\mu\partial_\mu)\theta = 0$$

$\Rightarrow \quad \Phi = \Phi(y, \theta) \qquad \Phi$ depends on $\bar\theta$, but only through y

Explicitly : $\Phi(y,\theta) = \phi(y) + \sqrt{2}\,\theta\psi(y) - \theta^2 F(y)$

$$\Phi(y) = \Phi(x) + i\theta\sigma^\mu\bar\theta\,\partial_\mu\Phi(x) + \frac{1}{2}(i\theta\sigma^\mu\bar\theta)(i\theta\sigma^\nu\bar\theta)\,\partial_\mu\partial_\nu\Phi$$

$$= \phi(x) + \sqrt{2}\,\theta\psi(x) - \theta^2 F(x) + i\theta\sigma^\mu\bar\theta\,\partial_\mu\phi(x) - \frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi\sigma^\mu\bar\theta - \frac{1}{4}\theta^2\bar\theta^2\partial^2\phi(x)$$

$$\left(\theta^\alpha\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\theta^\gamma\psi_\gamma = -\frac{1}{2}\theta^\alpha\theta_\alpha\psi^\gamma\sigma^\mu_{\gamma\dot\beta}\bar\theta^{\dot\beta}\right)$$

$$\left(\theta\sigma^\mu\bar\theta\,\theta\sigma^\nu\bar\theta = +\frac{1}{2}\theta^2\,\bar\theta\bar\sigma^\mu\sigma^\nu\bar\theta = \frac{1}{2}\theta^2\bar\theta^2\eta^{\mu\nu}\right)$$

(ϕ, ψ, F) components of chiral superfield

the other components are 0 or functions of these ...

(remember $\{\bar Q_{\dot\alpha}, \psi_\beta\} \sim \partial_\mu\phi$, $[\bar Q_{\dot\alpha}, F] \sim \partial_\mu\psi$, $[Q_\alpha, F] = 0$)

Anti-chiral superfield : $D_\alpha\bar\Phi = 0$.

<u>SUSY transformations of Φ</u>

$$\delta_{\epsilon,\bar\epsilon}\Phi = i(\epsilon Q + \bar\epsilon\bar Q)\Phi$$

in the new coordinate system $(x,\theta,\bar\theta) \longrightarrow (y,\theta,\bar\theta)$

$$\begin{cases} \dfrac{\partial}{\partial\theta^\alpha} = \dfrac{\partial}{\partial\theta^\alpha} + \dfrac{\partial y^\mu}{\partial\theta^\alpha}\dfrac{\partial}{\partial y^\mu} = \partial_\alpha + i\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\dfrac{\partial}{\partial y^\mu} \\ \dfrac{\partial}{\partial\bar\theta^{\dot\alpha}} = \dfrac{\partial}{\partial\bar\theta^{\dot\alpha}} + \dfrac{\partial y^\mu}{\partial\bar\theta^{\dot\alpha}}\dfrac{\partial}{\partial y^\mu} = \bar\partial_{\dot\alpha} - i\theta^\beta\sigma^\mu_{\beta\dot\alpha}\dfrac{\partial}{\partial y^\mu} \\ \dfrac{\partial}{\partial x^\mu} = \dfrac{\partial y^\nu}{\partial x^\mu}\dfrac{\partial}{\partial y^\nu} = \dfrac{\partial}{\partial y^\mu} \end{cases}$$

$$Q_\alpha \to Q'_\alpha = -i\partial_\alpha$$

$$\bar Q_{\dot\alpha} \to \bar Q'_{\dot\alpha} = i\bar\partial_{\dot\alpha} + 2\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\frac{\partial}{\partial y^\mu}$$

$$\delta_{\epsilon,\bar\epsilon}\Phi = i(\epsilon Q + \bar\epsilon\bar Q)\cdot\Phi = i\epsilon^\alpha\left(-i\frac{\partial\Phi}{\partial\theta^\alpha}\right) - i\bar\epsilon^{\dot\alpha}\left(i\cancel{\frac{\partial\Phi}{\partial\bar\theta^{\dot\alpha}}} + 2\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\frac{\partial\Phi}{\partial y^\mu}\right)$$

$\Phi(y,\theta)$

$$= \sqrt{2}\,\epsilon\psi - 2\epsilon\theta F + 2i(\theta\sigma^\mu\bar\epsilon)(\partial_\mu\phi + \sqrt{2}\,\theta\partial_\mu\psi)$$

$$\begin{cases} \delta\phi = \sqrt{2}\,\epsilon\psi \\ \delta\psi_\alpha = \sqrt{2}\,(i\sigma^\mu_{\alpha\dot\alpha}\bar\epsilon^{\dot\alpha}\partial_\mu\phi - \epsilon_\alpha F) \\ \delta F = \sqrt{2}\,i\,\partial_\mu\psi^\alpha\sigma^\mu_{\alpha\dot\alpha}\bar\epsilon^{\dot\alpha} \end{cases}$$