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Website: <http://www.physik.uzh.ch/en/teaching/PHY519/>

Exercise 1 [A spinning rod]

(a) The time dependent parts of the reduced quadrupole moment tensor are

$$Q_{xx} = \mu \cos^2(\omega t) \cdot 2 \int_0^{l/2} x^2 dx = \frac{\mu l^3}{12} \cos^2 \omega t = \frac{Ml^2}{24} \cos(\omega t) + C_1 \quad (1)$$

$$Q_{yy} = \frac{Ml^2}{24} \cos 2\omega t + C_2 \quad (2)$$

$$Q_{xy} = Q_{yx} = \frac{Ml^2}{24} \sin 2\omega t. \quad (3)$$

(b)

$$P_{GW} = L_{GW} = \frac{1}{5} \langle \ddot{Q}_{jk} \ddot{Q}_{jk} \rangle = \frac{(2\omega)^6}{5} \left(\frac{Ml^2}{24} \right)^2 \langle 2 \cos^2 2\omega t + 2 \sin^2 2\omega t \rangle = \frac{2}{45} \omega^6 M^2 l^4, \quad (4)$$

with $G = c = 1$.

(c) Balancing the electrostatic and centripetal forces on the charges,

$$e \nabla \phi = rm\omega^2, \quad (5)$$

where ϕ is the electrostatic potential, and e and m are the charge and mass of an electron. Maxwell's Gauss's law equation tells us that the negative gradient of the potential is the density, so

$$\rho \sim -\nabla^2 \phi \sim -\frac{m}{e} \omega^2. \quad (6)$$

So over the whole volume lA , the charge is $(m/e)\omega^2 lA$, where A is the rod's x-section. Thus the rod becomes an electric quadrupole with quadrupole moment $Q_E \sim (m/e)\omega^2 l^3 A$.

(d) The electric quadrupole radiation due to this induced moment is approximately $\omega^6 \cdot Q_E^2$, or $\omega^{10} (m/e)^2 l^6 A^2$.

(e) The ratio of electromagnetic damping to gravitational damping is then

$$\frac{L_{EM}}{L_{GW}} \sim \frac{\omega^{10} (m/e)^2 l^6 A^2}{\omega^6 \rho^2 A^2 l^6} = \left(\frac{(m/e)\omega^2}{\rho} \right)^2. \quad (7)$$

In $G = c = 1$ units, $(m/e) \approx 0.5 \cdot 10^{-21}$, $\rho (10g/cm^3) \approx 10^{-27}$, and $\omega (1kHz) \approx 1/3 \cdot 10^{-7}$, yielding

$$\frac{L_{EM}}{L_{GW}} \sim 10^{-18}. \quad (8)$$