PHY127 FS2026

Prof. Ben Kilminster
Lecture 11
May 15th, 2026

from PHY 117:
angular momentum
torque
magnetic moment

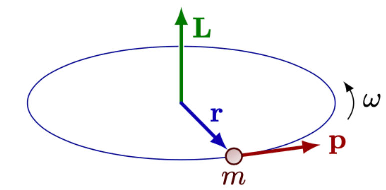
NMR → MRI
(spin precession of nuclei)

Angular momentum reminder

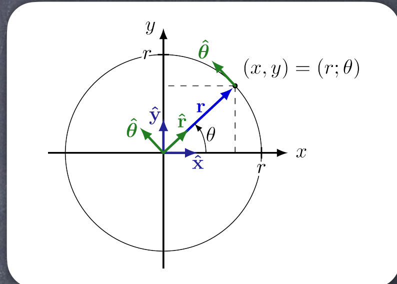
Linear motion

momentum $\vec{p} = m\vec{v}$

Angular motion



(a) Uniform circular motion.

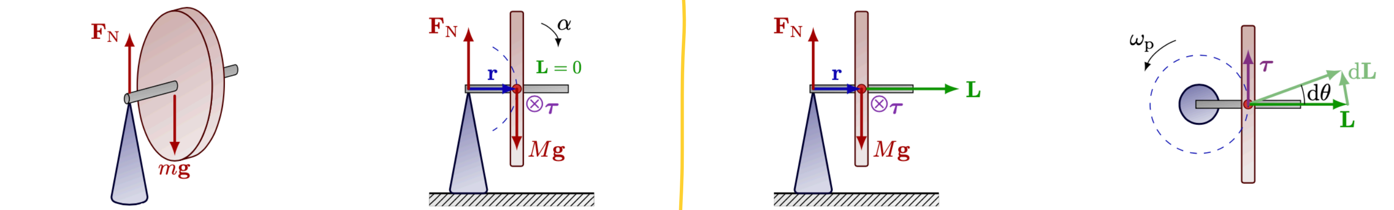


Interesting consequences

Interesting consequences

static | rotating

from above



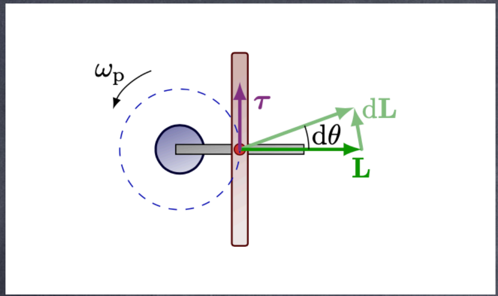
(a) The handle allows the disk to spin around its axis and around the pivot.

(b) The disk does not spin, and it will fall down due to an unbalanced torque $\boldsymbol{\tau}$.

(c) The disk spins, creating an angular momentum $\mathbf{L}$. Torque $\boldsymbol{\tau}$ will cause a precession.

(d) Torque $\boldsymbol{\tau}$ perpendicular to angular momentum $\mathbf{L}$, will only change its direction.

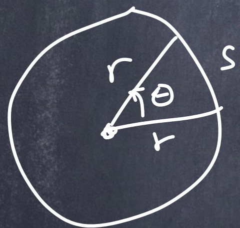
θ changing



Aside: why is $dL = L\,d\theta$?

Circumference: $C = 2\pi r$

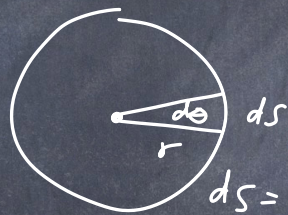
A part of the circumference, s



$s = r\theta$

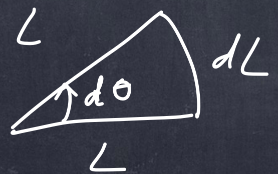
$\rightarrow$

For small angles, $d\theta$



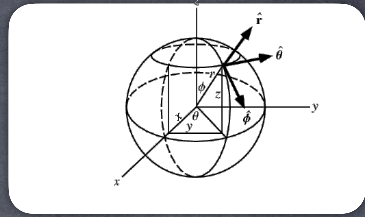
$ds = r\,d\theta$

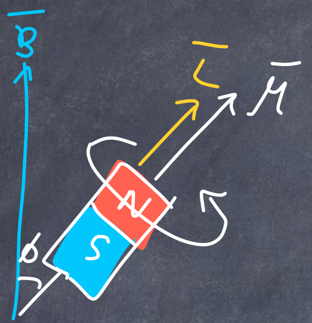
we can use the same arguments to get $dL = L\,d\theta$

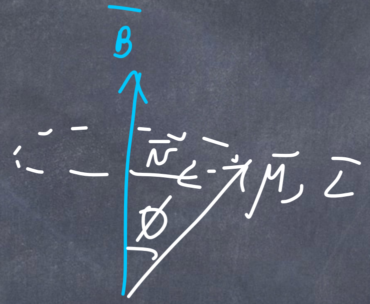


$dL = L\,d\theta$

The same effect occurs for a magnet spinning in a magnetic field

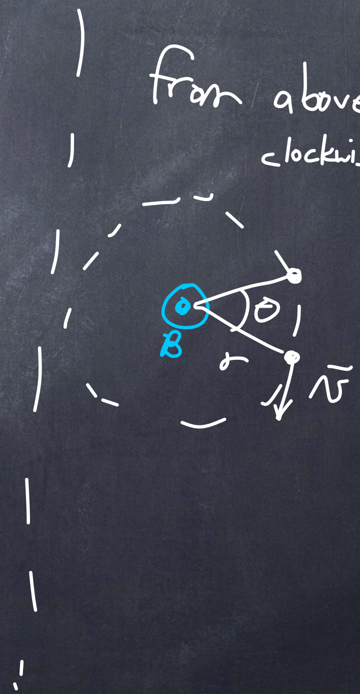


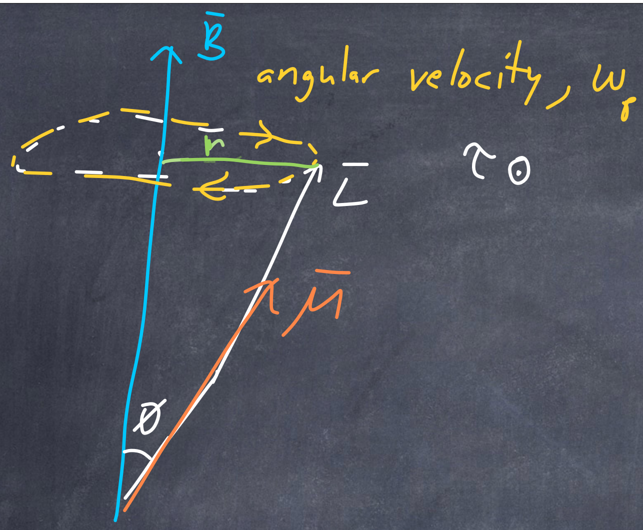




magnet has angular momentum and magnetic moment, which are parallel vectors

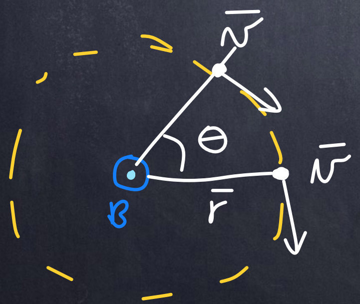
from above: clockwise



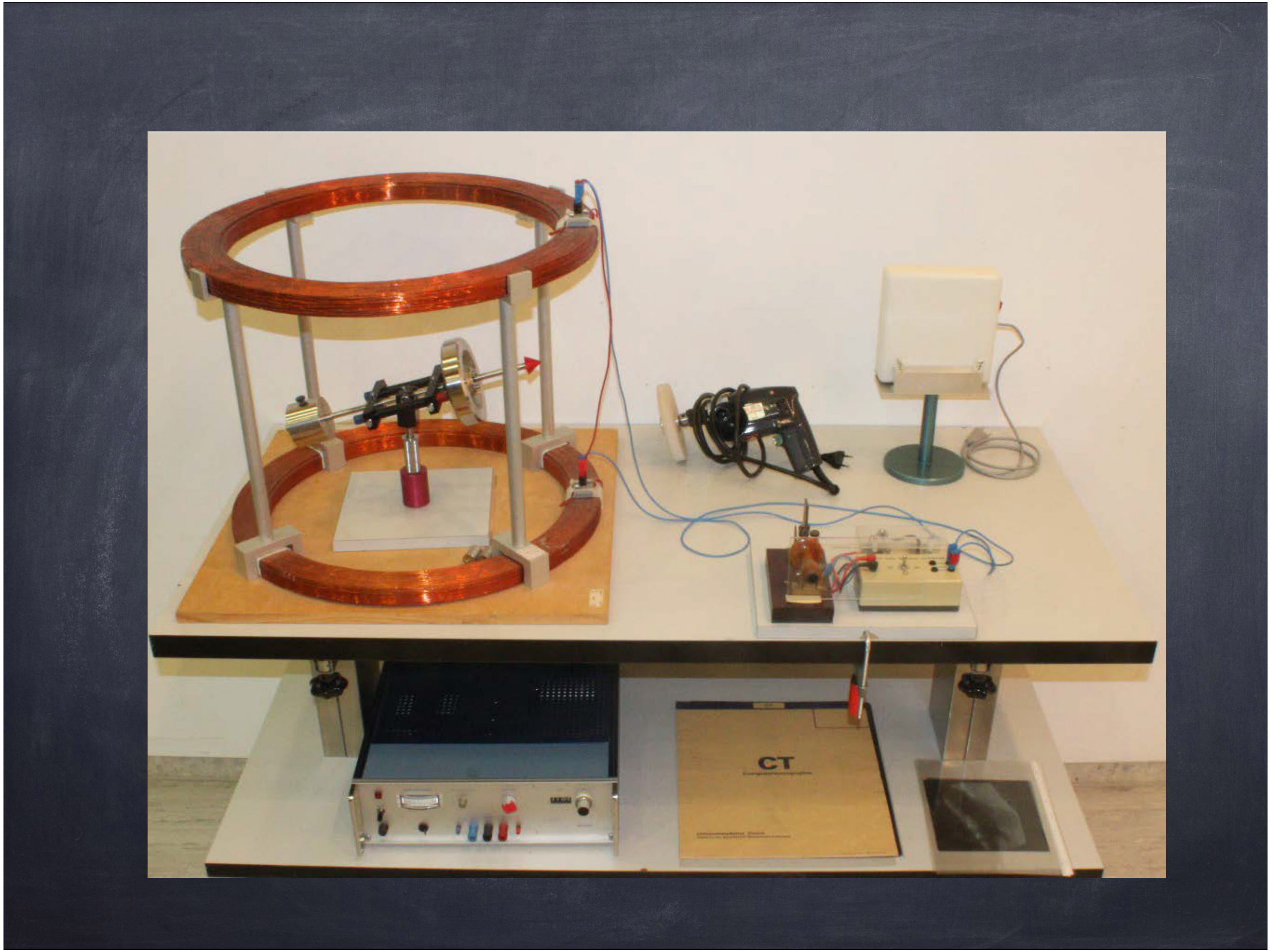


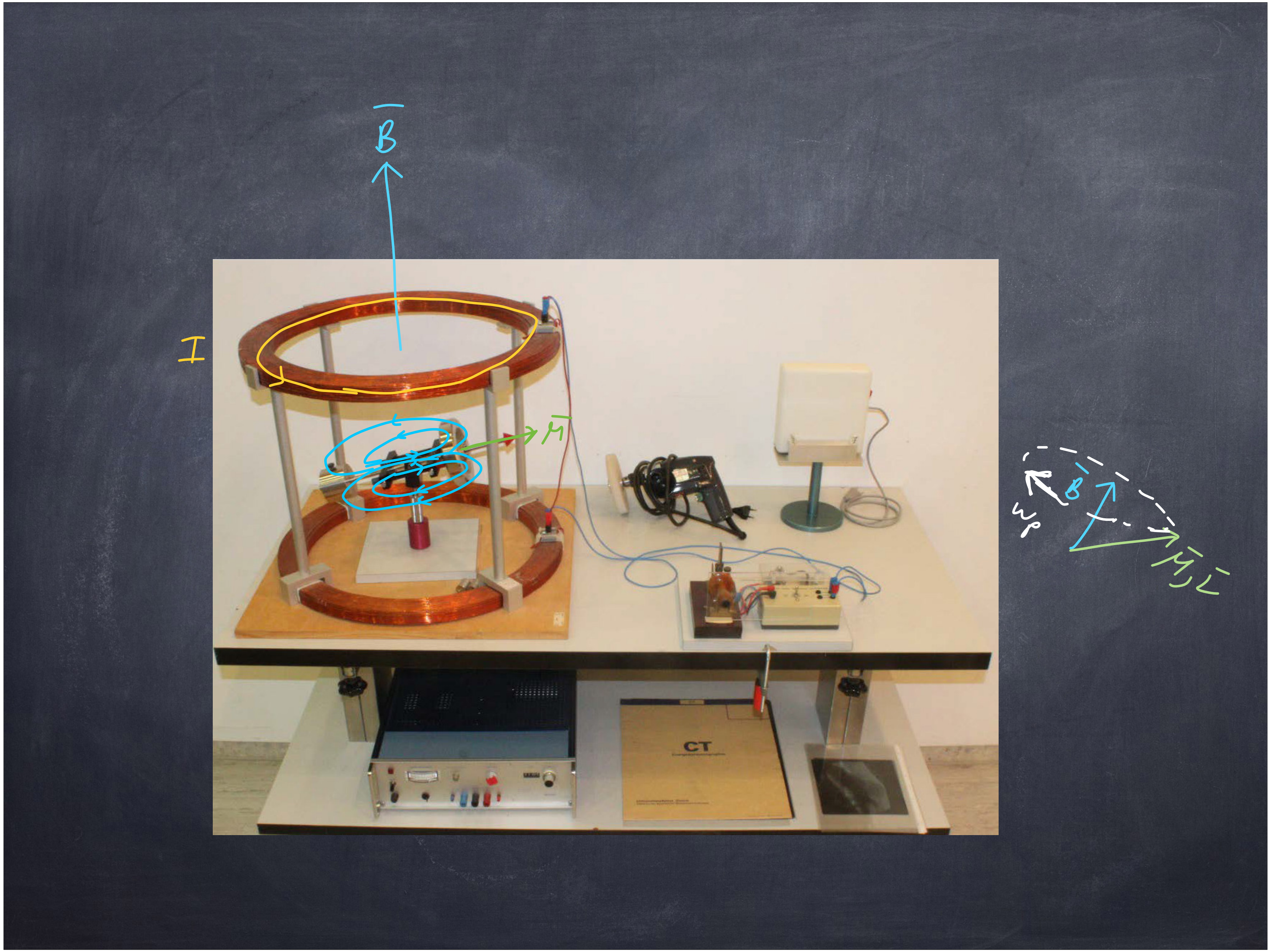
what is ω_p ?

From above:



$$v = r\frac{d\theta}{dt} = r\,\omega_p \quad (-\hat{\theta})$$


CT

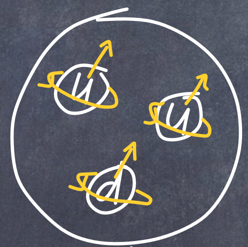

B
I
M
CT
B
M, L

But wait... we know that subatomic particles have angular momentum and electric charge (they have a magnetic moment)

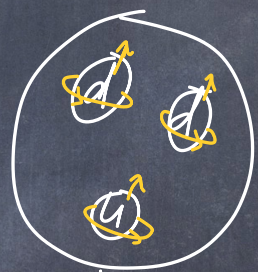
Could this precession happen at a quantum level?

Intrinsic angular momentum (spin)

① protons + neutrons are composed of quarks.



proton



neutron

angular momentum S_z quantized by an integer number of $\frac{1}{2}\hbar$ (h: Planck's constant, $\hbar = \frac{h}{2\pi}$

reminder:

$S_z = m_s \hbar$, $m_s = \pm\frac{1}{2}$

$|\bar{S}| = \sqrt{s(s+1)}\,\hbar$

S_z is z-component of S vector

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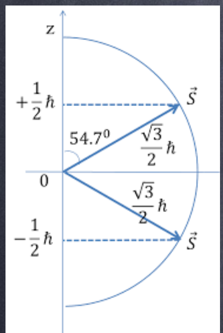
$|\bar{S}| = \sqrt{s(s+1)}\,\hbar$

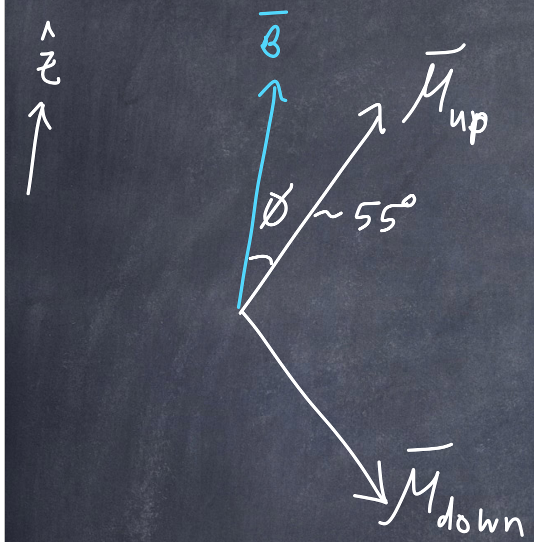
S_z is z-component of S vector

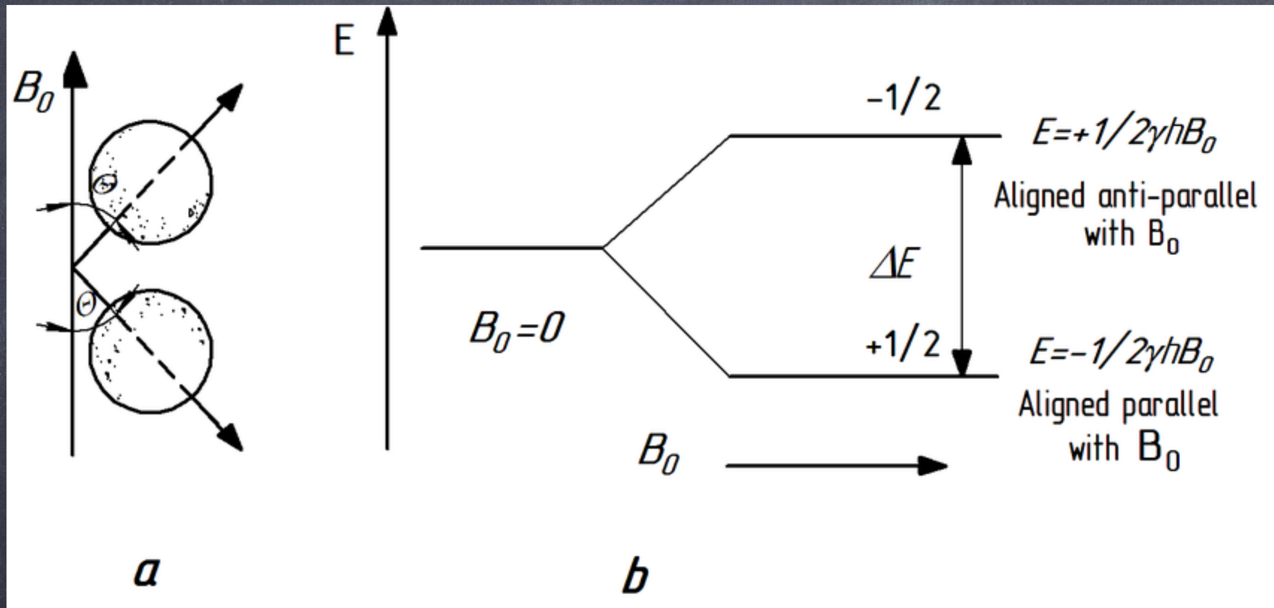
nucleus	(s) spin quantum number	(S_z) spin projection	($\lvert\bar{S}\rvert$) spin vector magnitude
proton	$\frac{1}{2}$	$\pm\frac{1}{2}\hbar$	$\frac{\sqrt{3}}{2}\hbar$
neutron	$\frac{1}{2}$	$\pm\frac{1}{2}\hbar$	$\frac{\sqrt{3}}{2}\hbar$
deuteron (^2H)	1	$\pm\hbar$	$\sqrt{2}\,\hbar$
Helium (He)	0	0	0
^{12}C	0	0	0
^{13}C	$1/2$	$\pm 1/2\,\hbar$	$\frac{\sqrt{3}}{2}\hbar$
^{14}N	1	$\pm\hbar$	$\sqrt{2}\,\hbar$
^{16}O	0	0	0
^{19}F	$1/2$	$\pm\frac{1}{2}\hbar$	$\frac{\sqrt{3}}{2}\hbar$
^{31}P	$1/2$	$\pm 1/2\,\hbar$	$\frac{\sqrt{3}}{2}\hbar$

If spinning charged particle, you have a magnetic moment.


z
$+\frac{1}{2}\hbar$
$\vec{S}$
54.7^0
$\frac{\sqrt{3}}{2}\hbar$
0
$\frac{\sqrt{3}}{2}\hbar$
$-\frac{1}{2}\hbar$
$\vec{S}$

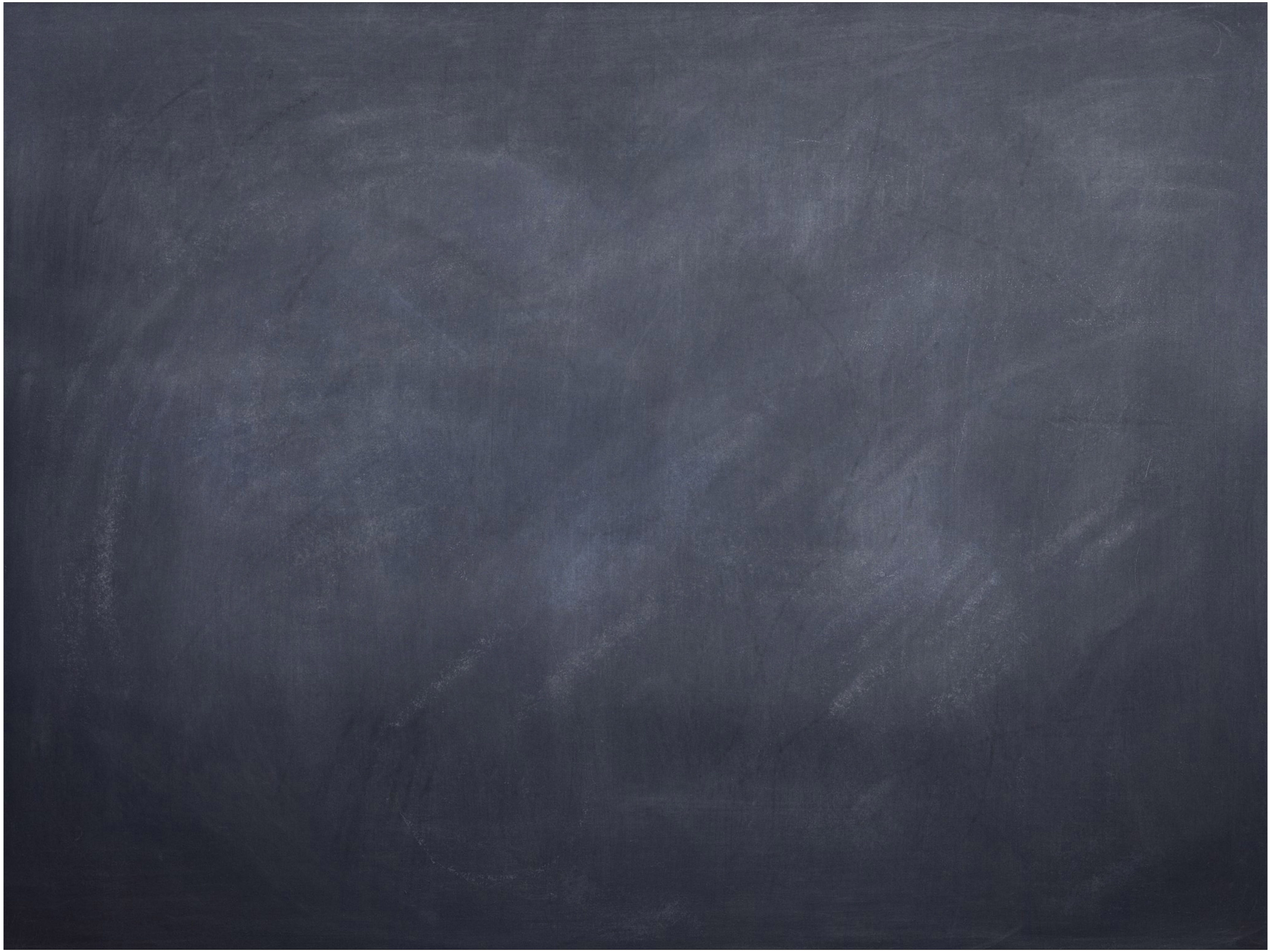
The fact that μ is not aligned with $\bar{B}$ (external) gives the nucleus a potential energy, U.



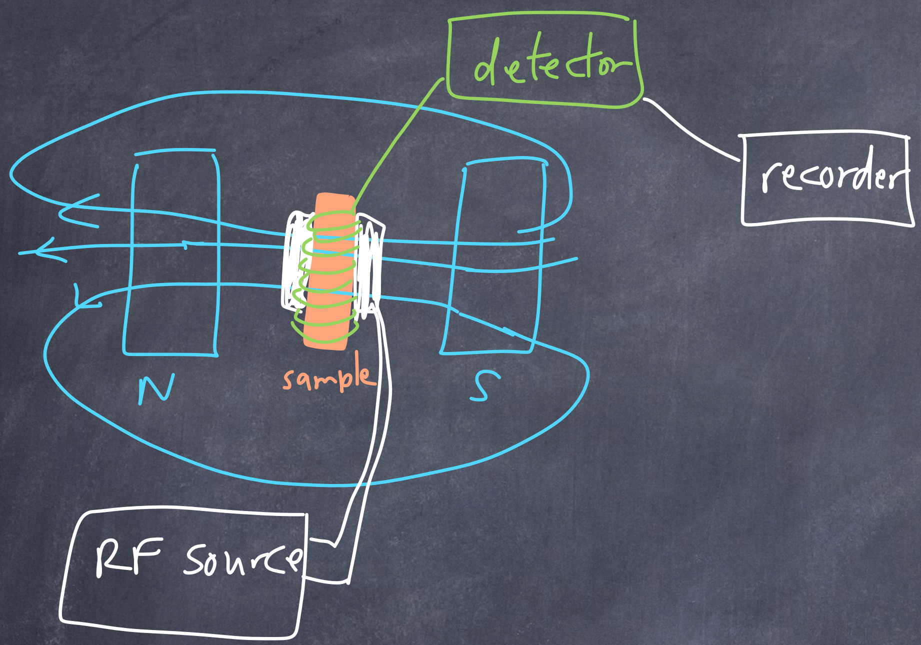


B_0
E
-1/2
$E=+1/2\gamma hB_0$
Aligned anti-parallel with B_0
ΔE
$B_0=0$
+1/2
$E=-1/2\gamma hB_0$
Aligned parallel with B_0
B_0
a
b

Some numbers: For $B = 1\,T$ + nucleus of hydrogen (1 proton) with nuclear spin of $\frac{1}{2}\hbar$



How to detect NMR radiation.



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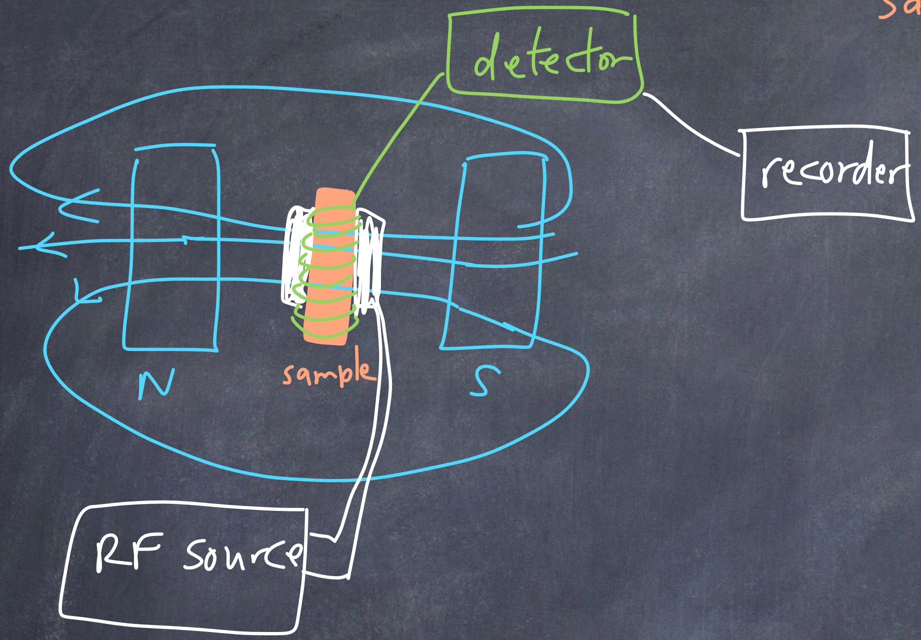

sample is placed in a magnetic field

detector is a solenoid that can measure changes in electric current

Next time: how to use NMR to determine molecular structure