University of Zurich[UZH]

Solid State Physics
Exercise Sheet 6

FS17
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Discussion on 5th April

Due on 12th April

Exercise 1 *Neutron and photon dispersion relations*

Particles have dispersion relations. For example, the energy E of electrons and neutrons is given by:

$$E = \frac{\hbar^2 k^2}{2m} \tag{1}$$

where m is the particle mass and $p = \hbar k$ is the momentum. Photons (light) by contrast have the following dispersion:

$$E = \hbar c k \tag{2}$$

where c is the speed of light and $\hbar = h/(2\pi)$ with h being Planck's constant.

a) For a neutron moving with 2 km/s, what is its kinetic energy E (in meV)? (Hint: look up the mass of a neutron) What is its wavelength $\lambda = 2\pi/k$? Derive the following relation for neutrons:

$$\lambda[\text{Å}] = \frac{9.045}{\sqrt{E[\text{meV}]}}. \tag{3}$$

b) With the wavelength calculated in (a), calculate the energy of a photon.

c) To experimentally study excitations such as phonons, meV energy resolution is needed. Let the instrumental resolving power be defined by $\Delta E/E$ where ΔE is the energy resolution. If $\Delta E = 1$ meV, what is the resolving power of neutrons and photons with a wavelength of 4 Å.

Exercise 2 *Measuring phonons*

In a previous lecture, we discussed the recent discovery of high-temperature superconductivity in H_2S. We found that under the high pressure needed to crystallize this gas, the crystal structure is bcc.

a) Is the (200) Bragg peak allowed (non-zero) or forbidden (zero) by the structure factor for a monoatomic crystal?

b) If the conventional lattice parameter is 3 Å, and we use neutrons moving with 2 km/s, what is the scattering angle of the (200) Bragg peak (hint: use Bragg's law) and what is the energy of the scattered neutrons?

c) What is the expectation for the phonon branches (dispersions) of a mono atomic bcc lattice? Can we expect optical phonons? What is the expectation for H_2S?

d) Let's assume that the phonon velocity of an acoustic branch is 4 meV per reciprocal lattice unit ($2\pi/a$) in the long wavelength limit $k \to 0$. What is the phonon energy at $\mathbf{Q} = (2.1,0,0)$ (where $\mathbf{Q}$ is in reciprocal units)?

e) If we fix the analyser at our triple axis instrument to measure neutrons with energy 7 meV, what should be the energy of the incident neutrons to measure the phonon at $\mathbf{Q} = (2.1,0,0)$?

Exercise 3 *Singularity in density of states*

a) From the dispersion relation derived in the lecture for a monoatomic linear lattice of N atoms with nearest neighbour interactions, show that the density of modes is

$$D(\omega) = \frac{2N}{\pi} \cdot \frac{1}{\sqrt{\omega_{\mathrm{m}}^2 - \omega^2}}, \tag{4}$$

where ω_{m} is the maximum frequency.

b) Make a plot of equation (4).

c) Suppose that an optical phonon branch has the form $\omega(K) = \omega_0 - AK^2$, near $K = 0$ in three dimensions. Show that $D(\omega) = \left(\frac{L}{2\pi}\right)^3 \left(\frac{2\pi}{A^{3/2}}\right) (\omega_0 - \omega)^{\frac{1}{2}}$ for $\omega < \omega_0$ and $D(\omega) = 0$ for $\omega > \omega_0$. Here the density of modes is discontinuous.