

PHY 117 HS2026

Week 3, Lecture 1

Sept. 29th, 2026

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Today:

Energy

Energy conservation

Kinetic energy

Potential energy

Work

New script 1, v3 : Thanks
to Dean Mark for
finding typos!

Please do Quiz #2
on OLAT if you
haven't yet.
Quiz #3 you can do after
tomorrow's lecture.

What is energy?

Today we learn about 3 types:

- 1) work
- 2) kinetic energy
- 3) potential energy

Let's start by examining units of energy.

reminder:
$$\underset{\text{force}}{[N]} = \left[\frac{\text{kg} \cdot \text{m}}{\text{s}^2} \right]$$

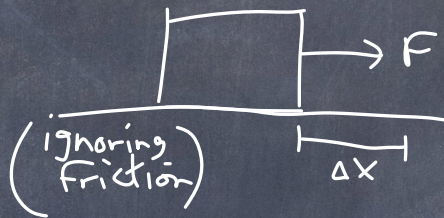
$$\underset{\substack{\text{energy} \\ \text{"Joules"}}}{[J]} = \left[\frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \right] = \underset{\text{force} \cdot \text{distance}}{[N \cdot \text{m}]}$$

So energy has units of force * distance

Work is exactly this:

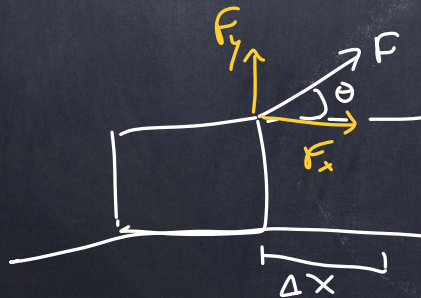
The "work" done by a force is $W = F \Delta x$
(for the case when $\vec{F}$ is in the direction of $\vec{x}$)

Simple case:



$$W = F \Delta x$$

If $\vec{F}$ is not parallel to $\Delta \vec{x}$, we need to find the component of $\vec{F}$ that is parallel.



$$W = F_x \Delta x = (F \cos \theta) \Delta x$$

(so $F_x \parallel \Delta x$)

when the force is in the same direction as the motion, W is (+)

Reminder: a net force causes acceleration: $\Sigma F = ma$

Remember $v^2 = v_0^2 + 2a\Delta x \Rightarrow a = \frac{v^2 - v_0^2}{2\Delta x}$

$$\text{work} = F_x \Delta x = (ma)\Delta x = m \left(\frac{v^2 - v_0^2}{2\Delta x} \right) \Delta x = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

the work-energy theorem:

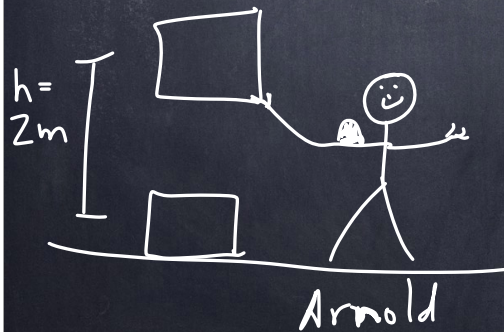
$$W_{\text{TOTAL}} = \underbrace{\frac{1}{2}mv_F^2}_{\text{final}} - \underbrace{\frac{1}{2}mv_i^2}_{\text{initial}} = K_f - K_i = \Delta K$$

$$K = \frac{1}{2}mv^2 = \text{kinetic energy}$$

Kinetic energy is the energy of movement

- Notes:
- K is a scalar, no direction
 - K is always positive or zero
 - ΔK can be negative (if object slows down)
 - consider each force separately, and it the work it does.

Example:



Arnold lifts a 5kg block to $h = 2\text{m}$, using 500 N of force then let's go of block.

- 1) what is the work done by Arnold?
- 2) what is the work done by gravity?
- 3) what is the final velocity of the block when he lets go?

There are 2 forces at work, Arnold + gravity.
Initially, velocity is zero. Distance is 2 m.

so far, we have a constant force.

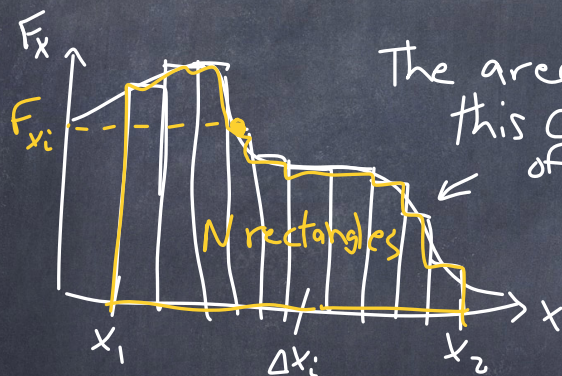
$$W = F \Delta x$$

Note: $F \Delta x$ is the area of the curve of F vs. x

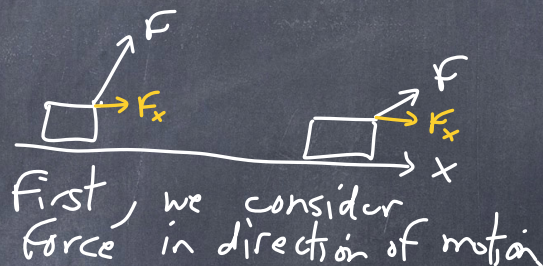


$$\text{Area} = F \Delta x = \text{Work}$$

what if our force is changing?



The area of this curve is approximation of work.



First, we consider force in direction of motion

The total work is

$$W = \sum F_{x_i} \Delta x_i \quad \left] \begin{matrix} N \\ \text{rectangles} \end{matrix} \right.$$

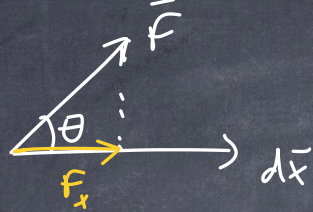
As $\lim_{\Delta x_i \rightarrow 0}$

(# rectangles goes to ∞)

$$W = \int_{x_1}^{x_2} F_x dx$$

Precise calculation of area = work

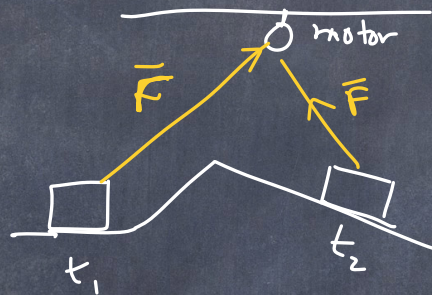
so far, we considered objects moving in x .



$$W = \int_{x_1}^{x_2} F_x dx = \int_{x_1}^{x_2} (F \cos \theta) dx$$

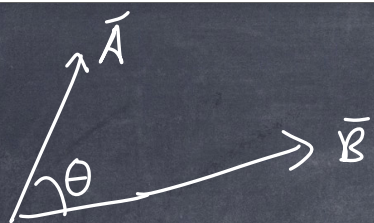
But movement direction can change:

Example:



we want to consider the force in direction of motion.

The "dot product" of force + direction gives us this.



The dot product of $\vec{A} + \vec{B}$ is
 $\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$ (It is a scalar quantity)

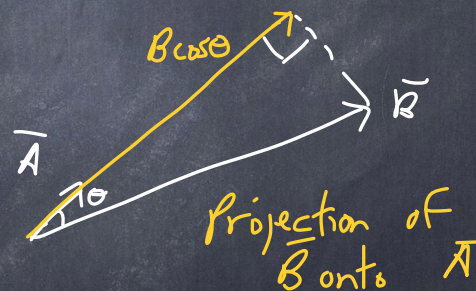
A dot product multiplies the parallel components of 2 vectors. $|\vec{A}| = A = \text{magnitude of } \vec{A} = \sqrt{A_x^2 + A_y^2 + A_z^2}$
 The dot product projects one vector onto another.

Two ways to think about it:

Projection of $\vec{A}$ onto $\vec{B}$



$$\vec{A} \cdot \vec{B} = (\underbrace{A \cos \theta}_{\text{magnitude}}) B = AB \cos \theta$$



$$\vec{A} \cdot \vec{B} = (B \cos \theta) (A) = AB \cos \theta$$

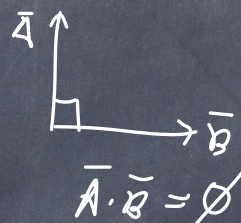
Both give the same answer: $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

$\vec{A} \cdot \vec{B}$ can also be calculated using vector components:

$$\boxed{\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z}$$

where $\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$ and $\vec{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}$

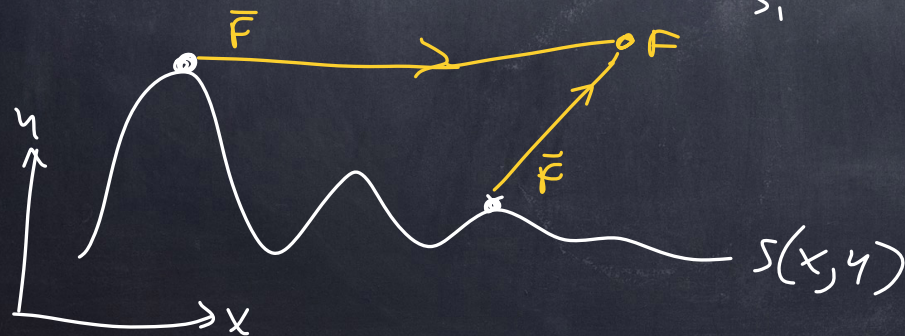
consider (1) $\vec{A}$
 $\vec{B}$
 $\vec{A} \cdot \vec{B} = AB$
 maximal



So our formula in general is

$$W = \int_{\vec{s}_1}^{\vec{s}_2} \vec{F} \cdot d\vec{s}$$

$\vec{s}$ is the path of the object.



($d\vec{s}$ would be $d\vec{x}$ if object moves only in $\hat{x}$ direction)

The work done on a system can be stored as potential energy, U .

Potential energy can change: $\Delta U = U_f - U_i = U_2 - U_1$

Potential energy and work are related.

$$-\Delta U = W = \int_{s_1}^{s_2} \vec{F} \cdot d\vec{s}$$

Notice that $\Delta U = -W$

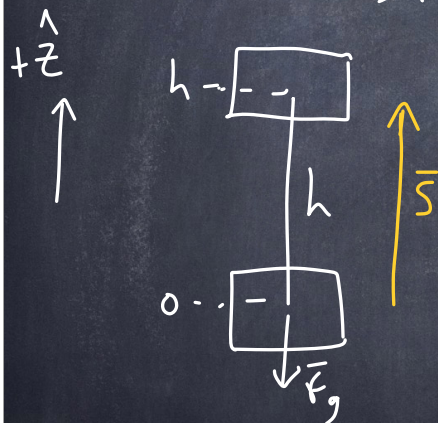
The potential energy is associated with a particular force. The work here is the work done by that force.

Different forces have different associated potential energies.

An example is the gravitational potential energy U_g that is associated with the gravitational force.

$$W = \int \vec{F} \cdot d\vec{s}$$

Example: Assume we lift an object of mass, m , to a height, h .



F_g is opposite $\vec{s}$,
so W is $(-)$.

Force of gravity is $F_g = mg$, $\vec{F}_g = -mg \hat{z}$
 $d\vec{s} = dz \hat{z}$

Work done by gravity: $W_g = \int \vec{F}_g \cdot d\vec{s}$

$$W_g = \int_0^h (-mg \hat{z}) \cdot dz \hat{z} = \int_0^h -mg dz$$

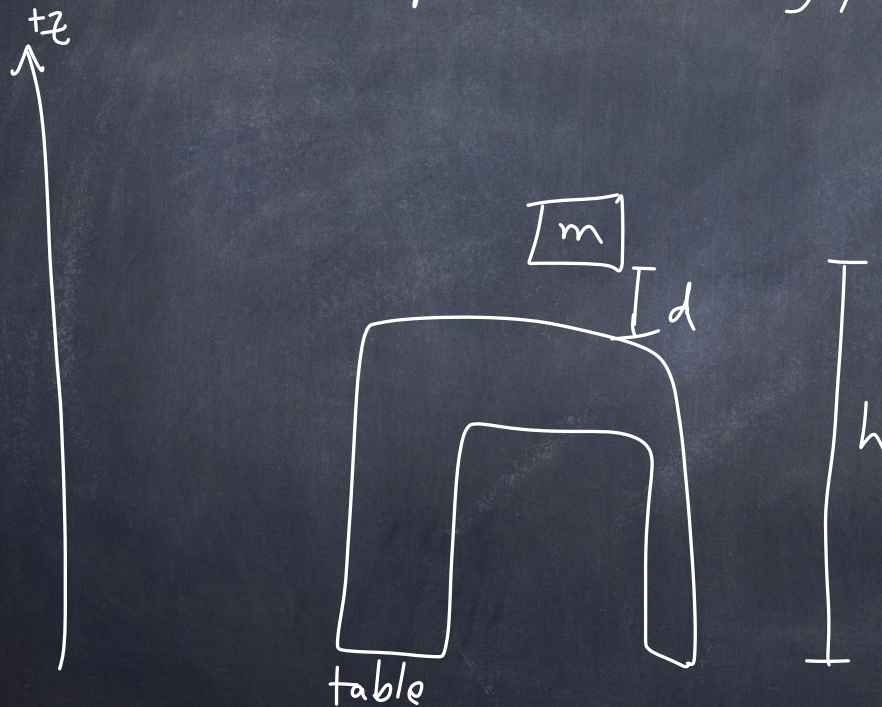
$$W_g = -mgz \Big|_0^h = -mgh \quad (-)$$

change in potential
energy

$$\Delta U_g = -W_g = -(-mgh) = mgh \quad (+)$$

In general, gravitational potential energy
is $U_g = mgz$
↑
height

But potential energy is relative.



$$U_g = mgz$$

U_g relative to the floor:
 $U_g = mgh$

U_g relative to the table:
 $U_g = mgd$

conservation of energy

$$E_{\text{before}} = E_{\text{after}}$$

always true.

If we consider all sources of energy, then the sum of energies is conserved, before & after anything happens.

For conservative forces, potential energy plus kinetic energy is conserved.

$$(K + U)_{\text{before}} = (K + U)_{\text{after}}$$

only true for conservative forces

summary so far,

Energy is conserved

$$E_{\text{before}} = E_{\text{after}}$$

$$\text{Potential energy from gravity} = mgh = U$$

$$\text{Kinetic energy} = \frac{1}{2}mv^2 = K$$

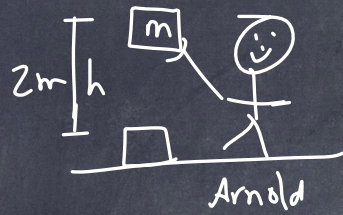
work-energy
theorem

$$W_{\text{TOT}} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}m_i v_i^2$$

potential energy
+
work relation

$$W = -\Delta U$$

$$W = \int_{s_i}^{s_f} \vec{F} \cdot d\vec{s}$$

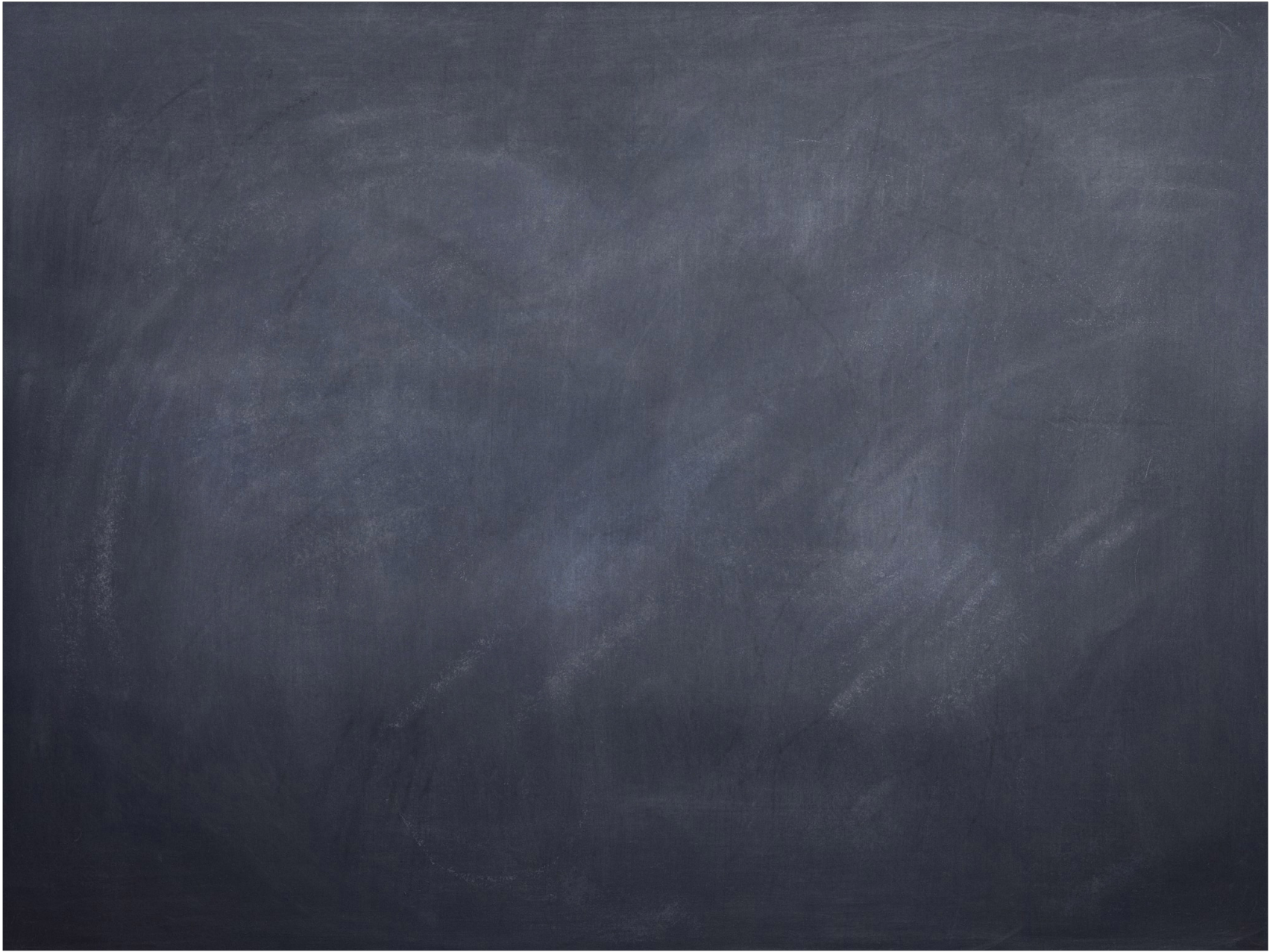


$$m = 5 \text{ kg}$$

Consider Arnold again. If he lifts the block with 50 N of force, to a height of $2m$, and then let go. How fast will it travel when it hits the ground?

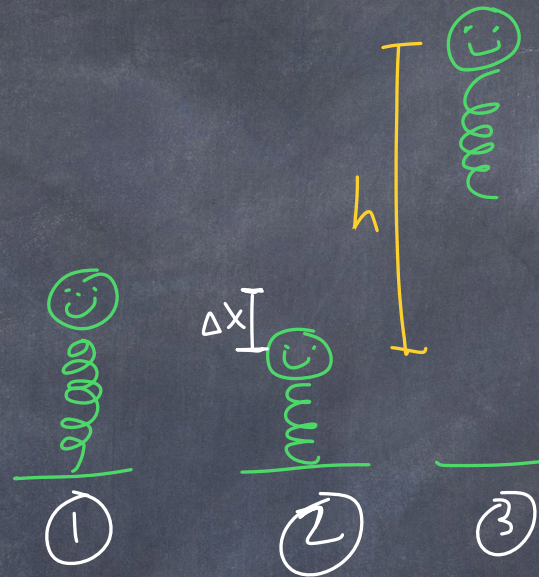
Arnold has done $+100\text{ J}$ of work, while gravity has done -100 J of work. Where did Arnold's 100 J go?

Now Arnold drops the weight, how fast is it
when (just before) it hits the ground?





How high does the grasshopper jump?



- ① grasshopper just sitting there at equilibrium.
- ② additional compression, Δx from equilibrium.
- ③ grasshopper releases spring, jumps a height, h .

Three types of energy here:

U_s : potential energy of spring (from spring force)

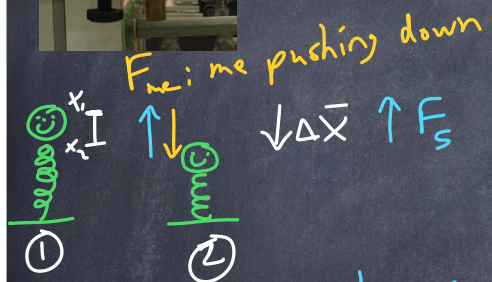
U_g : potential energy of gravity (from gravity force)

K : kinetic energy

work can change these energies.



Focus on spring energy, U_s .
we know the force on a spring:



$$\Delta x = x_2 - x_1 < 0$$

Force of grasshopper
spring pushing
back at me

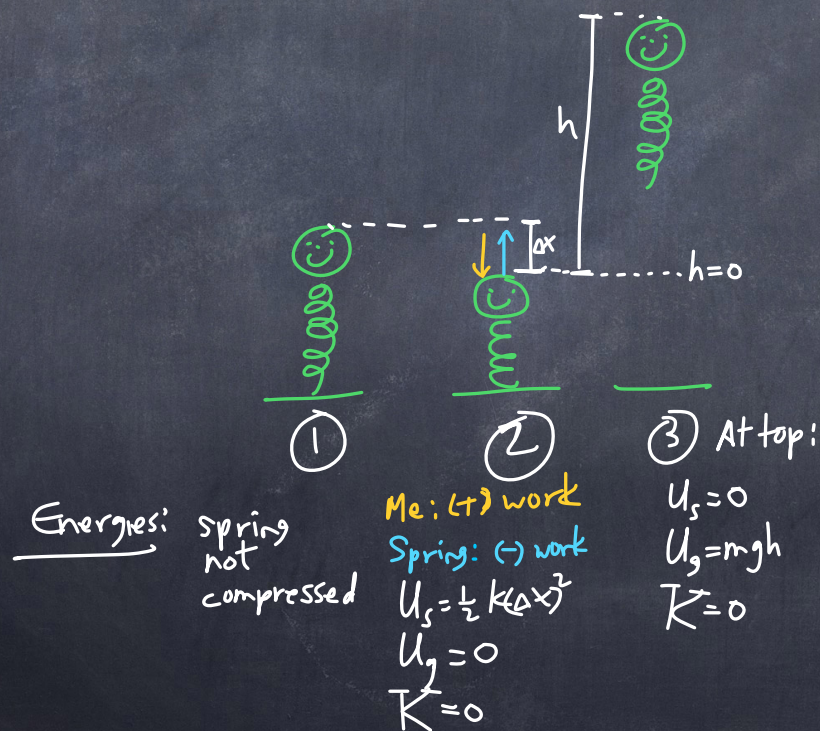
so far: work done by spring:
is $W_s = -\frac{1}{2}k(\Delta x)^2$

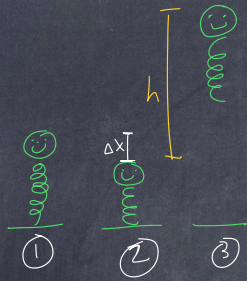
work done by me:
 $W_{me} = -W_s = +\frac{1}{2}k(\Delta x)^2$

Negative work by the spring
gives it positive potential
energy

$$\Delta U_{\text{spring}} = -W_{\text{spring}} = \frac{1}{2}k(\Delta x)^2$$

Keep in mind $\begin{cases} \Delta U = U_f - U_i \\ U_f = \Delta U + U_i \end{cases}$





Apply energy conservation:

$$\text{Total energy ②} = \text{Total energy ③}$$