

PHY 117 HS2026

Today: corrections to ideal gas law
heat and temperature and work
internal energy
equipartition theorem
pressure, volume and work
thermal energy transfer processes

Week 6, Lecture 2

Oct. 21st, 2026

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The ideal gas law is an approximation, assuming the molecules are point particles (no size) and it neglects the force between the molecules.

More accurate

$$\left(P + \frac{an^2}{V^2}\right) (V - bn) = nRT$$

Van der Waals equation

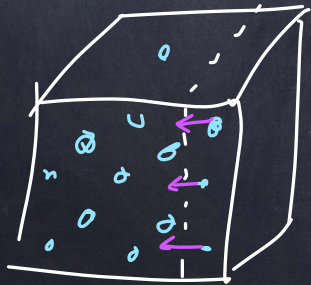
correction to the volume:

b : volume of 1 mole of molecules

bn : volume of n moles of molecules

a : constant that depends on the attractive molecular forces.

There is a decrease in the pressure against the walls of the container due to molecular forces



each molecule near the right wall feels a net attractive force from the molecules on the left.

The force pulling on one molecule near the wall is proportional to the density of molecules $\sim \frac{n}{V}$

The number of molecules near the wall is also proportional to the density $\sim \frac{n}{V}$

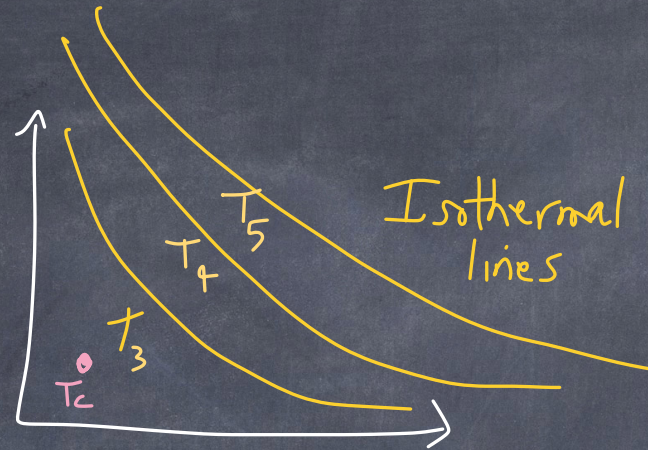
So the total force is ^{proportional to} $\frac{n}{V} \cdot \frac{n}{V} = \frac{n^2}{V^2}$
(and pressure is $\frac{\text{Force}}{\text{area}}$)

Note: Is the pressure more or less due to the attractive forces?

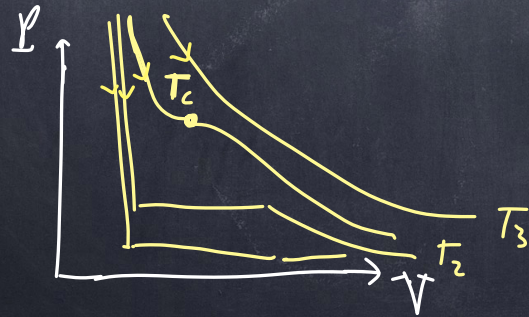
$$P = \frac{nRT}{V - bn} - \underbrace{\frac{an^2}{V^2}}_{\text{pressure decreases}}$$

pressure decreases

Ideal
gas
law

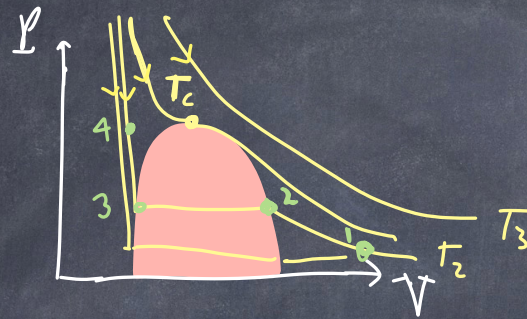


The ideal gas law + Van der Waal equation
are only valid above a certain temperature,
 T_c : critical temperature.



This is what we
see experimentally.

Above T_c , the behavior of the gas is described by the Van der Waals equation. But below T_c , we see something different.



Each curve is for a constant temp

Starting at low pressure & high volume at ①, we compress the gas. The pressure rises to ② as expected by the ideal gas law.

But between ② and ③, pressure stops rising, and between ③ + ④, it increases sharply. Why?

② → ③: gas begins to liquefy (both gas & liquid exist)

③ → ④: only liquid exists. Since liquid is incompressible, pressure increases rapidly, but the volume changes little.

Relationship between temperature, heat, internal energy.

Heat is energy transferred because of a temperature difference. We use the symbol Q for the energy transferred as heat.

- 1) No work done
- 2) No phase change

$$Q = \Delta U = mc\Delta T$$

↑ ↑ ↑
heat added change mass
in [J] in internal of a
 energy substance
 [kg]

c : specific heat of a substance

ΔT : temperature change

$$\Delta T = T_f - T_i$$

[K]
[°C]

$$c: \left[\frac{\text{J}}{\text{kg} \cdot \text{K}} \right]$$

(In a few slides, we will find $Q = \Delta U + W$
But, if no work is done, and no phase change, then $Q = \Delta U = mc\Delta T$)

Substances	specific heat $c \left[\frac{\text{J}}{\text{kg} \cdot \text{K}} \right]$	molar heat capacity $C_m \left[\frac{\text{J}}{\text{mol} \cdot \text{K}} \right]$
copper	386	24.5
aluminum	900	24.2
silicon	710	42.2
water	<u>4186</u>	75.3
pine wood	1500	
oak wood	2400	

↑
forests are also
good at moderating
temperature.

(same thing ; different units)

$$\left. \begin{array}{l} Q = \overset{\text{mass}}{m} c \Delta T \\ Q = \underset{\text{\# moles}}{n} C_m \Delta T \end{array} \right\} \text{so } mc = nC_m$$

This means that
water is good at
storing heat, and
temperature changes
only slightly

A big lake moderates the
temperature nearby.
Keeping summers cooler
+ winters warmer.



Note:

These constants assume
constant pressure, i.e., P_{atm}

IF one instead assume
the substance is forced to
have constant volume,
different constants needed.

Experiment:



Heat cold metals with room-temperature water.
Each block is 1 mole. Copper, aluminium, silicon.

liquid N_2 Temp: $-196^\circ C$

water is 60 mL: #moles = $\frac{1 \text{ mol}}{18 \text{ g}} \cdot \underbrace{\frac{1 \text{ g}}{\text{mL}}}_{\text{density}} \cdot 60 \text{ mL} = 3.3 \text{ mol}$

what is final equilibrium temperature?
which cools the most?

$$Q_{\text{cu}} + Q_{\text{water}} = 0$$

$$\underbrace{(nC_m)(T_f - T_i)_{\text{cu}}}_{\substack{\text{heat gain } T_f - T_i: (+) \\ n = 1 \text{ mol}}} + \underbrace{nC_m(T_f - T_i)_{\text{water}}}_{\substack{\text{heat lost } T_f - T_i: (-) \\ n = 3.3 \text{ mol}}} = 0$$

$$T_i = -196^\circ C$$

$$C_m = 24.5 \frac{\text{J}}{\text{mol} \cdot K}$$

$$T_i =$$

$$C_m = 75.3 \frac{\text{J}}{\text{mol} \cdot K}$$

$$\frac{(nC_m)_{\text{water}}}{(nC_m)_{\text{cu}}} = \frac{(T_f - T_i)_{\text{cu}}}{(T_f - T_i)_{\text{water}}} \Rightarrow T_f = \frac{(nC_m T_i)_{\text{water}} + (nC_m T_i)_{\text{cu}}}{(nC_m)_{\text{water}} + (nC_m)_{\text{cu}}}$$

$(T_f)_{\text{water}} = (T_f)_{\text{cu}}$

Measured:

T_f copper :
 T_f aluminium :
 T_f silicon :

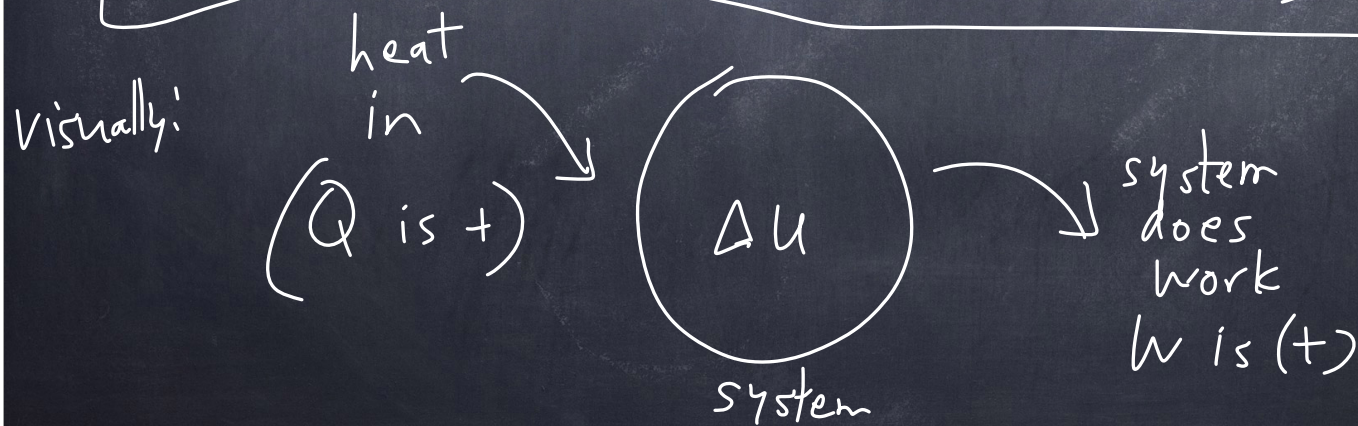
Observations:
which T_f is coldest? why?
Does expt match calculation?
If not, why?

So far, we changed a system's energy only by heating it. But there is another way to transfer energy that we know already, by doing work.

1st law of thermodynamics:
a statement of energy conservation!

$$Q = \Delta U + W$$

↑ ↑ ↑
heat added change work done
to a system in internal by the system
 energy of means $W > 0$
 the system.



For example:



Heating gas causes the piston to move out because the gas expands.

The work is (+), since the work is done by the system.

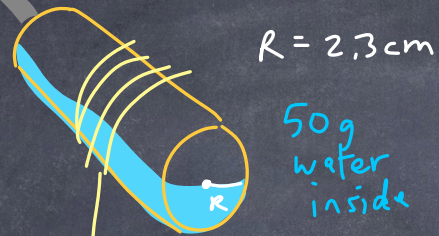
ΔU would increase (+)

Example: 3 kg of water at 80°C . We stir it with 15 kJ of energy. Also, we remove 50 kJ of heat. What is the final temperature?

The change in internal energy is (-).
The temperature will decrease.

Temperature change due
to $Q + W$ on water: the volume + pressure of water
do not change, so ΔU only changes the
temperature.

Can we use torque to increase temperature?
(friction just balancing gravity)



$F_g = Mg$
5 kg

$R = 2.3 \text{ cm}$

50g water inside

$$F_f = F_g$$

$$\vec{\tau} = \vec{r} \times \vec{F} = RMg$$

The work done by torque on the cylinder to rotate it N times $2\pi N$

$$W = \int_0^{2\pi N} \tau d\theta \quad (\text{analogue of: } W = \int \vec{F} \cdot d\vec{x})$$

$$W = \int_0^{2\pi N} RMg d\theta = [MgR\theta]_0^{2\pi N} = MgR 2\pi N$$

we want W by system!

$$W = -MgR 2\pi N$$

work done on the system.

1st law: $Q = \Delta U + W$

Q : no heat added = 0

$$\Delta U = -W = -(-MgR 2\pi N) = +MgR 2\pi N$$

ΔU is the increase in internal energy of water + copper

$$MgR 2\pi N = (mc\Delta T)_{\text{water}} + (mc\Delta T)_{\text{copper}}$$

$$\Delta U = (m_c \Delta T)_{\text{water}} + (m_c \Delta T)_{\text{copper}}$$

at equilibrium, $\Delta T_{\text{water}} = \Delta T_{\text{copper}}$ (both heat up)

$$C_w: 4186 \text{ J/kg}\cdot\text{K} \quad C_c: 386 \text{ J/kg}\cdot\text{K}$$

Approximate: $\Delta U = \Delta T (m_w C_w + m_c \overset{\text{small}}{C_c}) = mgR 2\pi N$

$$\Delta T = \frac{2\pi N mg R}{m_w C_w}$$

$$N = 100$$

$$R = 0.023 \text{ m}$$

$$m_w = 0.05 \text{ kg}$$

$$M = 5 \text{ kg}$$

$$\Delta T = 3^\circ\text{C} \quad \text{for } N=100 \text{ times}$$

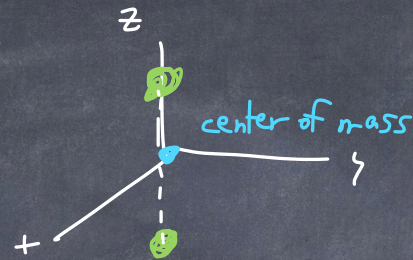
This works if W becomes internal energy that increases temperature.

Remember : a molecule has $\frac{1}{2}kT$ of kinetic energy per degree of freedom at a given temperature.
(or $\frac{1}{2}RT$ per mole)

Equipartition theorem:

when a substance is in equilibrium, there is an average energy of $\frac{1}{2}kT$ per molecule or $\frac{1}{2}RT$ per mole with each degree of freedom. The total is called the internal energy, U .

Consider a diatomic molecule in a gas ($N_2, O_2, N_2 \dots$)
(at constant volume)



z-direction has very little I_z , so we ignore it.

It can rotate around the x-axis or the y-axis so it has rotational kinetic energy.

$$K_{\text{rot}} = \frac{1}{2} I_x \omega_x^2 + \frac{1}{2} I_y \omega_y^2$$

The total kinetic energy is then:

for 1 molecule :

$$K = \underbrace{\frac{1}{2} m v_x^2}_{\frac{1}{2} kT} + \underbrace{\frac{1}{2} m v_y^2}_{\frac{1}{2} kT} + \underbrace{\frac{1}{2} m v_z^2}_{\frac{1}{2} kT} + \underbrace{\frac{1}{2} I_x \omega^2}_{\frac{1}{2} kT} + \underbrace{\frac{1}{2} I_y \omega^2}_{\frac{1}{2} kT}$$

For N molecules, with 5 degrees of freedom

$$U = \sum_i N kT = \frac{5}{2} nRT$$

Note! if we increase U , then we increase T

$$\Delta U = \frac{5}{2} nR \Delta T \Rightarrow \frac{\Delta U}{\Delta T} = \frac{5}{2} nR = C_v \text{ heat capacity of diatomic gas at constant volume}$$

Now, consider velocity of N_2 molecules at room temperature (like we did for Neon yesterday)

N_2 has 2 additional rotational degrees of freedom compared to Neon.

yesterday, Neon
only 3-d,
3 d.o.f.

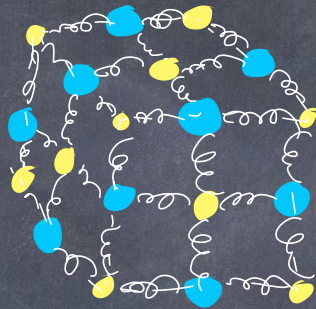
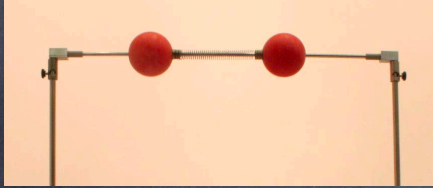
we found:

$$V_{rms} = \sqrt{\frac{3kT}{m}}$$

today, N_2
(including rotations,
 $3+2=5$ d.o.f.)

$$V_{rms} = \sqrt{\frac{5kT}{m}} \\ \approx 600 \frac{m}{s}$$

Likewise, for a solid, such as NaCl



Atoms are held together
in a solid, bound
like springs.

$$K = \underbrace{\frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2}_{\text{3-D translational}} + \underbrace{\frac{1}{2}K_s x^2 + \frac{1}{2}K_s y^2 + \frac{1}{2}K_s z^2}_{\text{springs in 3-D}}$$

For 6 degrees of freedom:

$$U = 6 \cdot \frac{1}{2}nRT = 3nRT$$

$$U = 6 \cdot \frac{1}{2}NkT = 3NkT$$

If we increase U , T increases

$$\Delta U = 3nR \Delta T \quad \frac{\Delta U}{\Delta T} = 3nR$$

So $\frac{\Delta U}{\Delta T} = 3nR = C_v$ for solids with 6 d.o.f.

$$\frac{\Delta U}{\Delta T} = 3R \text{ per mole}$$

$$\text{where: } R = 8.314 \text{ J/mol}\cdot\text{K}$$

$$= 24.9 \text{ J/mol}\cdot\text{K}$$

This is close to the value of C_m from our previous table

$$C_m \cong 3R \quad \text{and} \quad \frac{\Delta U}{\Delta T} = nC_m$$

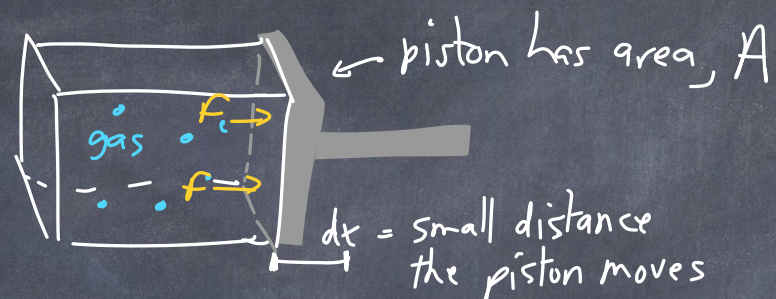
Aside note: Be careful with units on C

C can be given in several units:

$$\left[\frac{\text{J}}{\text{mol}\cdot\text{K}} \right], \left[\frac{\text{J}}{\text{kg}\cdot\text{K}} \right], \left[\frac{\text{kcal}}{\text{kg}\cdot^\circ\text{C}} \right]$$

Therefore, check units: $U: [\text{J}]$, $\Delta T: [\text{K}]$ or $[^\circ\text{C}]$

work done by a gas to move a piston



$$F = PA$$

Volume changes:

$$\Delta V = A \Delta x$$

$$dV = A dx$$

$$dW = F dx = PA dx = P dV$$

$$W = \int dW = \int P dV \Rightarrow W = \int P dV$$

$$W = \int_{V_1}^{V_2} P dV$$

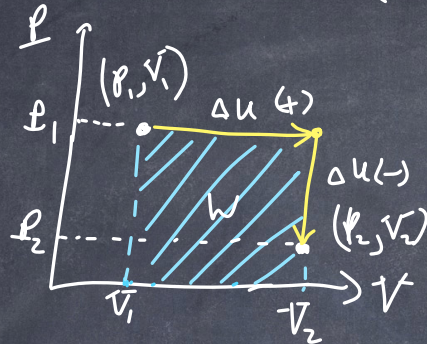
work done by
gas is the area
under the
 P vs. V curve

can be negative if $V_2 < V_1$

To go from (P_1, V_1) to (P_2, V_2)

work depends on how we do it.

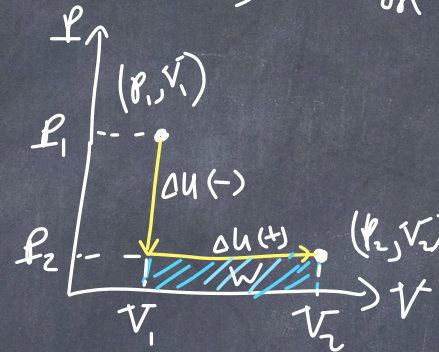
①



we heat gas allowing it to expand, then fix the volume and cool the gas.

$$\begin{aligned} W &= \text{area under } P \text{ vs. } V \\ &= P_1 (V_2 - V_1) \\ &= (+) \end{aligned}$$

②



cool the gas at constant volume, then heat the gas at constant pressure:

$$\begin{aligned} W &= P_2 (V_2 - V_1) \\ &= (+) \end{aligned}$$

work is $(+)$ because gas expands ($V_2 > V_1$)

Notice that each step can increase or decrease temperature (ΔT) and internal energy ΔU .

In a cycle, we return to the same P and V ,
and therefore to the same temperature.
So $\Delta u = 0$

cycle $a \rightarrow b \rightarrow a$:

work to complete a cycle

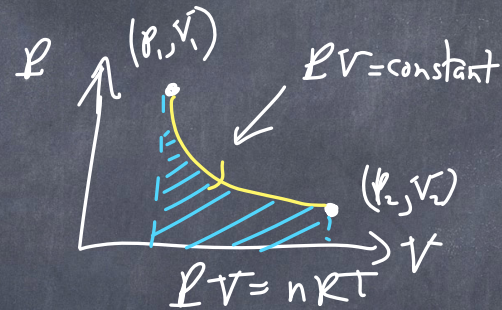


$$= P_1(V_2 - V_1) - P_2(V_2 - V_1)$$

This is negative
because it is opposite
our previous calculation.
Note: volume decreases, so $W < 0$

Another way to get From P_1, V_1 to P_2, V_2

using
the ideal
gas law
 $PV = nRT$



T stays
constant!

no ΔT , no $\Delta U = 0$

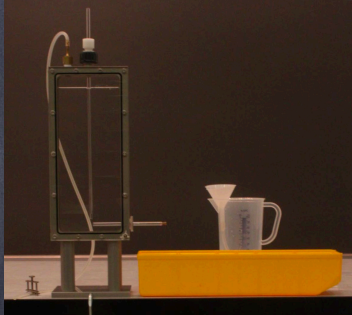
Heat the gas

$$W = \int_{V_1}^{V_2} P dV$$

$$P = \frac{nRT}{V} \Rightarrow W = nRT \int_{V_1}^{V_2} \frac{dV}{V}$$

$$W = nRT \ln\left(\frac{V_2}{V_1}\right)$$

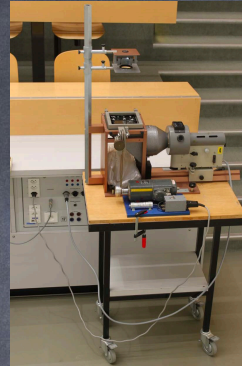
work to
expand an
ideal gas
with no temperature
change.



H21



Th57



Th36



Th58



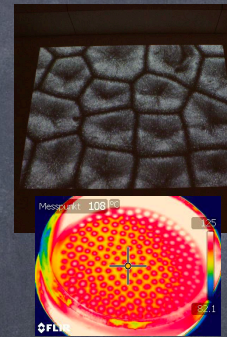
Th12



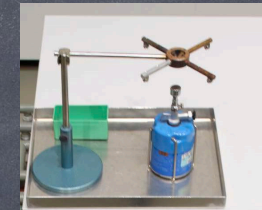
Th63



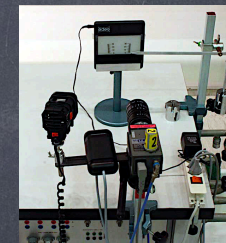
Th54



Th35



Th20



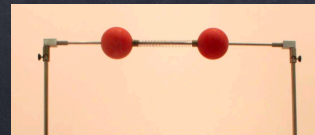
E12



Th19



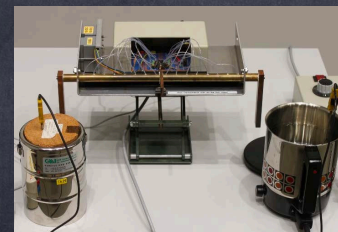
Th28



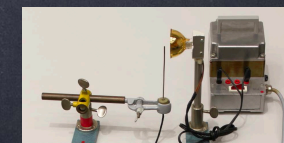
Th27



Th2



Th22



Th48