Applications of General Relativity

Spring 2021
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Solutions 7

Website: http://www.physik.uzh.ch/en/teaching/PHY519/

Exercise 1 [A spinning rod]

(a) The time dependent parts of the reduced quadrupole moment tensor are

$$Q_{xx} = \mu \cos^2(\omega t) \cdot 2 \int_0^{l/2} x^2 dx = \frac{\mu l^3}{12} \cos^2 \omega t = \frac{Ml^2}{24} \cos(2\omega t) + C_1 \tag{1}$$

$$Q_{yy} = -\frac{Ml^2}{24} \cos 2\omega t + C_2 \tag{2}$$

$$Q_{xy} = Q_{yx} = \frac{Ml^2}{24} \sin 2\omega t. \tag{3}$$

(b)

$$P_{GW} = L_{GW} = \frac{1}{5} < \dddot{Q}_{jk} \dddot{Q}_{jk} >= \frac{(2\omega)^6}{5} \left(\frac{Ml^2}{24}\right)^2 < 2\cos^2 2\omega t + 2\sin^2 2\omega t >= \frac{2}{45} \omega^6 M^2 l^4, \tag{4}$$

with $G = c = 1$.

(c) Balancing the electrostatic and centripetal forces on the charges,

$$e \nabla \phi = r m \omega^2, \tag{5}$$

where ϕ is the electrostatic potential, and e and m are the charge and mass of an electron. Maxwell's Gauss's law equation tells us that the negative gradient of the potential is the density, so

$$\rho \sim -\nabla^2 \phi \sim -\frac{m}{e} \omega^2. \tag{6}$$

So over the whole volume lA, the charge is $(m/e)\,\omega^2 lA$, where A is the rod's x-section. Thus the rod becomes an electric quadrupole with quadrupole moment $Q_E \sim (m/e)\,\omega^2 l^3 A$.

(d) The electric quadrupole radiation due to this induced moment is approximately $\omega^6 \cdot Q_E^2$, or $\omega^{10} (m/e)^2 l^6 A^2$.

(e) The ratio of electromagnetic damping to gravitational damping is then

$$\frac{L_{EM}}{L_{GW}} \sim \frac{45}{2} \frac{\omega^{10} (m/e)^2 l^6 A^2}{\omega^6 \rho^2 A^2 l^6} = \frac{45}{2} \left(\frac{(m/e)\,\omega^2}{\rho}\right)^2. \tag{7}$$

In $G = c = 1$ units, $(m/e) \approx 0.5 \cdot 10^{-21}$, $\rho\left(10g/cm^3\right) \approx 10^{-27}$, and $\omega\,(1kHz) \approx 1/3 \cdot 10^{-7}$, yielding

$$\frac{L_{EM}}{L_{GW}} \sim 10^{-17}. \tag{8}$$