uzh | eth | zürich

Applications of General Relativity

Sheet 6

Spring 2017
Prof. P. Jetzer

Y. Boetzel, M. Haney
Website: `http://www.physik.uzh.ch/en/teaching/PHY519/`

Issued: 31.03.2017
Due: 07.04.2017

Exercise 1 [Non-monochromatic gravitational waves]

In the lecture we considered monochromatic gravitational waves $h_{\mu\nu}(t, \boldsymbol{x}) = h_{\mu\nu}(\boldsymbol{x}) \exp[-i\omega t] +$ c.c.. A general gravitational wave can be a superposition of different frequencies. Start from the second order Ricci tensor, Eq. (5.54) in the script, to derive the general expression for the energy-momentum tensor of a gravitational wave imposing Lorentz gauge $\partial_\mu \gamma^{\mu\nu} = 0$ and tracelessness $\gamma = \eta^{\mu\nu}\gamma_{\mu\nu} = 0$

$$t_{\mu\nu}^{\text{grav}} = \frac{c^4}{32\pi G} \left\langle \partial_\mu \gamma^{\alpha\beta} \partial_\nu \gamma_{\alpha\beta} \right\rangle \tag{1}$$

Note that spacetime derivatives ∂_μ in the temporal average can be integrated by parts, neglecting the boundary terms. The three remaining gauge modes can be used to set $\gamma^{0i} = 0$, yielding the metric in the *transverse traceless gauge* $h_{\mu\nu}^{\text{TT}}$. Prove that $h_{00}^{\text{TT}} = 0$ and write down the time-time component of the above energy-momentum tensor in transverse traceless gauge.

Exercise 2 [Plugging in the numbers]

(i) In 2003, a double pulsar, a binary pulsar consisting of two neutron stars (each of mass roughly 1.4 $M_\odot$, with an orbital period of 1.4 h), was discovered. Estimate the timescale for coalescence via emission of gravitational radiation.

(ii) Suppose that two supermassive (10^9 $M_\odot$) black holes merge in a galaxy at a cosmological distance (1 Gpc). Consider a detector built to detect the gravitational waves from such events.

  (a) Estimate the frequency range that the detector would have to operate at.

  (b) Estimate the strain sensitivity (smallest dimensionless amplitude h) that would be necessary to see mergers at these distances.

  (c) Estimate the duration of these events.