

Website: <http://www.physik.uzh.ch/en/teaching/PHY519/>

Exercise 1 [Hamiltonian geodesic formulation in PN framework]

Geodesics are commonly expressed in terms of the Lagrangian,

$$L = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu, \quad (1)$$

where the dots are derivatives with respect to the affine parameter. An equivalent, yet generally more numerically stable formulation, exists in terms of the Hamiltonian

$$H = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu, \quad (2)$$

with the conjugate momenta

$$p_\mu = \frac{\partial L}{\partial \dot{x}^\mu}. \quad (3)$$

The solutions to Hamilton's equations

$$\dot{x}^\mu = \frac{\partial H}{\partial p_\mu}, \quad \dot{p}_\mu = -\frac{\partial H}{\partial x^\mu}, \quad (4)$$

are geodesics.

- a) Write down the Lagrangian and Hamiltonian of the Schwarzschild metric.
- b) Write down Hamilton's equations for a test particle trajectory in the Schwarzschild spacetime.
- c) Expand the equations of motion for both a massive and massless particle to first post-Newtonian order.
- d) Integrate these equations numerically. Produce a plot of a few trajectories. Compare the full solution to the expanded one.