PHY117 HS2026

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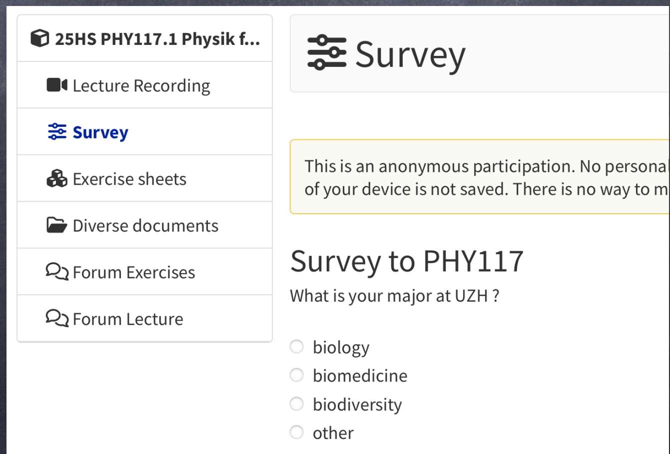
Sept. 16, 2026
Week 1, Lecture 2

Lecture notes here:
https://www.physik.uzh.ch/de/lehre/PHY117/HS2026.html

Make sure to complete survey:
https://lms.uzh.ch/
26HS PHY117.1
(takes just a few minutes)



Q: what are these: $\hat{x}, \hat{y}, \hat{z}$

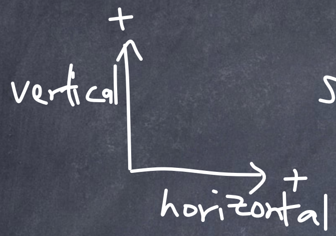
A: They are special vectors with length 1 called unit vectors

the "^" is called a "hat."

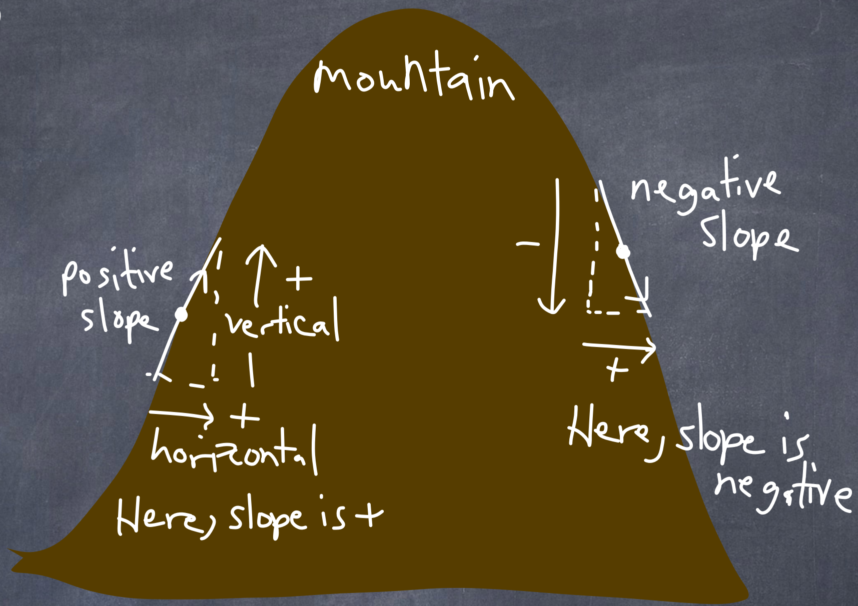
Unit vectors give us a coordinate system to define vectors.

Sometimes, people use $\hat{i}, \hat{j}, \hat{k}$ or $\hat{e}_1, \hat{e}_2, \hat{e}_3$.

Q: what is a slope?



$$\text{slope} = \frac{\text{vertical}}{\text{horizontal}}$$



Q: what is the difference between velocity & speed?

A: velocity is a vector
Speed is a scalar

$$\text{velocity} = \bar{v} \left(\begin{matrix}\text{magnitude} \\ + \\ \text{direction}\end{matrix}\right)$$

$$\text{Speed} = v = |\bar{v}| \left(\begin{matrix}\text{magnitude,} \\ \text{no direction}\end{matrix}\right)$$

Example:
$$\begin{cases} \bar{V} = 3\frac{m}{s}\hat{x} \\ V = 3\frac{m}{s} \end{cases}$$

Q: Reminder on derivatives: (useful for today)

start with function $f(x)$

derivative is $f'(x) = \frac{df(x)}{x}$

$$f(x) = \sin(x)$$

(Here, this is just a rule)

$$\frac{df}{dx} = \frac{df(x)}{dx} = f'(x) = \cos(x)$$

suppose $f(x) = \sin(kx)$. What is the derivative?

chain rule: if $y = f(u)$ but $u = g(x)$, then $\frac{dy}{dt} = \frac{dy}{\cancel{du}} \cdot \frac{\cancel{du}}{dx}$

Here, inside: $u = kx$ $\quad \frac{du}{dt} = k$

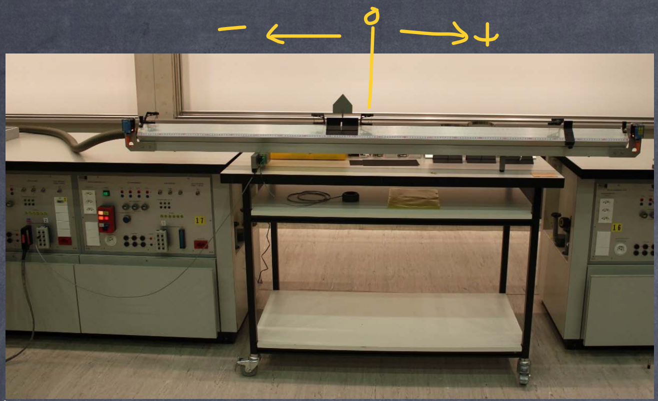
outside: $f(u) = \sin(u)$ $\quad \frac{df}{du} = \cos(u)$

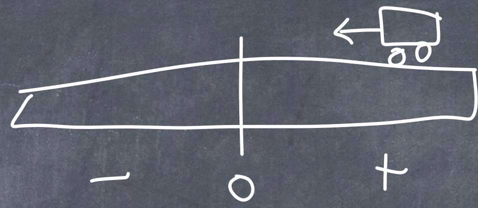
$$\frac{df}{dx} = \frac{df}{\cancel{du}} \cdot \frac{\cancel{du}}{dx}$$

$$= \cos(u) \cdot k$$

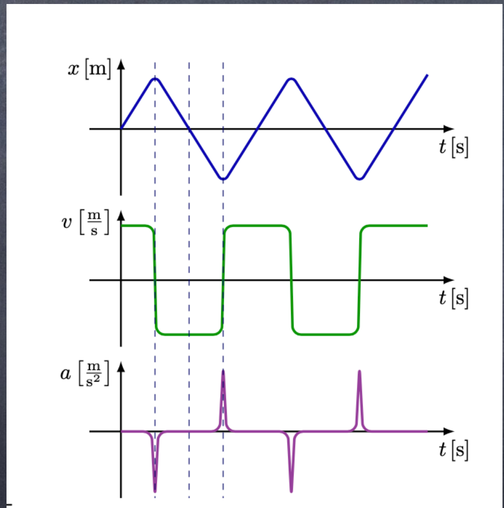
$$f(x) = \frac{df}{dx} = k\cos(kx)$$

Explanation of yesterday's experiment:





Better diagram in script



(f) Train bouncing back and forth with constant speed. The acceleration peaks when it quickly changed directions.

velocity is the slope of x vs. t

acceleration is the slope of v vs. t

velocity is +: moving in + direction

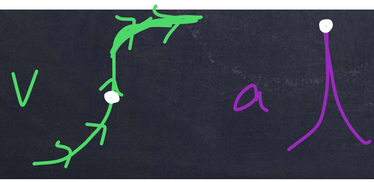
−: moving in − direction

$a = 0$: when velocity is constant

$a = (-)$: when velocity is decreasing

Here: acceleration is not constant. It has a peak structure. We will discuss this later when we talk about "impulse".

Notice spikes



Can we make our air-car accelerate more smoothly?

we can by making use of a "simple harmonic oscillator"

One example is a spring.

The position changes according to the formula

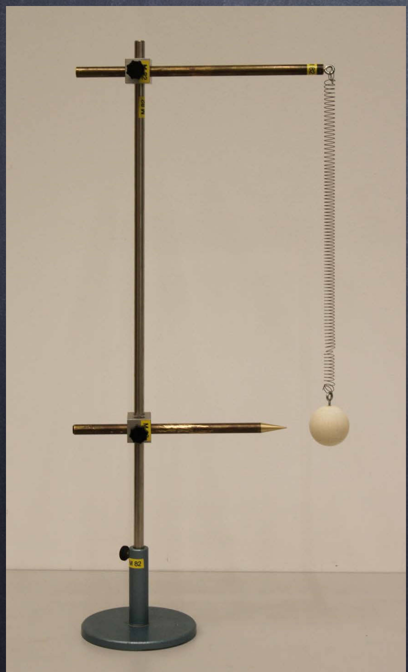
$$x(t) = x_0 \cos(\omega t)$$

x_0: starting position

ω is the "angular frequency" or "angular velocity"

$\omega = \frac{2\pi}{T}$, where

T is the "period", the time it takes to do one cycle.



Then: $v(t) = \frac{dx}{dt} = \underbrace{-x_0 \omega}_{v_0} \sin(\omega t)$ $\quad$ v_0 is max. speed

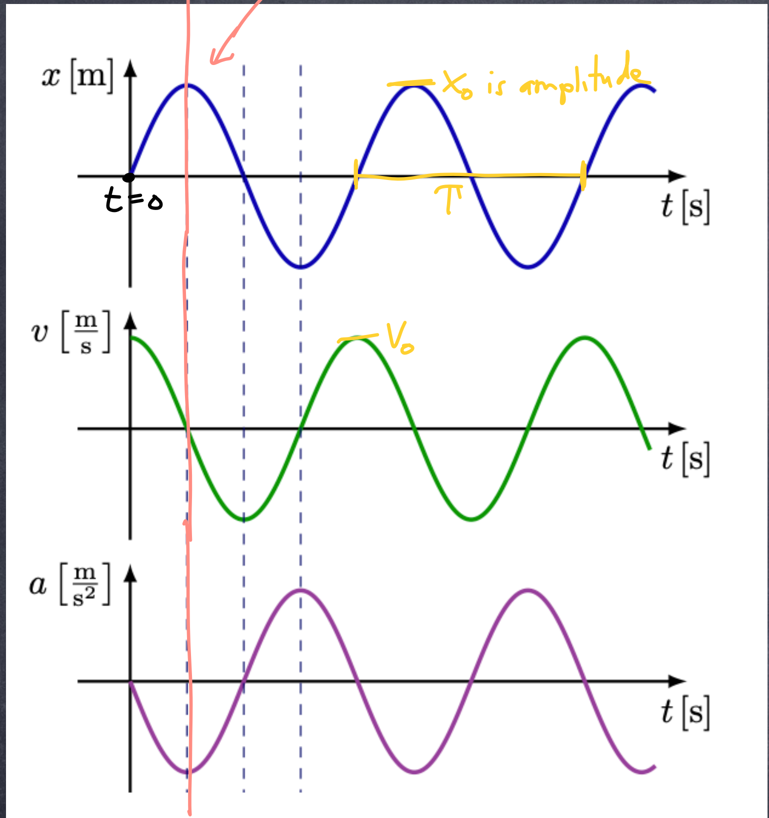
$$a(t) = \frac{dv}{dt} = -x_0 \omega^2 \cos(\omega t)$$

Notice $a(t) = x(t)(-\omega^2)$ $\quad$ This we will come back to.

Can we make our air-car accelerate more smoothly?

* $t=0$

$x(t)=x_0\cos(\omega t)$



(g) Periodic one dimensional motion of a mass on a spring moving back and forth. Velocity is largest when $x = 0 = a$.

$$x(t) = x_0 \sin(\omega t)$$

$$\omega = \frac{2\pi}{T}$$

where x_0 = initial displacement

$$V(t) = V_0 \cos(\omega t)$$

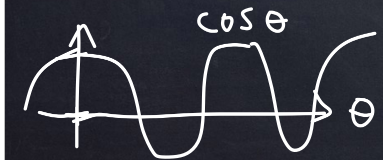
where $V_0 = \omega x_0$ is the maximum speed

$$a(t) = -a_0 \sin(\omega t)$$

where $a_0 = \omega^2 x_0$ is the maximum acceleration

Notice, we could start $t=0$ at a different time. Then we get a phase shift : $x(t) = x_0 \sin(\omega t + \phi)$.

ϕ: arbitrary angle



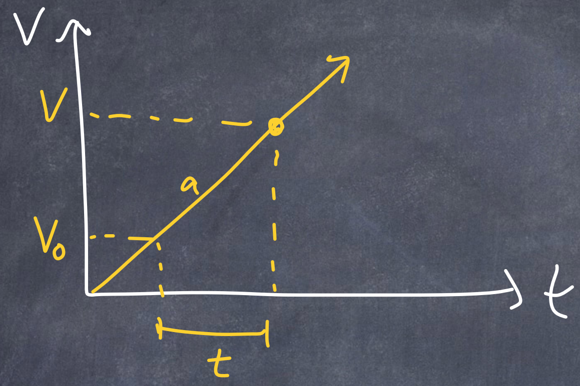
* Example:

start $t=0$ at $\frac{1}{4}$ of a period late: $\frac{1}{4}\cdot 2\pi = \frac{\pi}{2}$ or 90^0

so $\phi = \frac{\pi}{2}$:

$$x(t) = x_0 \sin\left(\omega t + \frac{\pi}{2}\right) = x_0 \cos(\omega t)$$

Motion with constant acceleration:



$a =$ slope of v vs. t

$= $ constant

$$a = \frac{dv}{dt} = \frac{\Delta v}{\Delta t}$$

Here: $\boxed{a = \frac{v - v_0}{t}}$ ①

we can determine V from a starting V_0 and the slope, a.

rewriting ① $\rightarrow$ $\boxed{V = V_0 + at}$ ②

final velocity

starting velocity

check units:

$$a \cdot t = \left[\frac{m}{s^{\cancel{2}}}\right]\left[\cancel{s}\right] = \frac{m}{s}$$

Average velocity is half-way between V_0 + V:

$$\boxed{V_{av} = \frac{1}{2}(V_0 + V)} \quad ③$$

Some other useful formulas for constant acceleration

from ③ and

$$\boxed{\Delta x = \frac{1}{2}(v_0 + v)t} \quad ④$$

$$\boxed{x = x_0 + v_0 t + \frac{1}{2} a t^2} \quad ⑤$$

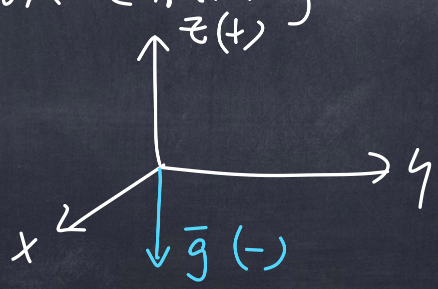
final position — starting position — starting velocity — acceleration (constant)

we will derive this later once we do integrals.

$$\boxed{v^2 = v_0^2 + 2a\Delta x} \quad ⑥$$

Most of the time, on earth, the acceleration is due to gravity.

$$\bar{a} = -g\hat{z}$$

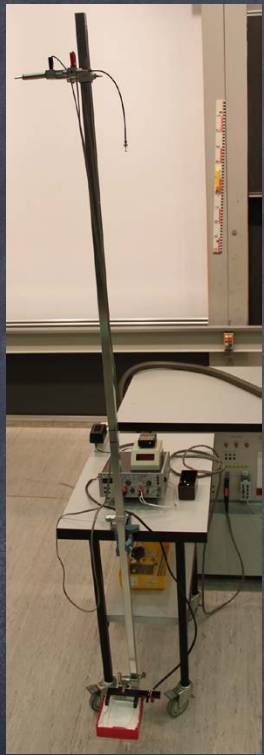


Does gravity accelerate everything the same?

yes (but sometimes other forces like air resistance)

what is g ? $\left[\frac{m}{s^2}\right]$

$+z$ $\uparrow$



z_0 : initial

$h = 2.00 \pm 0.01$ m

z : final

$$\bar{a} = -g\hat{z}$$

$$z = z_0 + v_{0z} t + \frac{1}{2} a t^2$$

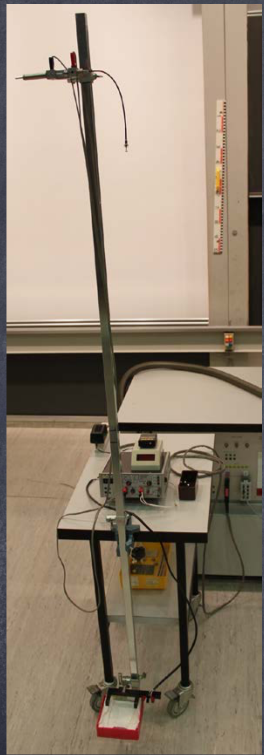
$$\underbrace{z - z_0}_{(-)} = \cancel{v_{0z}}^{0} t - \frac{1}{2} g t^2$$

$$\underbrace{z_0 - z}_{(+)} = \frac{1}{2} g t^2$$

$$h = \frac{1}{2} g t^2 \rightarrow g = \frac{2h}{t^2}$$

$+z$

$\uparrow$



measure the time:

$t = \boxed{}$

z_0

today we estimate uncertainty (or do calculation from lecture 1)

$h = 2.00 \pm 0.005\,\mathrm{m}$

$t = \boxed{} \pm \boxed{}$

Calculate g: $g = \dfrac{2h}{t^2} = \dfrac{2(2.00\,\mathrm{m})}{\left(\boxed{}\right)^2} =$

Uncertainty in g:

Example 2.4: For something more complicated like

$$f(x,y,a,b) = K\frac{xy^n}{ab^n}, \tag{2.16}$$

with a constant K, we find after some algebra

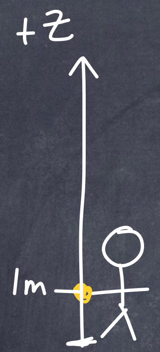
$$\sigma_f = |f|\sqrt{\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2 + \left(\frac{n\sigma_a}{a}\right)^2 + \left(\frac{n\sigma_b}{b}\right)^2}, \tag{2.17}$$

$$\sigma_g = |g|\sqrt{\left(\frac{\sigma_h}{h}\right)^2 + \left(\frac{2\cdot\sigma_t}{t}\right)^2}$$

$$\sigma_g = \boxed{}\sqrt{\left(\right)^2 + \left(\right)^2}$$

$$g = \boxed{} \pm \boxed{}\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

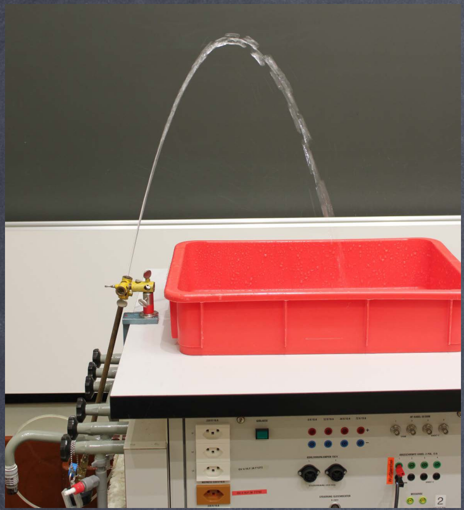
Actual value in Zurich: $g = 9.807\ \frac{\mathrm{m}}{\mathrm{s}^2}$



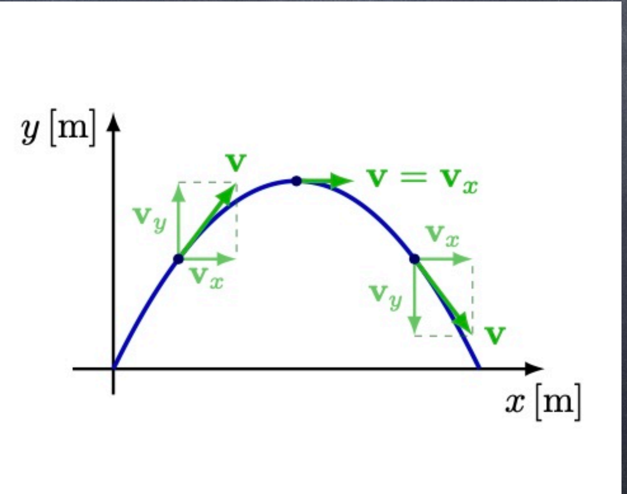
Assume we throw a ball up at 20 m/s, what is its position and velocity in 4 s?

Motion in 2D:

An object with an initial velocity $\bar{V} = V_{0x}\hat{x} + V_{0y}\hat{y}$, which feels gravity, will move in the shape of a parabola.

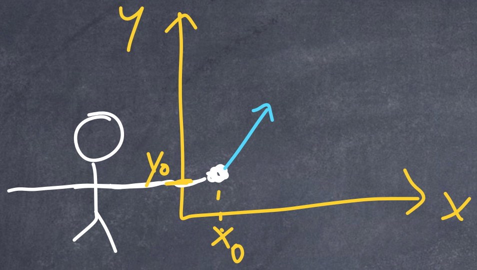


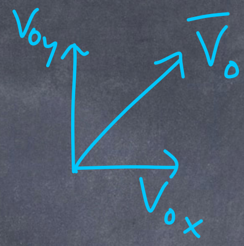
Its V_x does not change, but its V_y changes (like the ball).



(c) *y-x* diagram.

Let's calculate the formulas for parabolic motion:





Initial velocity:

$$\bar{V}_0 = V_{0x}\hat{x} + V_{0y}\hat{y}$$

Consider x and y separately using ② and ⑤:

x-direction: $\boxed{V_x = V_{0x}} + \cancelto{0}{a}t$

$\boxed{x = x_0 + V_{0x}t} + \frac{1}{2}\cancelto{0}{a}t^2$

(Neglect air resistance)

velocity in x-direction is constant

y-direction: $\boxed{V_y = V_{0y} + at} \Rightarrow V_y = V_{0y} - gt$

$$y = y_0 + V_{0y}t + \frac{1}{2}at^2 \Rightarrow$$

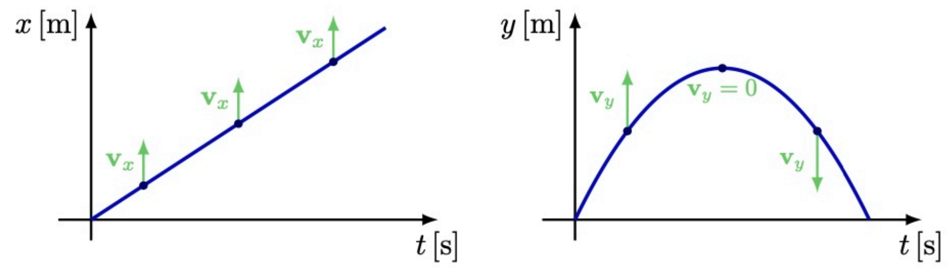
$$\boxed{y = y_0 + V_{0y}t - \frac{1}{2}gt^2}$$

$V_x = V_{0x}$

$x = x_0 + V_{0x} t$

$V_y = V_{0y} - gt$

$y = y_0 + V_{0y} t - \frac{1}{2} g t^2$

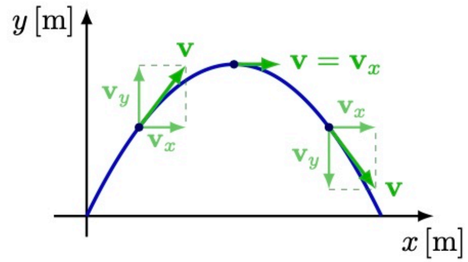


(a) x-t diagram.

(b) y-t diagram.

Diagram assumes $t_0 = y_0 = 0$

What we see (with water)

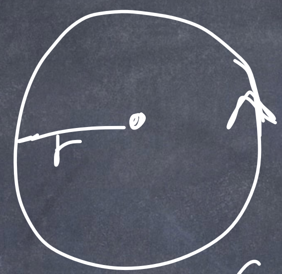


(c) y-x diagram.

y vs. x

Motion in a circle:

radius, r

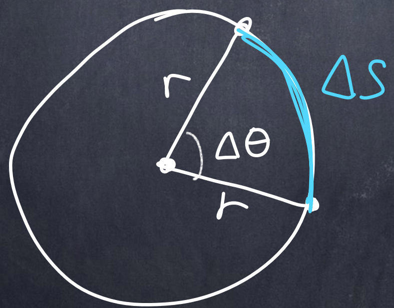


circle has $360^\circ = 2\pi$ radians

$90^\circ = \frac{\pi}{2}$ radians

Calculator choice : "deg" / "rad"

Circumference of a circle is $C = 2\pi r$



$\Delta S = r\,\Delta\theta$

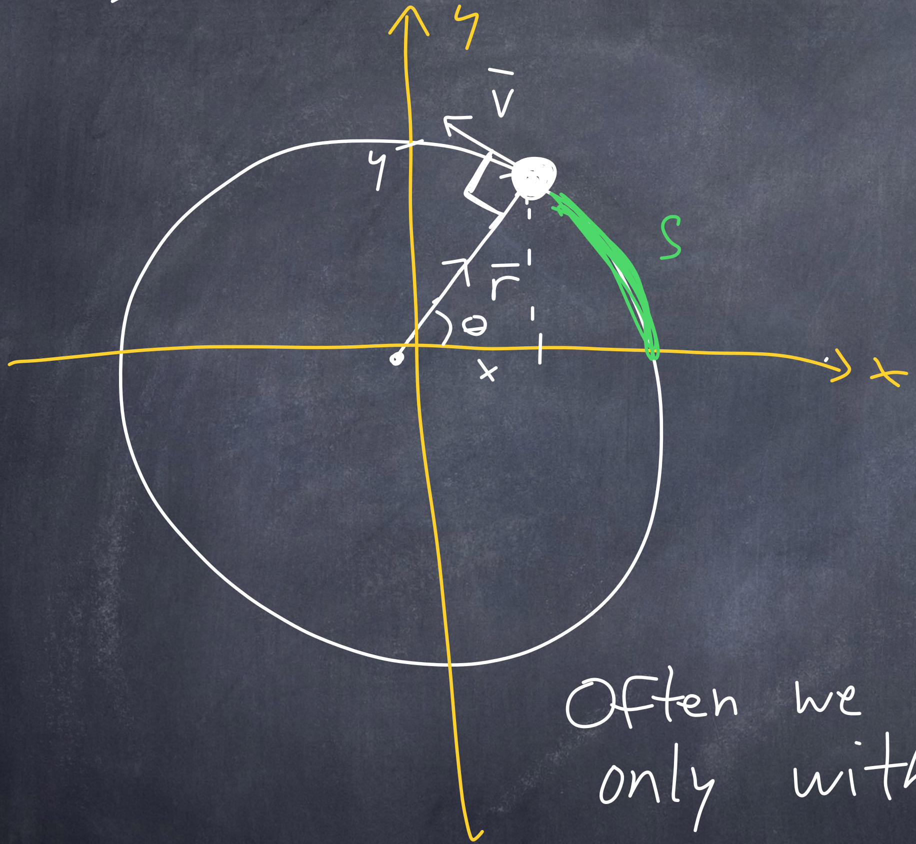
If $\Delta\theta = 2\pi$, then $\Delta S = 2\pi r$ (circumference)

Half a circle

$\Delta\theta = 180^\circ = \pi$ radians $=$

$\Delta S = r\pi$

Circular motion (in 2D):



position $\bar{r}$, velocity $\bar{v}$

$\bar{r} \perp \bar{v}$

Distance traveled $= s = r\theta$

$$\underset{\text{speed}}{v} = |\bar{v}| = \frac{\text{circumference}}{\text{time to do a circle}}$$

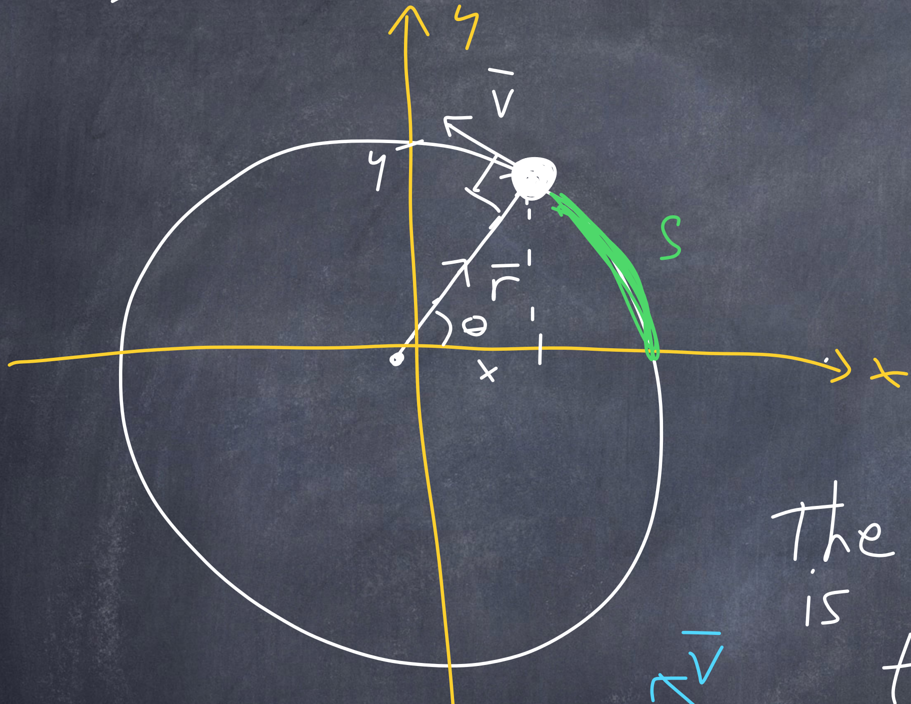
$$v = \frac{2\pi r}{T}$$ T: period of a cycle

Often we describe the position only with θ, since $|\bar{r}|$ is a constant

For the velocity, we often use $\frac{\Delta\theta}{\Delta t} = \omega$: angular velocity

$$\omega = \frac{v}{r} \quad \left[\frac{\cancel{m}/s}{\cancel{m}}\right] = \left[\frac{1}{s}\right] = \text{radians per second}$$

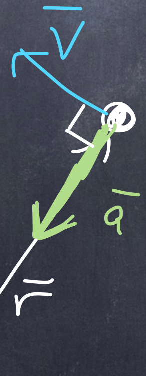
Circular motion (in 2D):



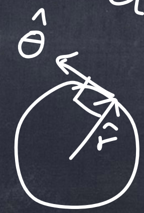
$$a = \frac{v^2}{r} = \text{centripetal acceleration}$$

(Derivation of $a = \frac{v^2}{r}$ on the next page if you are interested)

The direction of $\bar{a}$ is toward the center of the circle.



$$\bar{a} = -\frac{v^2}{r}\hat{r}$$

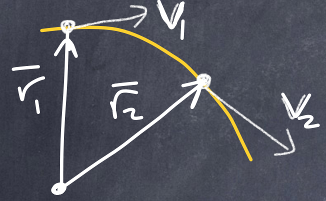


skip

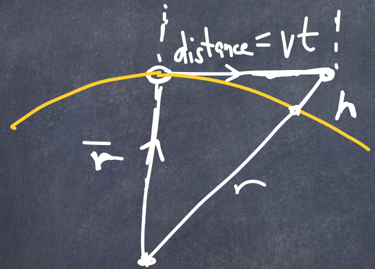
Motion in a circle. Ball on a string

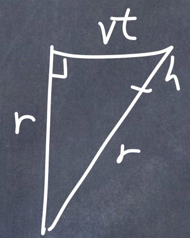
assumptions: The speed is the same vs. time

The velocity is changing (direction changes)



If the ball's velocity stayed the same, it would be at a height of $r+h$ after a time t.





Pythagorean theorem:

$$(vt)^2 + r^2 = (r+h)^2$$

$$(vt)^2 + \cancel{r^2} = \cancel{r^2} + 2rh + h^2$$

For small time t, h is small

h is very small compared to r $(h \ll r)$

so $h^2 \ll rh$

Therefore, $(vt)^2 \simeq 2rh + \cancel{h^2}^{\,0}$

so $h \approx \frac{1}{2}\left(\frac{v^2}{r}\right)t^2$

Compare this to $x = \frac{1}{2}at^2$, we see that $\boxed{a = \frac{v^2}{r}}$

This is the centripetal acceleration. Movement in circle.

A satellite is moving around the earth at a height of 100 km above the ground. How long does it take to go around the earth?

$C_{earth} = 40{,}000 \text{ km}$

$g \sim 9.5 \frac{m}{s^2}$ at 100 km

Experiments