



MMP I

Exercise Sheet 10

HS 21
Prof. Ph. Jetzer

L. Buonocore, M. Loechner, X. Liu, M. Ebersold
<https://www.physik.uzh.ch/en/teaching/PHY312>

Issued: 25.11.2021
Due: 02.12.2021

Exercise 1 [Geodesic on a cone (4 points)]

Find the geodesic (the curve representing the shortest path between two points in a surface) on the cone $z^2 = 8(x^2 + y^2)$. Introduce cylindrical coordinates (R, θ, z) and minimize the line element $ds^2 = dx^2 + dy^2 + dz^2$. You should find a relation between θ and R .

Hint: $\int \frac{kdr}{r\sqrt{r^2-k^2}} = \cos^{-1} \frac{k}{r}$.

Exercise 2 [Hermitian operators (4 points)]

Let A and B be two hermitian operators:

- Under which conditions is their product AB hermitian?
- For which values of the number α is the operator C hermitian?
 - $C = \alpha(AB + BA)$,
 - $C = \alpha(AB - BA)$,
 - $C = \alpha(AB + iBA)$.

Exercise 3 [Eigenvalues and eigenspaces (4 points)]

Consider the the linear operator \mathcal{P} :

$$\mathcal{P}f(x) = f(x) + k \int_0^c f(t)dt$$

in the space \mathcal{R} of the functions $f(x)$, complex and continuous in $[0, c]$, with the hermitian metric

$$(f, g) = \int_0^c f(x)\bar{g}(x)dx$$

- For which values of k is the operator \mathcal{P} hermitian? Which are its eigenvalues and the corresponding eigenspace?
- For which values of k is \mathcal{P} a projector? Which are the corresponding eigenvalues and eigenspaces?