Exercise 1. ***Hund's Rules***

Hund's rules are a set of three empirical rules that allow to determine the electronic configuration of the ground state of multi-electron atoms.

The notation used to identify the state is ${}^{(2S+1)}L_J$, where S, L are respectively the total spin and orbital angular momenta, and $J = L + S$. According to the spectroscopic notation the possible values of the L quantum number are identified by the letters $L = S, P, D, F, G, \ldots$ for $L = 0, 1, 2, 3, 4, \ldots$.

Use the Hund's rules to find the ground states of Nitrogen (N), Aluminium (Al) and Titanium (Ti) starting from the electronic configuration that you can find on any periodic table.

Exercise 2. ***Identical particles***

(a) The Hilbert space $\mathcal{H}_\mathcal{N}$ of a system of identical particles can be decomposed into a symmetric component $\mathcal{H}_\mathcal{N}^{(+)}$ and an antisymmetric component $\mathcal{H}_\mathcal{N}^{(-)}$. Show that for any physical observable A_N

$$\langle \phi_N^{(+)} | A_N | \phi_N^{(-)} \rangle = 0 \qquad \text{for} \quad |\phi_\mathrm{N}^{(\pm)}\rangle \in \mathcal{H}_\mathcal{N}^{(\pm)}$$

(b) Consider two identical non-interacting particles in a one-dimensional potential well with infinitely high walls, i. e.,

$$V(x) = \begin{cases} 0, & \text{if } |x| < a \\ \infty, & \text{otherwise} \end{cases}$$

Assume the two particles have the same maximal magnetic quantum number m_s.

(i) Write down the Hamiltonian of the two-particle system and show that the energy eigenstates factorize into a space and a spin component. Discuss the symmetry properties of the various components of the eigenstates if the two particles are bosons or fermions.

(ii) Calculate the possible eigenstates and eigenenergies for bosons or fermions.

(iii) Give the ground-state energy for two bosons and two fermions, respectively.