

Website: <http://www.physik.uzh.ch/en/teaching/PHY519/>

Exercise 1 [Non-monochromatic gravitational waves]

In the lecture we considered monochromatic gravitational waves $h_{\mu\nu}(t, \mathbf{x}) = h_{\mu\nu}(\mathbf{x}) \exp[-i\omega t] + \text{c.c.}$. A general gravitational wave can be a superposition of different frequencies. Start from the second order RICCI tensor, Eq. (5.54) in the script, to derive the general expression for the energy-momentum tensor of a gravitational wave imposing LORENTZ gauge $\partial_\mu \gamma^{\mu\nu} = 0$ and tracelessness $\gamma = \eta^{\mu\nu} \gamma_{\mu\nu} = 0$

$$t_{\mu\nu}^{\text{grav}} = \frac{c^4}{32\pi G} \left\langle \partial_\mu \gamma^{\alpha\beta} \partial_\nu \gamma_{\alpha\beta} \right\rangle \quad (1)$$

Note that spacetime derivatives ∂_μ in the temporal average can be integrated by parts, neglecting the boundary terms. The three remaining gauge modes can be used to set $\gamma^{0i} = 0$, yielding the metric in the *transverse traceless gauge* $h_{\mu\nu}^{\text{TT}}$. Prove that $h_{00}^{\text{TT}} = 0$ and write down the time-time component of the above energy-momentum tensor in transverse traceless gauge.

Exercise 2 [Plugging in the numbers]

- (i) In 2003, a double pulsar, a binary pulsar consisting of two neutron stars (each of mass roughly $1.4 M_\odot$, with an orbital period of 1.4 h), was discovered. Estimate the timescale for coalescence via emission of gravitational radiation.
- (ii) Suppose that two supermassive ($10^9 M_\odot$) black holes merge in a galaxy at a cosmological distance (1 Gpc). Consider a detector built to detect the gravitational waves from such events.
 - (a) Estimate the frequency range that the detector would have to operate at.
 - (b) Estimate the strain sensitivity (smallest dimensionless amplitude h) that would be necessary to see mergers at these distances.
 - (c) Estimate the duration of these events.