

Website: <http://www.physik.uzh.ch/en/teaching/PHY519/>

Exercise 1 [The cosmological constant in the black hole context]

a) Show that the Einstein Field Equations with cosmological constant

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}, \quad (1)$$

reduce to

$$R_{\mu\nu} + \Lambda g_{\mu\nu} = 0, \quad (2)$$

in vacuum.

- b) Assume spherical symmetry, and solve the equations to find the black hole solution.
- c) Given a photon passing the black hole, show if and how the formula for the deflection of light is modified. Do not assume a weak field.

Exercise 2 [Age and size of the universe]

Calculate age and size of a flat Λ CDM universe. In order to find the size, consider the comoving distance covered by a photon traveling on a radial geodesic from the big bang to the present time. The Planck 2013 best fit density parameters are $\Omega_{m,0} = 0.30$, $\Omega_{\Lambda,0} = 0.70$ and the Hubble constant is measured to be $H_0 = 68 \text{ km s}^{-1}\text{Mpc}^{-1}$ ($1 \text{ Mpc} = 3.085 \times 10^{19} \text{ km}$). For simplicity, the expansion is normalized such that present time corresponds to $a_0 = 1$.

Hint: Show that the first Friedmann equation can be written as

$$H(a) = H_0 \sqrt{\Omega_{m,0} a^{-3} + \Omega_{\Lambda,0}}. \quad (3)$$