Spontaneous SUSY breaking

If SUSY is spontaneously broken, it means that the ground state is not invariant under SUSY transformations

$$\Rightarrow \quad Q_\alpha |0\rangle \neq 0 \quad \& \quad \bar{Q}_{\dot\alpha} |0\rangle \neq 0$$

($|0\rangle$ ↑ ground state)

Since $H = P^0 = \frac{1}{4}\left(Q_1 \bar{Q}_1 + Q_2 \bar{Q}_2 + \bar{Q}_1 Q_1 + \bar{Q}_2 Q_2 \right)$

this means that $\boxed{\langle 0 | H | 0 \rangle = \frac{1}{4} \sum_i \left(|Q_i |0\rangle|^2 + |\bar{Q}_i |0\rangle|^2 \right) > 0}$

$\Rightarrow$ we must have an effective potential such that the ground state has positive energy

$\langle 0 | V_{eff} | 0 \rangle$
- → non-perturbative option (e.g. fermion condensate)
- → perturbative option
  - → $F \neq 0$ — most natural choice (Higgs-like possibility)
  - → $D \neq 0$

$$\left(\text{Recall} \quad V(\varphi_i, \bar{\varphi}_i) = \sum_i \left| \frac{\partial W}{\partial \varphi_i} \right|^2 = \sum_i F_i \bar{F}_i \right)$$

The O'Raifeartaigh model

$$W = -\lambda^2 \Phi_1 + m \Phi_1 \Phi_3 + \frac{y}{2} \Phi_1 \Phi_3^2$$

Φ_i = chiral superfields (not charged under gauge symm.)

N.B.: The linear term in Φ_1 is possible ONLY if Φ_1 has no charges $\Rightarrow$ not possible in the MSSM $\Rightarrow$ need for extra fields (general conclusion for SUSY breaking)

$$V(\varphi_i) = \sum_{i=1}^{3} |F_i|^2$$

$$F_i = \left.\frac{\partial V}{\partial \Phi_i}\right|_{\Phi_i = \varphi_i} \qquad F_1 = -\Lambda^2 + \frac{y}{2}\varphi_3^2$$

$$F_2 = m\varphi_3$$

$$F_3 = m\varphi_2 - y\varphi_1\varphi_3$$

It is clear that we cannot simultaneously have $F_1 = F_2 = F_3 = 0$
$\Rightarrow$ good prototype model for SUSY breaking.

$$V\Big|_{\substack{\varphi_2 = 0 \\ \varphi_1 = 0}} = \left|\Lambda^2 - \frac{y}{2}\varphi_3\right|^2 + m^2|\varphi_3|^2 \xrightarrow[\text{assume } m^2 > y\Lambda^2]{} V \geq \Lambda^2$$

(min. configuration obtained for $\varphi_2 = \varphi_3 = 0$ any φ_1)

$$\boxed{\sqrt{F}\Big|_{min} = \Lambda}$$

Expanding around $\varphi_1 = \varphi_2 = \varphi_3 = 0$ one gets the following (tree-level) spectrum:

Scalar masses

- 0 } masses associate to φ_1 (flat-direction)
- 0 }
- m^2
- m^2
- $m^2 - y\Lambda^2$
- $m^2 + y\Lambda^2$

Fermion masses

- 0 ← GOLDSTINO
- m
- m

Some comments :

(A) The different mass spectrum between fermions & bosons is a clear signal that SUSY has been broken

(B) The "flat direction" in ϕ_1 is an accident of the tree-level potential → removed by quantum corrections → non-vanishing masses for all scalars beyond the tree-level

(C) The massless fermion, on the other hand, is a general consequence of the spontaneous symmetry breaking of SUSY (analog of Goldstone boson)

↳ it can be removed only gauging SUSY (supergravity) ⇒ mass to the <u>GRAVITINO</u>

(D) Note that $\mathrm{Tr}\left[M^2_{\text{scalars}} \right] = 2\,\mathrm{Tr}\left[M^2_{\text{fermions}} \right]$

This property also remains valid in general

⇓

From (D) + The fact that ϕ_1 cannot be charged we deduce that SUSY breaking must occur in a different "sector" of the theory (new fields) and is "communicated" in some way to the "SM sector"

"Hidden sector"
(non-SM chiral fields)
↓
SUSY breaking

- - - Flavor-blind interactions - - -

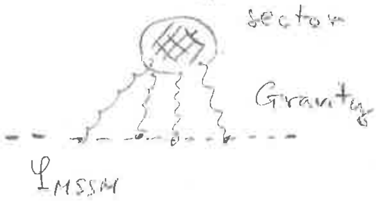
MSSM
"Visible sector"

Two classical way to "mediate" the breaking of SUSY from the hidden sector to the visible sector is via

⊙ gravitational interactions (gravity-mediated MSSM)

⊙ gauge interactions (gauge-mediated MSSM)

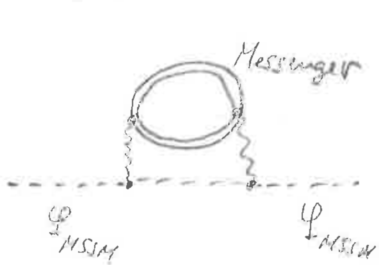
Gravity mediation is always present, by naive dim. analysis we should expect



$$m^2_{soft}\Big|_{Grav.} \sim \frac{(\sqrt{F})^4}{M^2_{Planck}}$$

$$\hookrightarrow \quad (\sqrt{F}) \gtrsim 10^{10} \text{ GeV} \quad \Rightarrow \quad m_{soft} \gtrsim \text{few } \mathcal{O}(100 \text{ GeV})$$

Gauge mediation is possible only if some of the particles in the hidden sector (called "messengers") have non-zero gauge interactions :



$$m^2_{soft} \sim \left(\frac{g_{SM}}{16\pi^2}\right)^2 \frac{(\sqrt{F})^4}{M^2_{mess}}$$

↗ mass of the messenger

$$m_{soft} \sim \frac{\alpha_{SM}}{4\pi}\frac{(\sqrt{F})^2}{M_{mess}} \longrightarrow \frac{\alpha_{SM}}{4\pi}\sqrt{F} \quad \Rightarrow \quad \sqrt{F} \gtrsim 10^6 \text{ GeV}$$

(if $M_{mess} \sim \sqrt{F}$)

$\hookrightarrow m_{soft} \gtrsim 1$ TeV

The key point is that in both cases we expect flavor-degenerate soft-breaking terms (up to corrections induced by the Yukawa couplings) → no problem with flavor.

The simplest scenario proposed in the literature is the so-called minimal/constrained gravity mediation framework, where one assumes (in an EFT-type inspired approach):

$$\mathcal{L}_{soft}^{mSUGRA} = -\frac{1}{M_{Pl}} F_S \sum_a \frac{1}{2} f_\lambda \lambda^a \lambda^a + h.c.$$

$$- \frac{1}{M_{Pl}^2} |F_S|^2 \sum_i f_\phi (\phi_i \phi_i^*)$$

↓

$$m_3 = m_2 = m_1 = f \frac{F_S}{M_{Pl}} \stackrel{def}{=} m_{1/2}$$

$$m_Q^2 = m_{u_c}^2 = m_{d_c}^2 = m_L^2 = m_{e_c}^2 = m_0^2 \cdot \mathbb{1} \qquad m_0^2 \stackrel{def}{=} f_\phi \frac{F_S^2}{M_{Pl}^2}$$

and similarly for the A terms
and the soft Higgs mass term (→ only 4 parameters!)

But many other options are possible ⇒ data will tell
(if SUSY will be discovered ...)